Question 01

Suppose that $u = 1$ and $v = 3$. Evaluate the following expressions using MATLAB:

(a) $\dfrac{4u}{3v}$

(b) $\dfrac{2v^{-2}}{(u+v)^2}$

(c) $\dfrac{v^3}{v^3 - u^3}$

(d) $\dfrac{4}{3}\pi v^2$

Question 02

Define the variables x and y as $x = 6.5$ and $y = 3.8$; then evaluate:

(a) $\left(x^2 + y^2\right)^{2/3} + \dfrac{xy}{y-x}$ (b) $\dfrac{\sqrt{x+y}}{(x-y)^2} + 2x^2 - xy^2$

Question 03

Define the variables a, b, c, and d as:

$c = 4.6$, $d = 1.7$, $a = cd^2$, and $b = \dfrac{c+a}{c-d}$; then evaluate:

(a) $e^{d-b} + \sqrt[3]{c+a} - (ca)^d$ (b) $\dfrac{d}{c} + \left(\dfrac{ct}{b}\right)^2 - c^d - \dfrac{a}{b}$

Question 04

Two trigonometric identities are given by:

(a) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$ (b) $\dfrac{\tan x}{\sin x - 2\tan x} = \dfrac{1}{\cos x - 2}$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = \pi / 10$.

Question 05

Two trigonometric identities are given by:

(*a*) $(\sin x + \cos x)^2 = 1 + 2\sin x\cos x$ (*b*) $\dfrac{1 - 2\cos x - 3\cos^2 x}{\sin^2 x} = \dfrac{1 - 3\cos x}{1 - \cos x}$

For each part, verify that the identity is correct by calculating the values of the left and right sides of the equation, substituting $x = 20°$.

Question 06

A rectangular box has the dimensions shown.

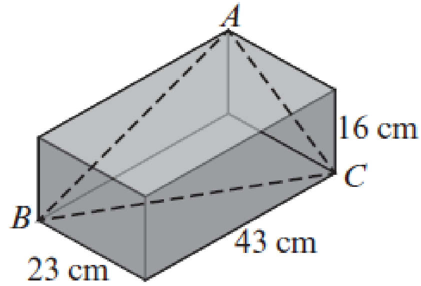
(*a*) Determine the angle *BAC* to the nearest degree.

(*b*) Determine the area of the triangle *ABC* to the nearest tenth of a centimeter.

Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Heron's formula for triangular area:

$A = \sqrt{p(p-a)(p-b)(p-c)}$, where $p = (a+b+c)/2$.



Question 07

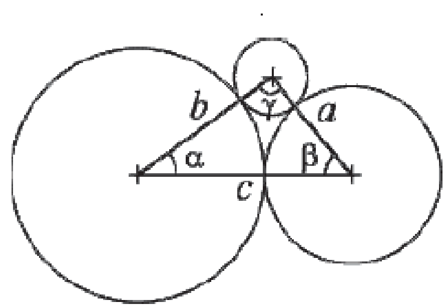
The three shown circles, with radius 15 in., 10.5 in., and 4.5 in., are tangent to each other.

(*a*) Calculate the angle γ (in degrees) by using the law of cosines.

(Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)

(*b*) Calculate the angles γ and α (in degrees) using the law of sines.

(*c*) Check that the sum of the angles is 180°.

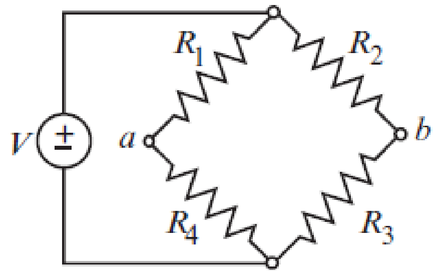


Question 08

A total of 4217 eggs have to be packed in boxes that can hold 36 eggs each. By typing one line (command) in the Command Window, calculate how many eggs will remain unpacked if every box that is used has to be full.

Question 09

The voltage difference V_{ab} between points a and b in the Wheatstone bridge circuit is given by:

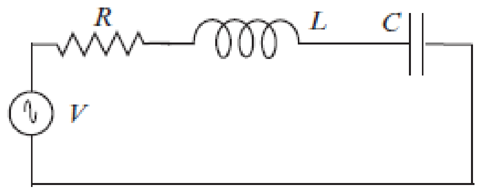


$$V_{ab} = V\left(\frac{c-d}{(c+1)(d+1)}\right)$$

where $c = R_2 / R_1$ and $d = R_3 / R_4$. Calculate the V_{ab} if $V = 15$ V, $R_1 = 119.8\ \Omega$, $R_2 = 120.5\ \Omega$, $R_3 = 121.2\ \Omega$, and $R_4 = 119.3\ \Omega$.

Question 10

The current in a series RCL circuit is given by:



$$I = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

where $\omega = 2\pi f$. Calculate I for the circuit shown if the supply voltage is 80 V, $f = 50$ Hz, $R = 6\ \Omega$, $L = 400 \times 10^{-3}$ H, and $C = 40 \times 10^{-6}$ F.

