CSE 211: Discrete Mathematics **(Due: 17/11/20)**

Homework #1

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1: Conditional Statements (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
(Solution)

Converse: If I will stay at home, then it snows tonight.

Contrapositive: If I will not stay at home, then it doesn't snow tonight.

Inverse: If it doesn't snow tonight,then I won't stay at home.

(b) I go to the beach whenever it is a sunny summer day.
(Solution)

Converse: If I go to the beach, then it is a sunny summer day

Contrapositive: If I don't go to the beach, then it is not a sunny summer day.

Inverse: If it is not a sunny day,then I don't go to the beach.

(c) If I stay up late, then I sleep until noon.
(Solution)

Converse: If I sleep until, I stay up late.

Contrapositive: If I don't sleep until, I don't stay up late.

Inverse: If I don't stay up late, then I don't sleep until.

Problem 2: Truth Tables For Logic Operators (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.
(a) $(p \oplus \neg q)$
(Solution)

p	**q**	**¬ q**	**p ⊕ ¬ q**	
0	0	1	1	
0	1	0	0	
1	0	1	0	
1	1	0	1	

(b) $(p \iff q) \oplus (\neg p \iff \neg r)$
(Solution)

p	**q**	**r**	**¬ p**	**¬ r**	**(p ⟺ q)**	**(¬ p ⟺ ¬ r)**	**(p ⟺ q) ⊕ (¬ p ⟺ ¬ r)**
0	0	0	1	1	1	1	0
0	1	0	1	1	0	1	1
1	0	0	0	1	0	0	0
1	1	0	0	1	1	0	1
0	0	1	1	0	1	0	1
0	1	1	1	0	0	0	0
1	0	1	0	0	0	1	1
1	1	1	0	0	1	1	0

(c) $(p \oplus q) \Rightarrow (p \oplus \neg q)$

p	**q**	**¬ q**	**(p ⊕ q)**	**(p ⊕ ¬ q)'**	**(p ⊕ q) ⇒ (p ⊕ ¬ q)**
0	0	1	0	1	1
0	1	0	1	0	0
1	0	1	1	0	0
1	1	0	0	1	1

Problem 3: Predicates and Quantifiers (21 points)

There are three predicate logic statements which represent English sentences as follows.

- P(x): "x can speak English."
- Q(x): "x knows Python."
- H(x): "x is happy."

Express each of the following sentences in terms of P(x), Q(x), H(x), quantifiers, and logical connectives or vice versa. The domain for quantifiers consists of all students at the university.

(a) There is a student at the university who can speak English and who knows Python.
(Solution)
$\exists x(P(x) \wedge Q(x))$
(b) There is a student at the university who can speak English but who doesn't know Python.
(Solution)
$\exists x(P(x) \wedge \neg Q(x))$
(c) Every student at the university either can speak English or knows Python.
(Solution)
$\forall x(P(x) \vee Q(x))$
(d) No student at the university can speak English or knows Python.
(Solution)
$\neg \exists x(P(x) \vee Q(x))$
(e) If there is a student at the university who can speak English and know Python, then she/he is happy.
(Solution)
$\exists x(P(x) \wedge Q(x)) \rightarrow H(x)$
(f) At least two students are happy.
(Solution)
$\exists x H(x)$
(g) $\neg \forall x(Q(x) \wedge P(x))$
(Solution)
Not everybody is a student at the university who knows Phyton and who can speak English.

Problem 4: Mathematical Induction (21 points)

Prove that $3 + 3\ .\ 5 + 3\ .\ 5^2 + \ .\ .\ .\ + 3\ .\ 5^n = \frac{3(5^{n+1}-1)}{4}$ whenever n is a nonnegative integer.
(Solution)
P(n)=$3+3.5+3.\ 5^2+...+3\ .\ 5^n=\frac{3(5^{n+1}-1)}{4}$

Basic Step:Apply n=1 on the equation.We prove that equation is true for n=1
$\frac{3(5^2-1)}{4}=18$

