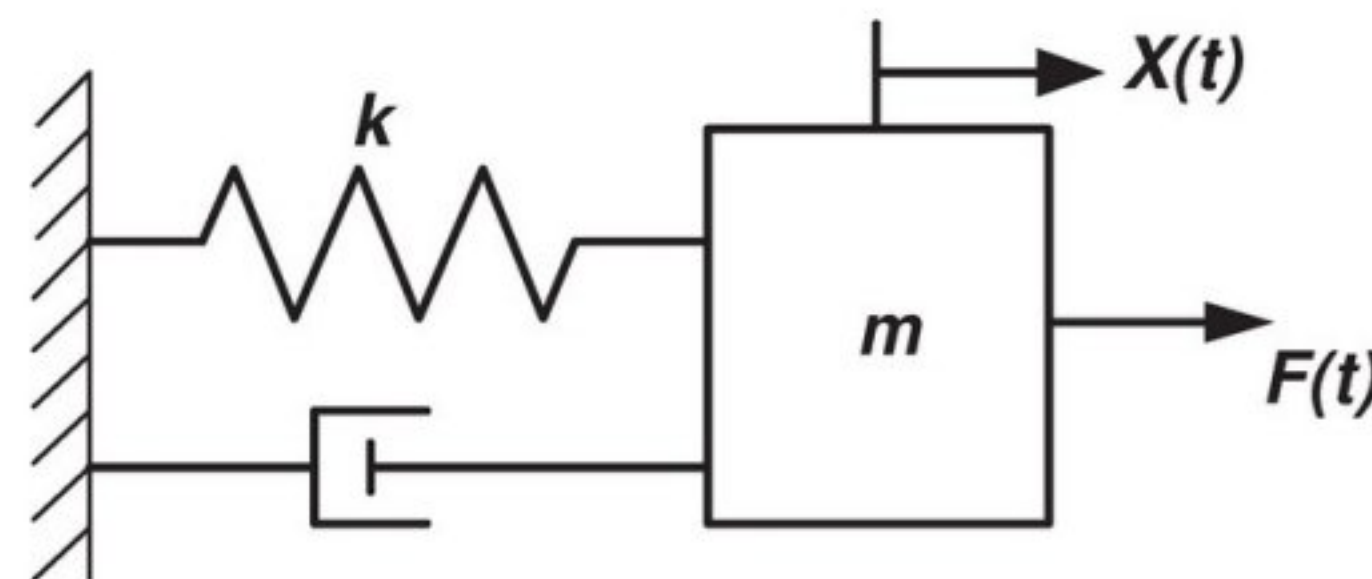


Lab Session 07

Exercise:

Question 1:

Write Transfer function $X(s)/F(s)$ for the system shown below. Find Rise time, Peak time, Overshoot, and Settling time for the transfer function using formulae derived in Chapter 4. Verify your results using a MATLAB script present at the end of Lab 7 in the manual. Change the time vector to $[0:0.001:300]$ in the script.



System Parameters:

$$M = 705$$

$$B = 30$$

$$K = 15$$

Nothing to write below this line. Use blank A4 sheets to solve the given problem and attach them with this document.

Solution

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At equilibrium,

$$\begin{aligned} F(t) &= f_k(t) + f_B(t) + f_M(t) \\ &= Kx(t) + B \frac{dx(t)}{dt} + M \frac{d^2x(t)}{dt^2} \end{aligned}$$

Taking Laplace transform

\Rightarrow

$$F(s) = Kx(s) + BSx(s) + MS^2x(s)$$

$$\frac{F(s)}{x(s)} = K + BS + MS^2$$

and

$$\frac{x(s)}{F(s)} = \frac{1}{MS^2 + BS + K} \quad \text{--- (1)}$$

We know that general equation

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{--- (A)}$$

Compare both eqn and rewrite eq (1) in form of general eqn

$$G(s) = \frac{x(s)}{F(s)} = \frac{1/M}{s^2 + \frac{B}{M}s + \frac{K}{M}} \quad \text{--- (2)}$$

Now by comparison of eq (A) and (2) we get

$$\omega_n^2 = \frac{K}{M} \quad \text{and} \quad 2\zeta\omega_n = \frac{B}{M}$$

NATURAL FREQUENCY :-

$$\omega_n^2 = \frac{K}{M} \Rightarrow \omega_n = \sqrt{\frac{K}{M}} = \sqrt{\frac{15}{705}}$$

$$\boxed{\omega_n = 0.14586 \text{ } 1/s}$$

DAMPING RATIO :-

$$2\zeta\omega_n = \frac{B}{M} \Rightarrow \zeta = \frac{B}{2\omega_n M} = \frac{B}{2\sqrt{MK}} = \frac{30}{2\sqrt{705(15)}}$$

$$\boxed{\zeta = 0.14586}$$

RISE TIME :-

$$\begin{aligned}\omega_n T_r &= 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1 \\ &= 1.76(0.14586)^3 - 0.417(0.14586)^2 + 1.039(0.14586) + 1\end{aligned}$$

$$T_r = \frac{1.1481}{\omega_n} = \frac{1.1481}{0.14586}$$

$$\boxed{T_r = 7.87 \text{ sec}}$$

PEAK TIME :-

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$= \frac{\pi}{0.14586 \sqrt{1-(0.14586)^2}}$$

$$\boxed{T_p = 21.77 \text{ sec}}$$

% OVERSHOOT :-

$$\%OS = e^{\left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)}$$

$$\%OS = e^{\left(\frac{-0.14586(\pi)}{\sqrt{1-(0.14586)^2}}\right)}$$

$$\boxed{\%OS = 0.629}$$

OR $\boxed{\%OS = 62.9\%}$

PLOT SCRIPT:

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```
1- clc, clear;
2- zeta = input('Enter the value of damping ratio ');
3- wn = input('Enter the value of Natural frequency ');
4- n = wn*wn;
5- d = [1 2*zeta*wn wn*wn];
6- disp('The transfer function is: ')
7- printsys(n,d);
8- t = 0:0.01:300;
9- [y,x,t] = step(n,d,t);
10- plot(t,y); grid on; title('step response');
11- k = 1; % rise time
12- while y(k) <= 0.1;
13-     k = k + 1;
14- end; tenpercent = t(k);
15- while y(k) <= 0.9;
16-     k = k + 1;
17- end
18- ninetypercent = t(k);
19- rtime = ninetypercent - tenpercent
20- fprintf('The rise time is: %.f sec \n', rtime);
21- for k = 1:length(t)-1; % maximum overshoot
22-     if y(k+1) <= y(k);
23-         break;
24-     end;
25- end;
26- Oshoot = y(k) - y(length(t)-1);
27- fprintf('The overshoot is: %.f sec \n', overst Oshoot)
```



```

28- tp = t(k);
29- fprintf('the peak time is : %f sec\n', tp);
30- % settling time
31- tot = 0.02;
32- for k = length(t)-1:-1:2;
33-     if (abs(y(k) - y(length(t)-1)) > tot)
34-         break;
35-     end;
36- end;
37- stime = t(k);
38- fprintf('the settling time is : %f sec\n', stime);

```

RESULT:-

The rise time is: 7.86800 sec
 The overshoot is: 0.630416 sec
 the peak time is: 21.770000 sec
 the settling time is: 177.940000 sec

CONCLUSION:-

Values of rise time, peak time, % overshoot are same in both theoretical and MATLAB calculations and plot also verifies all values.

Transfer Function from MATLAB we get:

The transfer function is:

$$0.021275$$

$$s^2 + 0.04255s + 0.021275$$

