

### Question 01

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For the function  $y = x^4 e^{-x}$ , calculate the value of  $y$  for the following values of  $x$  using element-by-element operations: 1.5, 2, 2.5, 3, 3.5, 4 .

### Question 02

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A unit vector  $\mathbf{u}_n$  in the direction of the vector  $\mathbf{u} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  is given by  $\mathbf{u}_n = \mathbf{u} / |\mathbf{u}|$  where  $|\mathbf{u}|$  is the length (magnitude) of the vector, given by  $|\mathbf{u}| = \sqrt{x^2 + y^2 + z^2}$ . Given the vector  $\mathbf{u} = 4\mathbf{i} + 13\mathbf{j} - 7\mathbf{k}$ , determine the unit vector in the direction of  $\mathbf{u}$  using the following steps:

- (a) Assign the vector to a variable  $\mathbf{u}$ .
- (b) Using element-by-element operation and the MATLAB built-in functions `sum` and `sqrt` calculate the length of  $\mathbf{u}$  and assign it to the variable `Lu`.
- (c) Use the variables from parts (a) and (b) to calculate  $\mathbf{u}_n$ .
- (d) Verify that the length of  $\mathbf{u}_n$  is 1 using the same operations as in part (b).

### Question 03

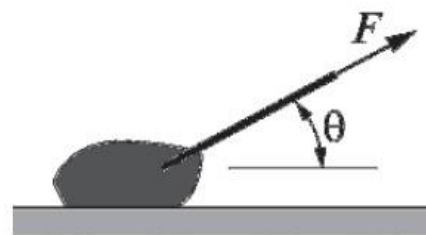
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A 70 lb-bag of rice is being pulled by a person by applying a force  $F$  at an angle  $\theta$  as shown. The force required to drag the bag is given by:

$$F(\theta) = \frac{70\mu}{\mu \sin\theta + \cos\theta}$$

where  $\mu = 0.35$  is the friction coefficient.

- (a) Determine  $F(\theta)$  for  $\theta = 5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ$ , and  $35^\circ$ .
- (b) Determine the angle  $\theta$  where  $F$  is minimum. Do it by creating a vector  $\theta$  with elements ranging from  $5^\circ$  to  $35^\circ$  and spacing of 0.01. Calculate  $F$  for each value of  $\theta$  and then find the maximum  $F$  and associated  $\theta$  with MATLAB's built-in function `max`.



### Question 04

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For the function  $y = (x + x\sqrt{x+3})(1+2x^2) - x^3$ , calculate the value of  $y$  for the following values of  $x$  using element-by-element operations:  $-2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5, 2$ .

### Question 05

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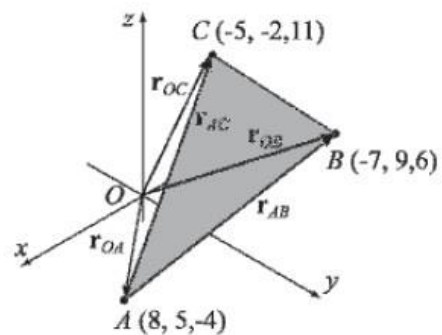
The angle between two vectors  $\mathbf{u}_1 = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$  and  $\mathbf{u}_2 = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$  can be determined by  $\cos \theta = \frac{x_1x_2 + y_1y_2 + z_1z_2}{|\mathbf{u}_1||\mathbf{u}_2|}$ , where  $|\mathbf{u}_i| = \sqrt{x_i^2 + y_i^2 + z_i^2}$ .

Given the vectors  $\mathbf{u}_1 = 3.2\mathbf{i} - 6.8\mathbf{j} + 9\mathbf{k}$  and  $\mathbf{u}_2 = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$ , determine the angle between them (in degrees) by writing one MATLAB command that uses element-by-element multiplication and the MATLAB built-in functions `acosd`, `sum`, and `sqrt`.