Inductive Step: Equation of n=k is true
n=k
$3+3.5+3.\ 5^2+...+3\ .\ 5^k=\frac{3(5^{k+1}-1)}{4}=a$

Apply n=k+1

3+3.5+3. 5^2+...+3 . 5^k+(3 . 5^{k+1})=$\frac{3(5^{k+2}-1)}{4}$

a+(3 . 5^{k+1})=$\frac{3(5^{k+2}-1)}{4}$

(3 . 5^{k+1})=$\frac{3(5^{k+2}-1)}{4}$-$\frac{3(5^{k+1}-1)}{4}$

(3 . 5^{k+1}) =$\frac{3(5^{k+1}-1)(5-1)}{4}$

(3 . 5^{k+1})=(3 . 5^{k+1})

Problem 5: Mathematical Induction (20 points)

Prove that n^2 - 1 is divisible by 8 whenever n is an odd positive integer.
(Solution)
Let's write 2m + 1 instead of n

$(2m+1)^2$-1=4.m^2+4m+1-1 =4.m^2+4m

m=1 4.m^2+4m=8 (It is divisible by 8)

m=2 4.m^2+4m=24 (It is divisible by 8)

m=3 4.m^2+4m=48 (It is divisible by 8)

It is concluded that 4.m^2+4m is divisible by 8 for all natural numbers.

Hence, $\mathrm{n}^{\mathbf{2}}$ -1 is divisible by 8 for all odd value of n .

Problem 6: Sets (8 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of x^2 − 6x + 8 = 0}
(b) {y : y is a real number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}

(Solution)
a) x^2-4x-2x+8=0
x = 2 or x = 4
a={2, 4}

b) b=[2, 3]

c) c={4, 2, 5} (It can be written as {4, 2, 5} Because we do not repeat the elements while writing the elements of a set.)

d) d={4, 5, 7, 2} - {5, 7} ={4, 2} (The difference of A and B)

e) e={4, 2}

Thus, the set A, D and E are equal.

Problem Bonus: Logic in Algorithms (20 points)

Let p and q be the statements as follows.

- **p:** It is sunny.
- **q:** The flowers are blooming.

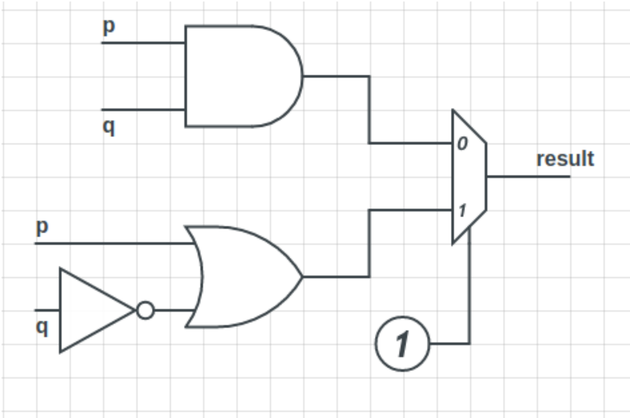

Figure 1: Combinational Circuit

In Figure 1, the two statements are used as input. The circuit has 3 gates as AND, OR and NOT operators. It has also a 2x1 multiplexer[1] which provides to select one of the two options.
(a) Write the sentence that "result" output has.
(Solution)
$p \wedge q$ =It is sunny and the flowers are blooming.
$p \vee \neg q$ =It is sunny or the flowers are not blooming.
The value coming to the mux determines what action it will do. Since the value 1 comes to the mux, it determines the result by performing the operation coming from the 1.
Result = $p \vee \neg q$ =It is sunny or the flowers are not blooming.
(b) Convert Figure 1 to an algorithm which you can write in any programming language that you prefer (including pseudocode).
(Solution)

```
int main()
{
int mux;
cout<<"Please input selection( 0 or 1 ) :";
cin >> mux;
if(mux==0)
{
cout<<"It is sunny and the flowers are blooming.";
}
if(mux==1)
{
cout<<"It is sunny or the flowers are not blooming.";
}
}
```

[1]https://www.geeksforgeeks.org/multiplexers-in-digital-logic/