Final Project

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Due: Friday, May 9, 2014 at 11:59PM

Directions

For the the final project, please work alone. Your programs should be written in MatLab or C++.

Assume that the risk-free rate of interest is zero. Consider the Merton model for a stock $S = e^X$:

$$dX = \mu dt + \sigma dW_t + \int_{\mathbb{R}} z d\widetilde{N}_t(dz),$$
$$d\widetilde{N}_t(dz) = dN_t(dz) - \nu(dz)dt.$$
$$\nu(dz) = \lambda \frac{1}{\sqrt{2\pi s^2}} \exp\left(\frac{-(z-m)^2}{2s^2}\right) dz.$$

Find the price $u(t,x) := \mathbb{E}_x \left(e^{X_t} - K\right)^+$ with $K = S_0$ using the following methods:

(1) the explicit finite difference scheme,

(2) Fourier transforms/characteristic functions,

(3) Monte Carlo methods.

That is, you must compute the price using all three methods. You must determine the appropriate risk-neutral drift μ. For the finite difference scheme, you should choose an appropriate time-step and space discretization and boundary conditions. For the Fourier transform, you should choose appropriate limits and discretization for your numerical integral. For the Monte Carlo method, you may use the Euler discretization. You should choose an appropriate time step, number of path realizations and report the uncertainty (i.e. standard deviation) of your obtained price. You will be graded primarily on your ability to use all three methods to obtain the correct price – not on the above details (i.e., if you choose a smaller time-step than your classmate, but you both obtain the correct price, you would receive the same grade). For all three parts, use the following parameters:

$$t = 1.0, \qquad \sigma = 0.2, \qquad m = -0.3, \qquad s = 0.3, \qquad \lambda = 1.0 \qquad X_0 = 10 \qquad K = e^{X_0}.$$

Have a great summer!