### Question 06

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The area of a triangle  $ABC$  can be calculated by  $|\mathbf{r}_{AB} \times \mathbf{r}_{AC}| / 2$ , where  $\mathbf{r}_{AB}$  and  $\mathbf{r}_{AC}$  are vectors connecting the vertices  $A$  and  $B$ , and  $A$  and  $C$ , respectively. Determine the area of the triangle shown in the figure. Use the following steps in a script file to calculate the area. First, define the vectors  $\mathbf{r}_{OA}$ ,  $\mathbf{r}_{OB}$ , and  $\mathbf{r}_{OC}$  from knowing the coordinates of points  $A$ ,  $B$ , and  $C$ . Then determine the vectors  $\mathbf{r}_{AB}$  and  $\mathbf{r}_{AC}$  from  $\mathbf{r}_{OA}$ ,  $\mathbf{r}_{OB}$ , and  $\mathbf{r}_{OC}$ . Finally, determine the area by using MATLAB's built-in functions `cross`, `sum`, and `sqrt`.



### Question 07

Define  $r$  and  $s$  as scalars  $r = 1.6 \times 10^3$  and  $s = 14.2$ , and,  $t$ ,  $x$ , and  $y$  as vectors  $t = [1, 2, 3, 4, 5]$ ,  $x = [2, 4, 6, 8, 10]$ , and  $y = [3, 6, 9, 12, 15]$ . Then use these variables to calculate the following expressions using element-by-element calculations for the vectors.

(a)  $G = xt + \frac{r}{s^2}(y^2 - x)t$                       (b)  $R = \frac{r(-xt + yt^2)}{15} - s^2(y - 0.5x^2)t$

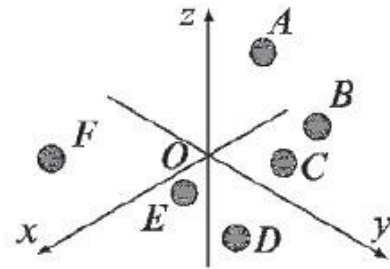
### Question 08

The center of mass,  $(\bar{x}, \bar{y}, \bar{z})$ , of  $n$  particles can be calculated by:

$$\bar{x} = \frac{\sum_{i=1}^{i=n} m_i x_i}{\sum_{i=1}^{i=n} m_i}, \quad \bar{y} = \frac{\sum_{i=1}^{i=n} m_i y_i}{\sum_{i=1}^{i=n} m_i}, \quad \bar{z} = \frac{\sum_{i=1}^{i=n} m_i z_i}{\sum_{i=1}^{i=n} m_i}$$

where  $x_i$ ,  $y_i$ , and  $z_i$  and  $m_i$  are the coordinates

and the mass of particle  $i$ , respectively. The coordinates and mass of six particles are listed in the following table. Calculate the center of mass of the particles.



Particle	Mass (kg)	Coordinate x (mm)	Coordinate y (mm)	Coordinate z (mm)
A	0.5	-10	8	32
B	0.8	-18	6	19
C	0.2	-7	11	2
D	1.1	5	12	-9
E	0.4	0	-8	-6
F	0.9	25	-20	8



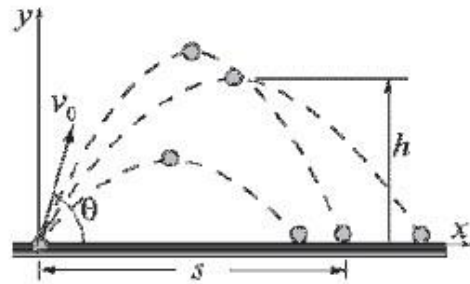
### Question 09

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The maximum distance  $s$  and the maximum height  $h$  that a projectile shot at an angle  $\theta$  are given by:

$$s = \frac{v_0^2}{g} \sin 2\theta \quad \text{and} \quad h = \frac{v_0^2 \sin^2 \theta}{2g}$$

where  $v_0$  is the shooting velocity and  $g = 9.81 \text{ m/s}^2$ . Determine  $s(\theta)$  and  $h(\theta)$  for  $\theta = 15^\circ, 25^\circ, 35^\circ, 45^\circ, 55^\circ, 65^\circ, 75^\circ$  if  $v_0 = 260 \text{ m/s}$ .



