Elementary Differential Equations (MATH200)
10^{th} Week Homework

Problem 1(5.3.4). Determine a lower bound for the radius of convergence of series solution about each given point x_0 for the given differential equation.

$$y'' + 4y' + 6xy = 0; \quad x_0 = 0, \; x_0 = 3$$

Problem 2(5.3.5). Determine a lower bound for the radius of convergence of series solution about each given point x_0 for the given differential equation.

$$(x^2 - 2x - 3)y'' + xy' + 4y = 0; \quad x_0 = 5, \; x_0 = -5, \; x_0 = 0$$

Problem 3(5.3.6). Determine a lower bound for the radius of convergence of series solution about each given point x_0 for the given differential equation.

$$xy'' + y = 0; \quad x_0 = 2$$

Problem 4(5.3.8). Find the first four nonzero terms in each of two power series solutions about the origin. Show that they form a fundamental set of solutions. What do you expect the radius of convergence to be for each solution?

$$e^x y'' + xy = 0$$

Problem 5(5.4.1). Determine the general solution of the given differential equation that is valid in any interval not including the singular point.

$$x^2y'' + 5xy' + 3y = 0$$

Problem 6(5.4.3). Determine the general solution of the given differential equation that is valid in any interval not including the singular point.

$$x^2y'' - 5xy' + 10y = 0$$

Problem 7(5.4.4). Determine the general solution of the given differential equation that is valid in any interval not including the singular point.

$$x^2y'' + 3xy' + 10y = 0$$

Problem 8(5.4.8). Determine the general solution of the given differential equation that is valid in any interval not including the singular point.

$$(x-1)^2y'' + 8(x-1)y' + 12y = 0$$

Problem 9(5.4.11). Find the solution of the given initial value problem. Plot the graph of the solution and describe bow the solution behaves as $x \to 0$.

$$x^2y'' - 3xy' + 4y = 0, \quad y(-1) = 2, \; y'(-1) = 2$$

Problem 10(5.5.3). In the problem,

a. Show that the given differential equation has a regular singular point at $x = 0$.

b. Determine the indicial equation, the recurrence relation, and the roots of the indicial equation.

c. Find the series solution ($x > 0$) corresponding to the larger root.

d. If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.

$$xy'' + 2y = 0$$

Problem 11(5.5.4). In the problem,

a. Show that the given differential equation has a regular singular point at $x = 0$.

b. Determine the indicial equation, the recurrence relation, and the roots of the indicial equation.

c. Find the series solution ($x > 0$) corresponding to the larger root.

d. If the roots are unequal and do not differ by an integer, find the series solution corresponding to the smaller root also.

$$2xy'' + 2y' - y = 0$$

Problem 12(5.6.2). In the problem,

a. Find all the regular singular points of the given differential equation.

b. Determine the indicial equation and the exponents at the singularity for each regular singular point.

$$x^2y'' - x(2 + x)y' + (2 + x^2)y = 0$$

Problem 13(5.6.4). In the problem,

a. Find all the regular singular points of the given differential equation.

b. Determine the indicial equation and the exponents at the singularity for each regular singular point.

$$3x(x+2)y'' + y' - xy = 0$$

Problem 14(5.6.6). In the problem,

a. Find all the regular singular points of the given differential equation.

b. Determine the indicial equation and the exponents at the singularity for each regular singular point.

$$x^2(1-x)y'' - (1+x)y' + 3xy = 0$$

□