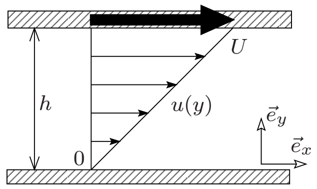
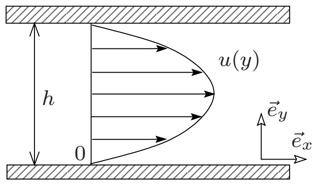
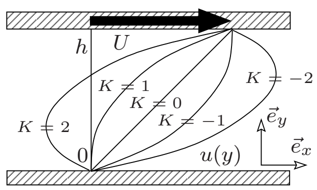
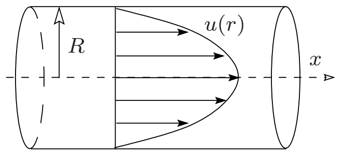
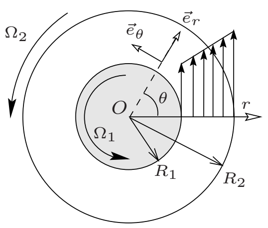
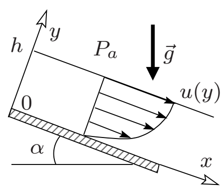
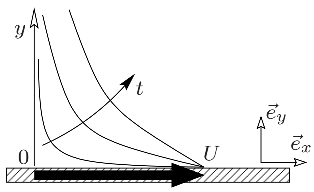
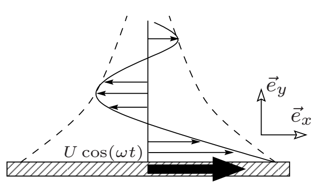
(Quelques) solutions exactes des équations de Navier-Stokes

Schéma	Hypothèses	Solution	Nom de l'écoulement
	$\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$	$\vec{u} = u(y)\,\vec{e}_x \quad \text{avec} \quad u(y) = \dfrac{U\,y}{h}$ $p = C^{te}$	Couette plan sans gradient de pression $(\partial p/\partial x = 0)$
	$\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$	$\dfrac{dp}{dx} = C^{te} \quad \text{et} \quad \vec{u} = u(y)\,\vec{e}_x$ $\text{avec} \quad u(y) = \dfrac{1}{2\mu}\dfrac{dp}{dx}y(h-y)$	Poiseuille plan
	$\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$	$\dfrac{dp}{dx} = C^{te} \quad \text{et} \quad \vec{u} = u(y)\,\vec{e}_x$ $\text{avec} \quad u(y) = U\left[\dfrac{y}{h} - K\dfrac{y}{h}\left(1 - \dfrac{y}{h}\right)\right]$ $\text{où} \quad K = \dfrac{h^2}{2\mu U}\dfrac{dp}{dx}$	Couette plan avec gradient de pression $(\partial p/\partial x \neq 0)$ = "Couette-Poiseuille"
	$\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial \theta} = \vec{0}$	$\dfrac{dp}{dx} = C^{te} \quad \text{et} \quad \vec{u} = u(r)\,\vec{e}_x$ $\text{avec} \quad u(r) = -\dfrac{1}{4\mu}\dfrac{dp}{dx}\left(R^2 - r^2\right)$	Poiseuille cylindrique
	$\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial \theta} = \vec{0}$ $\dfrac{\partial p}{\partial x} = 0$	$\vec{u} = v(r)\vec{e}_\theta \quad \text{avec} \quad v(r) = Ar + \dfrac{B}{r}$ $\text{où} \quad A = \dfrac{\Omega_2 R_2^2 - \Omega_1 R_1^2}{R_2^2 - R_1^2}$ $\text{et} \quad B = \dfrac{(\Omega_1 - \Omega_2)\,R_1^2 R_2^2}{R_2^2 - R_1^2}$	Couette cylindrique = "Couette-Taylor"
	$\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$	$\vec{u} = u(y)\vec{e}_x$ $\text{avec} \quad u(y) = \dfrac{g\sin\alpha}{2\nu}y(2h - y)$ $p(y) = \rho g(h-y)\cos\alpha + P_a$	Film tombant
	$\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ $\dfrac{\partial p}{\partial x} = 0$	$\vec{u} = u(y,t)\vec{e}_x$ $\text{avec} \quad u(y,t) = U\left[1 - \text{erf}\left(\dfrac{y}{2\sqrt{\nu\,t}}\right)\right]$ $\tau_p = \mu\left.\dfrac{\partial u}{\partial y}\right\rvert_{y=0} = -\rho U\sqrt{\dfrac{\nu}{\pi t}}$	Premier problème de Stokes
	$\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ $\dfrac{\partial p}{\partial x} = 0$	$\vec{u} = u(y,t)\vec{e}_x$ $u(y,t) = Ue^{-y/\delta}\cos(\omega t - \dfrac{y}{\delta});\ \delta = \sqrt{\dfrac{2\nu}{\omega}}$ $\tau_p = \mu\left.\dfrac{\partial u}{\partial y}\right\rvert_{y=0} = -\dfrac{\mu U\sqrt{2}}{\delta}\cos(\omega t - \pi/4)$	Second problème de Stokes