Elementary Differential Equations (MATH200)
3^{rd} Week Homework

Problem 1(3.1-3). Find the general solution of the given differential equation.

$$12y'' - y' - y = 0$$

Problem 2(3.1-4). Find the general solution of the given differential equation.

$$y'' + 6y' = 0$$

Problem 3(3.1-9). Find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

$$y'' + 3y' = 0,\ y(0) = 0,\ y'(0) = 3$$

Problem 4(3.2-7). Determine the longest interval in which the given initial value problem is certain to have a unique twice-differentiable solution. Do not attempt to find the solution.

$$t(t-4)y'' + 3ty' + 5y = 2,\ y(3) = 0,\ y'(3) = -1$$

Problem 5(3.2-18). Consider the equation $y'' - y' - 2y = 0$.

a. Show that $y_1(t) = e^{-t}$ and $y_2(t) = e^{2t}$ form a fundamental set of solutions.

b. Let $y_3(t) = -2e^{2t}$, $y_4(t) = y_1(t) + y_2(t)$, and $y_5(t) = 2y_1(t) - 2y_3(t)$. Are $y_3(t)$, $y_4(t)$, and $y_5(t)$ also solutions of the given differential equation?

c. Determine whether each of the following pairs form a fundamental sef of solutions: $\{y_1(t), y_3(t)\}$; $\{y_2(t), y_3(t)\}$.

Problem 6(3.2-20). Find the fundamental set of equations specified by Theorem 3.2.5 for the given differential equation and initial point.

$$y'' + 5y' + 4y = 0,\ t_0 = 1$$

Problem 7(3.3-6). Find the general solution of the given differential equation.

$$y'' - 2y' + 8y = 0$$

Problem 8(3.3-13). Find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior for increasing t.

$$y'' - 2y' + 5y = 0,\ y(\pi/2) = 0,\ y'(\pi/2) = 4$$

Problem 9(3.4-4). Find the general solution of the given differential equation.

$$4y'' - 12y' + 9y = 0$$

Problem 10(3.4-19). Use method of reduction of order to find a second solution of the given differential equation.

$$t^2y'' + ty' - 4y = 0,\ t > 0;\ y_1(t) = t^2$$

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