Question 11

Radioactive decay of carbon-14 is used for estimating the age of organic material. The decay is modeled with the exponential function $f(t) = f(0)e^{kt}$, where t is time, $f(0)$ is the amount of material at $t = 0$, $f(t)$ is the amount of material at time t, and k is a constant. Carbon-14 has a half-life of approximately 5,730 years. A sample taken from the ancient footprints of Acahualinca in Nicaragua shows that 77.45% of the initial ($t = 0$) carbon-14 is present. Determine the estimated age of the footprint. Solve the problem by writing a program in a script file. The program first determines the constant k, then calculates t for $f(t) = 0.7745 f(0)$, and finally rounds the answer to the nearest year.

Question 12

According to the Doppler effect of light, the perceived wavelength λ_p of a light source with a wavelength of λ_s is given by:

$$\lambda_P = \lambda_S \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

where c is the speed of light (about 300×10^6 m/s) and v is the speed the observer moves toward the light source. Calculate the speed the observer has to move in order to see a red light as green. Green wavelength is 530 nm and red wavelength is 630 nm.

Question 13

Newton's law of cooling gives the temperature $T(t)$ of an object at time t in terms of T_0, its temperature at $t = 0$, and T_s, the temperature of the surroundings.

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

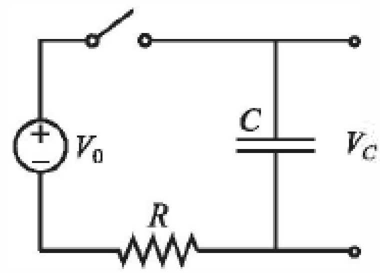
A police officer arrives at a crime scene in a hotel room at 9:18 PM, where he finds a dead body. He immediately measures the body's temperature and finds it to be 79.5°F. Exactly one hour later he measures the temperature again and finds it to be 78.0°F. Determine the time of death, assuming that victim body temperature was normal (98.6°F) prior to death and that the room temperature was constant at 69°F.

Question 14

The voltage V_C t seconds after closing the switch in the circuit shown is:

$$V_C = V_0(1 - e^{-t/(RC)})$$

Given $V_C = 36$ V, $R = 2500\ \Omega$, and $C = 1600\ \mu$F, calculate the current 8 seconds after the switch is closed.

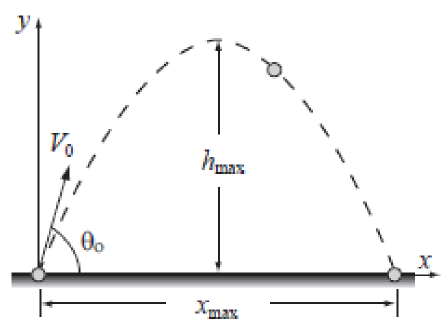


Question 15

A projectile is launched at an angle θ and speed of V_0. The projectile's travel time t_{travel}, maximum travel distance x_{max}, and maximum height h_{max} are given by:

$$t_{travel} = 2\frac{V_0}{g}\sin\theta_0, \quad x_{max} = 2\frac{V_0^2}{g}\sin\theta_0\cos\theta_0,$$

$$h_{max} = 2\frac{V_0^2}{g}\sin^2\theta_0$$

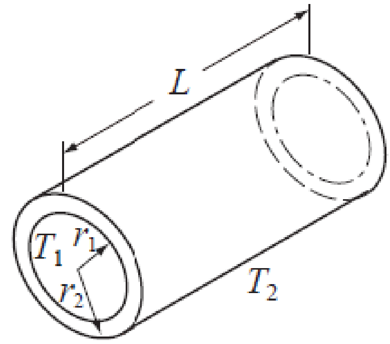


Consider the case where $V_0 = 600$ ft/s and $\theta = 54°$. Define V_0 and θ as MATLAB variables and calculate t_{travel}, x_{max}, and h_{max} ($g = 32.2$ ft/s^2).

Question 16

The steady-state heat conduction q from a cylindrical solid wall is determined by:

$$q = 2\pi Lk\frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)}$$



where k is the thermal conductivity. Calculate q for a copper tube ($k = 401$ Watts/^{o}C/m) of length $L = 300$ cm with an outer radius of $r_2 = 5$ cm and an inner radius of $r_1 = 3$ cm. The external temperature is $T_2 = 20°$C and the internal temperature is $T_1 = 100°$C.

Question 17

According to special relativity, a rod of length L moving at velocity v will shorten by an amount δ, given by:

$$\delta = L\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

where c is the speed of light (about 300×10^6 m/s). Calculate how much a rod 2 m long will contract when traveling at 5,000 m/s.

Question 18

In the triangle shown $a = 5$ in., $b = 7$ in., and $\gamma = 25°$. Define a, b, and γ as variables, and then:

(*a*) Calculate the length of c by substituting the variables in the Law of Cosines.

(Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$)

(*b*) Calculate the angles α and β (in degrees) using the Law of Sines.

(*c*) Verify the Law of Tangents by substituting the results from part (*b*) into the right and left sides of the equation.

Law of Tangents: $$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha - \beta)\right]}{\left[\frac{1}{2}(\alpha + \beta)\right]}$$