### Question 10

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The Hazen Williams equation can be used to calculate the pressure drop,  $P_d$  (psi/ft of pipe) in pipes due to friction:

$$P_d = 4.52 Q^{1.85} / (C^{1.85} d^{4.87})$$

where  $Q$  is the flow rate (gpm),  $C$  is a design coefficient determined by the type of pipe, and  $d$  is pipe diameter in inches. Consider a 3.5-in.-diameter steel pipe with  $C = 120$ . Calculate the pressure drop in a 1000-ft-long pipe for flow rates of 250, 300, 350, 400, and 450 gpm. To carry out the calculation, first create a five-element vector with the values of the flow rates (250, 300, ...). Then use the vector in the formula using element-by-element operations.

### Question 11

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Solve the following system of six linear equations:

$$\begin{aligned} 2a - 4b + 5c - 3.5d + 1.8e + 4f &= 52.52 \\ -1.5a + 3b + 4c - d - 2e + 5f &= -21.1 \\ 5a + b - 6c + 3d - 2e + 2f &= -27.6 \\ 1.2a - 2b + 3c + 4d - e + 4f &= 9.16 \\ 4a + b - 2c - 3d - 4e + 1.5f &= -17.9 \\ 3a + b - c + 4d - 2e - 4f &= -16.2 \end{aligned}$$

### Question 12

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A food company manufactures five types of 8-oz trail mix packages using different mixtures of peanuts, almonds, walnuts, raisins, and M&Ms. The mixtures have the following compositions:

	Peanuts (oz)	Almonds (oz)	Walnuts (oz)	Raisins (oz)	M&Ms (oz)
Mix 1	3	1	1	2	1
Mix 2	1	2	1	3	1
Mix 3	1	1	0	3	3
Mix 4	2	0	3	1	2
Mix 5	1	2	3	0	2

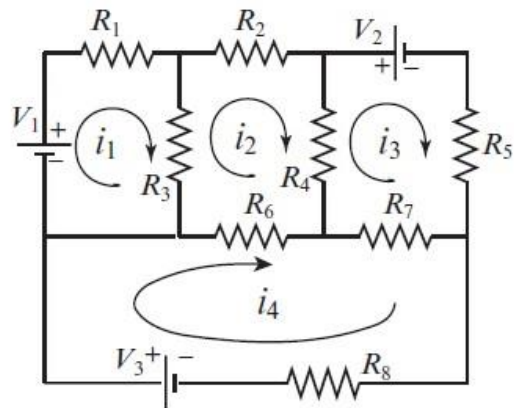
How many packages of each mix can be manufactured if 105 lb of peanuts, 74 lb of almonds, 102 lb of walnuts, 118 lb of raisins, and 121 lb of M&Ms are available? Write a system of linear equations and solve.

### Question 13

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The electrical circuit shown consists of resistors and voltage sources. Determine  $i_1, i_2, i_3$  and  $i_4$ , using the mesh current method based on Kirchhoff's voltage law (see Sample Problem 3-4).

$$\begin{aligned}V_1 &= 28 \text{ V}, \quad V_2 = 36 \text{ V}, \quad V_3 = 42 \text{ V} \\R_1 &= 16 \Omega, \quad R_2 = 10 \Omega, \quad R_3 = 6 \Omega \\R_4 &= 12 \Omega, \quad R_5 = 8 \Omega, \quad R_6 = 14 \Omega \\R_7 &= 4 \Omega, \quad R_8 = 5 \Omega.\end{aligned}$$



### Question 14

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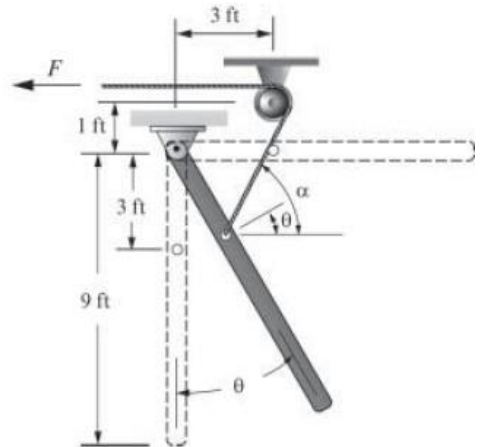
A 300-lb garage door is being opened by pulling on the cable as shown. As the door is lifted the force,  $F$ , in the cable, as a function of the angle  $\theta$ , is given by:

$$F = \frac{300 \cdot 4.5 \sin \theta}{3 \cos(\alpha - \theta)}$$

where

$$\sin \alpha = \frac{1 + 3 \cos \theta}{\sqrt{(1 + 3 \cos \theta)^2 + (3 - 3 \sin \theta)^2}}$$

