Elementary Differential Equations (MATH200)
1^{st} Week Homework

Problem 1(2.1-3c). Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \to \infty$.

$$y' + y = te^{-t} + 2$$

Problem 2(2.1-12). Find the solution of the given initial value problem.

$$ty' + 2y = 2\sin t,\ y(\pi/2) = 1,\ t > 0$$

Problem 3(2.2-5). Solve the given differential equation.

$$\frac{dy}{dx} = \frac{3x - e^{-x}}{2y + e^{y}}$$

Problem 4(2.2-9). For given differential equation and initial condition

$$y' = (1 - 2x)y^2, \; y(0) = -1/12$$

a. Find the solution of the given initial value problem in explicit form.
b. Plot the graph of the solution.
c. Determine (at least approximately) the interval in which the solution is defined.

Problem 5(2.3-1). Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains $150L$ of a dye solution with a concentration of $1g/L$. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of $2L/min$, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

Problem 6(2.3-10). Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 90°C when freshly poured, and 1 min later has cooled to 85°C in a room at 20°C, determine when the coffee reaches a temperature of 65°C. □