CSE 211: Discrete Mathematics **(Due: 17/01/21)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- The homeworks (both latex and pdf files in a zip file) will be submitted into the course page of Moodle.
- The latex, pdf and zip files of the homeworks should be saved as "Name_Surname_StudentId".{tex, pdf, zip}.
- If the answers of the homeworks have only calculations without any formula or any explanation -when needed- will get zero.
- Writing the homeworks on Latex is strongly suggested. However, hand-written paper is still accepted **IFF** hand writing of the student is clear and understandable to read, and the paper is well-organized. Otherwise, the assistant cannot grade the student's homework.

Problem 1 (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.
(Solution)
$= a_{n-1} = -2^{n+1-1} \rightarrow a_{n-1} = -2^n$
$= 3a_{n-1} + 2^n = 3(-2^n)+2^n$
$= -3.2^n+2^n$
$= 2^n(-3+1) = 2^n.-2$
$= -2^{n+1} = a_n$
$a_n = -2^{n+1}$ is a solution of the recurrence relation $a_n = 3a_{n-1} + 2^n$
(b) Find the solution with $a_0 = 1$.
(Solution)
a_n=r implies $a_n = 3a_{n-1} + 2^n$
r=$3.r^0+0$
r=3 the solution is $a_n=\alpha.3^n$
$a_n(h)=\alpha.3^n-2^{n+1}$
$a_0=\alpha.3^0-2^1$ then $a_0=1$
$1=\alpha - 2$
$= \alpha = 3$

Problem 2 (35 points)

Solve the recurrence relation f(n) = 4f(n-1) - 4f(n-2) + n^2 for f(0) = 2 and f(1) = 5.
(Solution)

g(n)=an2+bn+c which must satisfy g(n)=4g(n1)4g(n2)+n^2
We convert function f to function g.
g(n)=4g(n1)4g(n2)+n2
an^2+bn+c=4(a(n1)^2+b(n1)+c)4(a(n2)^2+b(n2)+c)+n^2
=(4(an^2+(b-2a)n+(a-b+c))-4(an^2+(b-4a)n+(4a-2b+c)+n^2
=(4a-4a+1)n^2+(4b-8a-4b+16a)n+(4a-4b+4c-16a+8b-4c)
n^2+8an+(-12a+4b)
so,a=1("a"n^2),b=8a(a=1)=8,c=-12.(1)+4*(8)=20=c
then g(n)=n^2+8n+20
The characteristic equation of this recurrence equation is t^2–4t+4=0 , which has a double root of t=2 . Therefore, the general solution must be 2n(dn+e).
f(n)=2n(dn+e)+n2+8n+20.
f(0)=2^0(d.0+e)+0^2+8.0+20= e+20=2
e=-18
f(1)=2^1(d.1+e)+1^2+8.1+20= 2d-7 = 5
d=6
Therefore, f(n)=2^n(6n-18)+n^2+8n+20

Problem 3 (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.
(Solution)
We find the characteristic equations
a_n=r^2 implies $a_n = 2a_{n-1}$ - $2a_{n-2}$
r^2=2r-2
r^2-2r+2 = 0
$r = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$
$r = \frac{-(-2)\pm\sqrt{(-2)^2-4.1.2}}{2.1}$
r = 1 $\pm i$
a_n=A$(1+i)^n$+B$(1-i)^n$

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.
(Solution)