Calculate  $F$  for  $\theta = 0^\circ$  through  $90^\circ$  with increments of  $10^\circ$ . Display the results in a two-column table.



### Question 15

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Early explorers often estimated altitude by measuring the temperature of boiling water. Use the following two equations to make a table that modern-day hikers could use for the same purpose.

$$p = 29.921(1 - 6.8753 \times 10^{-6}h), \quad T_b = 49.161 \ln p + 44.932$$

where  $p$  is atmospheric pressure in inches of mercury,  $T_b$  is boiling temperature in  $^\circ\text{F}$ , and  $h$  is altitude in feet. The table should have two columns, the first altitude and the second boiling temperature. The altitude should range between  $-500$  ft and  $10,000$  ft at increments of  $500$  ft.



### Question 16

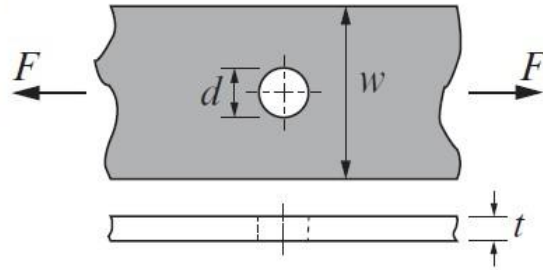
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The maximum stress  $\sigma_{\max}$  at the edge of a hole (diameter  $d$ ) in a thin plate, with width  $w$  and thickness  $t$ , loaded by a tensile force  $F$  as shown is given by:

$$\sigma_{\max} = K_t \sigma_{\text{nom}}$$

where  $\sigma_{\text{nom}} = \frac{F}{t(w-d)}$  and  $K_t = 3 - 3.14\left(\frac{d}{w}\right) + 3.667\left(\frac{d}{w}\right)^2 - 1.527\left(\frac{d}{w}\right)^3$ .

Write a program in a script file that calculates  $\sigma_{\max}$ .



The output should be in the form of a paragraph combining text and numbers—i.e., something like: “The maximum stress in a plate with a width of XX in. and thickness of XX in. and a hole of XX in. in diameter, due to a tensile force of XXX lb is XXXX psi, where XX stands for numerical values.” The stress should be rounded to the nearest integer. Use the program to calculate  $\sigma_{\max}$  when  $w = 2.5$  in.,  $d = 1.375$  in.,  $t = 0.1875$  in., and  $F = 8000$  lb.

### Question 17

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The intrinsic electrical conductivity  $\sigma$  of a semiconductor can be approximated by:

$$\sigma = e^{(C - \frac{E_g}{2kT})}$$

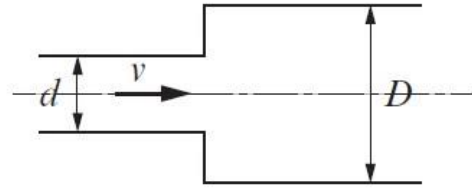
where  $\sigma$  is measured in  $(\Omega - m)^{-1}$ ,  $E_g$  is the band gap energy,  $k$  is Boltzmann's constant ( $8.62 \times 10^{-5}$  eV/K), and  $T$  is temperature in kelvins. For germanium,  $C = 13.83$  and  $E_g = 0.67$  eV. Write a program in a script file that calculates the intrinsic electrical conductivity for germanium for various temperatures.

The output should be presented as a table where the first column is the temperature and the second column is the intrinsic electrical conductivity. Use the following values for temperature: 400, 435, 475, 500, 520, and 545 K.

