Elementary Differential Equations (MATH200) 9th Week Homework

Problem 1(6.5.2a). Find the solution of the given initial value problem.

$$y'' - y = -5\delta(t - 3); \quad y(0) = 1, \ y'(0) = 0$$

Problem 2(6.6.4). Find the Laplace transform of the given function.

$$f(t) = \int_0^t (t - \tau)^2 \cos(3\tau) d\tau$$

Problem 3(6.6.5). Find the Laplace transform of the given function.

$$f(t) = \int_0^t e^{-(t-\tau)} \sin(2\tau) d\tau$$

Problem 4(6.6.6). Find the Laplace transform of the given function.

$$f(t) = \int_0^t \sin(t - \tau) \cos(2\tau) d\tau$$

Problem 5(6.6.8). Find the inverse Laplace transform of the given function by using the convolution theorem.

$$F(s) = \frac{1}{s^4(s^2 + 4)}$$

Problem 6(6.6.9). Find the inverse Laplace transform of the given function by using the convolution theorem.

$$F(s) = \frac{s}{(s+1)(s^2+9)}$$

Problem 7(6.6.10). Find the inverse Laplace transform of the given function by using the convolution theorem.

$$F(s) = \frac{1}{(s+1)^3(s^2+4)}$$

Problem 8(6.6.14). Express the solution of the given initial value problem in terms of a convolution integral.

$$4y'' + 4y' + 17y = g(t); \quad y(0) = 0, \quad y'(0) = 1$$

Problem 9(5.2.1). In the problem,

- **a.** Seek power series solutions of the given differential equation about the given point x_0 ; find the recurrence relation that the coefficients must satisfy.
- **b.** Find the first four nonzero terms in each of two solutions y_1 and y_2 (unless the series terminates sooner).

$$y'' - xy' - y = 0, \quad x_0 = 1$$

Problem 9(5.2.2). In the problem,

- **a.** Seek power series solutions of the given differential equation about the given point x_0 ; find the recurrence relation that the coefficients must satisfy.
- **b.** Find the first four nonzero terms in each of two solutions y_1 and y_2 (unless the series terminates sooner).
- d. If possible, find the general term in each solution.

$$y'' - xy' - y = 0, \quad x_0 = 0$$