

Elementary Differential Equations (MATH200)

7th Week Homework

Problem 1(6.2.2). Find the inverse Laplace transform of the given function.

$$F(s) = \frac{5}{(s-1)^3}$$

Problem 2(6.1.6). Find the inverse Laplace transform of the given function.

$$F(s) = \frac{8s^2 - 6s + 12}{s(s^2 + 4)}$$

Problem 3(6.1.7). Find the inverse Laplace transform of the given function.

$$F(s) = \frac{3 - 2s}{s^2 + 4s + 5}$$

Problem 4(6.2.14). Use the Laplace transform to solve the given initial value problem.

$$y^{(4)} - 4y''' + 6y'' - 4y' + y = 0; \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1$$

Problem 5(6.2.15). Use the Laplace transform to solve the given initial value problem.

$$y'' - 2y' + 2y = \cos t; \quad y(0) = 1, \quad y'(0) = 1$$

Problem 6(6.3.7). In the problem,

- a. Sketch the graph of the given function.
- b. Express $f(t)$ in terms of the unit step function $u_c(t)$.

$$f(t) = \begin{cases} t, & 0 \leq t < 1 \\ t - 1, & 1 \leq t < 2 \\ t - 2, & 2 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

Problem 7(6.3.9). Find the Laplace transform of the given function.

$$f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^3, & t \geq 2 \end{cases}$$

Problem 8(6.3.15). Find the inverse Laplace transform of the given function.

$$F(s) = \frac{(s-2)e^{-2s}}{s^2 - 4s + 3}$$

Problem 9(6.3.21). Find the Laplace transform of the given function.

$$f(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 0, & 1 \leq t < 2 \\ 1, & 2 \leq t < 3 \\ 0, & t \geq 3 \end{cases}$$

Problem 10(6.4.2). In the problem,

- a. Find the solution of the forcing function on an appropriate interval.
- b. Find the solution of the given initial value problem.

$$y'' + y = g(t); \quad y(0) = 0, \quad y'(0) = 2;$$

$$g(t) = \begin{cases} t/2, & 0 \leq t < 6 \\ 3, & t \geq 6 \end{cases}$$

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