Elementary Differential Equations (MATH200)
5^{th} Week Homework

Problem 1(7.5-5b). The coefficient matrix has a zero eigenvalue. As a result, the pattern of trajectories is different from those in the examples in the text. Find the general solution of the given system of equations.

$$\mathbf{x}' = \begin{pmatrix} 3 & -1 \\ 6 & -2 \end{pmatrix} \mathbf{x}$$

Problem 2(7.5-11). Solve the given initial value problem. Describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x},\ \mathbf{x}(0) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Problem 3(7.5-12). Solve the given initial value problem. Describe the behavior of the solution as $t \to \infty$.

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix} \mathbf{x}, \ \mathbf{x}(0) = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

Problem 4(7.6-4). For the given system

$$\mathbf{x}' = \begin{pmatrix} 3 & 4 \\ -2 & -1 \end{pmatrix} \mathbf{x}$$

a. Draw a direction field and sketch a few trajectories.

b. Express the general solution of the given system of equations in terms of real-valued functions.

c. Describe the behavior of the solutions as $t \to \infty$

Problem 5(7.7-2). For the given system,

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 \\ -5 & -2 \end{pmatrix} \mathbf{x}$$

a. Find a fundamental matrix for the given system of equations.

b. Find the fundamental matrix $\mathbf{\Phi}(t)$ satisfying $\mathbf{\Phi}(0) = \mathbf{I}$.

Problem 6(7.8-2). Find the general solution of the given system of equations.

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \mathbf{x}$$

Problem 7(7.8-4c). For the given system of equations, find the general solution.

$$\mathbf{x}' = \begin{pmatrix} 3 & 1 \\ -4 & -1 \end{pmatrix} \mathbf{x}$$

Problem 8(7.8-6a). Find the solution for the given initial value problem.

$$\mathbf{x}' = \begin{pmatrix} 3 & -1 \\ 9 & -3 \end{pmatrix} \mathbf{x}, \ \mathbf{x}(0) = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

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