### Question 18

The pressure drop  $\Delta p$  in pascals (Pa) for a fluid flowing in a pipe with a sudden increase in diameter is given by:

$$\Delta p = \frac{1}{2} \left[ 1 - \left( \frac{d}{D} \right)^2 \right]^2 \rho v^2$$



where  $\rho$  is the density of the fluid,  $v$ , the velocity of the flow, and  $d$  and  $D$  are defined in the figure. Write a program in a script file that calculates the pressure drop  $\Delta p$ . When the script file is executed, it requests the user to input the density in  $\text{kg/m}^3$ , the velocity in  $\text{m/s}$ , and values of the nondimensional ratio  $d/D$  as a vector. The program displays the inputted values of  $\rho$  and  $v$  followed by a table with the values of  $d/D$  in the first column and the corresponding values of  $\Delta p$  in the second column.

Execute the program assuming flow of gasoline ( $\rho = 737 \text{ kg/m}^3$ ) at  $v = 5 \text{ m/s}$  and the following ratios of diameters  $d/D = 0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3$ .

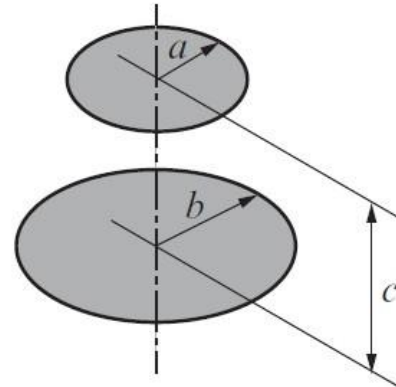
### Question 19

The net heat exchange by radiation from plate 1 with radius  $b$  to plate 2 with radius  $a$  that are separated by a distance  $c$  is given by:

$$q = \sigma \pi b^2 F_{1-2} (T_1^4 - T_2^4)$$

where  $T_1$  and  $T_2$  are the absolute temperatures of the plates,  $\sigma = 5.669 \times 10^{-8} \text{ W/(m}^2\text{-K}^4\text{)}$  is the Stefan-Boltzmann constant, and  $F_{1-2}$  is a shape factor which, for the arrangement in the figure, is given by:

$$F_{1-2} = \frac{1}{2} \left[ Z - \sqrt{Z^2 - 4X^2Y^2} \right]$$





where  $X = a/c$ ,  $Y = c/b$ , and  $Z = 1 + (1 + X^2)Y^2$ . Write a script file that calculates the heat exchange  $q$ . For input the program asks the user to enter values for  $T_1$ ,  $T_2$ ,  $a$ ,  $b$ , and  $c$ . For output the program prints a summary of the geometry and temperatures and then prints the value of  $q$ . Use the script to calculate the results for  $T_1 = 400\text{K}$ ,  $T_2 = 600\text{K}$ ,  $a = 1\text{ m}$ ,  $b = 2\text{ m}$ , and  $c = 0.1, 1$ , and  $10\text{ m}$ .

### Question 20

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A truss is a structure made of members joined at their ends. For the truss shown in the figure, the forces in the seven members are determined by solving the following system of seven equations.

$$F_1 \cos 28.5^\circ + F_2 - 3000 = 0,$$

$$F_1 \sin 28.5^\circ + 6521 = 0,$$

$$-F_1 \cos 28.5^\circ - F_3 \cos 58.4^\circ + F_5 \cos 58.4^\circ + F_6 \cos 28.5^\circ + 3000 = 0,$$

$$-F_1 \sin 28.5^\circ - F_3 \sin 58.4^\circ - F_5 \sin 58.4^\circ - F_6 \sin 28.5^\circ = 0$$

$$-F_4 - F_5 \cos 58.4^\circ + F_7 = 0, \quad F_6 \sin 28.5^\circ + 7479 = 0 \quad -F_7 - F_6 \cos 28.5^\circ = 0$$

Write the equations in matrix form and use MATLAB to determine the forces in the members. A positive force means tensile force and a negative force means compressive force. Display the results in a table where the first column displays the member number and the second column displays the corresponding force.

