Elementary Differential Equations (MATH200)
4^{th} Week Homework

Problem 1(3.5-4). Find the general solution of the given differential equation.

$$y'' - 2y' - 3y = -6te^{-t}$$

Problem 2(3.5-5). Find the general solution of the given differential equation.

$$y'' + 2y' = 5 + 4\sin(2t)$$

Problem 3(3.5-6). Find the general solution of the given differential equation.

$$y'' + 2y' + y = 4e^{-t}$$

Problem 4(3.5-16a). Consider the differential equation

$$y'' + 3y' = 2t^4 + t^2e^{-3t} + 2\sin(3t).$$

Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used.

Problem 5(3.5-18a). Consider the differential equation

$$y'' + 2y' + 2y = 2e^{-t} + 2e^{-t}\cos t + 4e^{-t}t^2 \sin t.$$

Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used.

Problem 6(3.5-21a). Consider the differential equation

$$y'' - 4y' + 4y = 4t^2 + 4te^{2t} + t\sin(2t).$$

Determine a suitable form for $Y(t)$ if the method of undetermined coefficients is to be used.

Problem 7(3.6-2). Use the method of variation of parameters to find a particular solution of the given differential equation. Then check your answer by the method of undermined coefficients.

$$y'' - y' - 2y = 4e^{-t}$$

Problem 8(3.6-11). Verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation.

$$t^2y'' - t(t+2)y' + (t+2)y = 6t^3,\ t > 0,;\ y_1(t) = t,\ y_2(t) = te^t$$

Problem 9(7.1-3). Transform the given equation into a system of first-order equations.

$$u^{(4)} - 3u = 0$$

Problem 10(7.1-4). Transform the given initial value problem into an initial value problem for two first-order equations.

$$u'' + 2u' + 4u = 2\cos(3t),\ \ u(0) = 1,\ \ u'(0) = -2$$

Problem 9(7.4-2). For the given homogeneous system of first-order linear differential equations and two vector-valued functions($\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$)

$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \mathbf{x}; \ \mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t}, \ \mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{-t}$$

a. Show that the given functions are solutions of the given system of differential equations.

b. Show that $\mathbf{x} = c_1\mathbf{x}^{(1)} + c_2\mathbf{x}^{(2)}$ is also a solution of the given system for any values of c_1 and c_2.

c. Show that the given functions form a fundamental set of solutions of the given system.

d. Find the solution of the given system that satisfies the initial condition $\mathbf{x}(0) = (1, 2)^T$.

e. Find $W\left[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}\right](t)$.

f. Show that the Wronskian, $W = W\left[\mathbf{x}^{(1)}, \mathbf{x}^{(2)}\right]$, found in e is a solution of Abel's equation: $W' = (p_{11}(t) + p_{22}(t))W$.

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