| Schéma | Hypothèses | Solution | Nom de l'écoulement |
|---|---|---|---|
|  | $\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ | $\vec{u} = u(y)\,\vec{e}_x$ avec $u(y) = \dfrac{Uy}{h}$ $p = C^{te}$ | Couette plan sans gradient de pression $(\partial p/\partial x = 0)$ |
|  | $\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ | $\dfrac{dp}{dx} = C^{te}$ et $\vec{u} = u(y)\,\vec{e}_x$ avec $u(y) = \dfrac{1}{2\mu}\dfrac{dp}{dx}y(h-y)$ | Poiseuille plan |
|  | $\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ | $\dfrac{dp}{dx} = C^{te}$ et $\vec{u} = u(y)\,\vec{e}_x$ avec $u(y) = U\left[\dfrac{y}{h} - K\dfrac{y}{h}\left(1 - \dfrac{y}{h}\right)\right]$ où $K = \dfrac{h^2}{2\mu U}\dfrac{dp}{dx}$ | Couette plan avec gradient de pression $(\partial p/\partial x \neq 0)$ = "Couette-Poiseuille" |
|  | $\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial \theta} = \vec{0}$ | $\dfrac{dp}{dx} = C^{te}$ et $\vec{u} = u(r)\,\vec{e}_x$ avec $u(r) = -\dfrac{1}{4\mu}\dfrac{dp}{dx}\left(R^2 - r^2\right)$ | Poiseuille cylindrique |
|  | $\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial \theta} = \vec{0}$ $\dfrac{\partial p}{\partial x} = 0$ | $\vec{u} = v(r)\vec{e}_\theta$ avec $v(r) = Ar + \dfrac{B}{r}$ où $A = \dfrac{\Omega_2 R_2^2 - \Omega_1 R_1^2}{R_2^2 - R_1^2}$ et $B = \dfrac{(\Omega_1 - \Omega_2)\,R_1^2 R_2^2}{R_2^2 - R_1^2}$ | Couette cylindrique = "Couette-Taylor" |
|  | $\dfrac{\partial \vec{u}}{\partial t} = \vec{0}$ $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ | $\vec{u} = u(y)\vec{e}_x$ avec $u(y) = \dfrac{g\sin\alpha}{2\nu}y(2h-y)$ $p(y) = \rho g(h-y)\cos\alpha + P_a$ | Film tombant |
|  | $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ $\dfrac{\partial p}{\partial x} = 0$ | $\vec{u} = u(y,t)\vec{e}_x$ avec $u(y,t) = U\left[1 - \text{erf}\left(\dfrac{y}{2\sqrt{\nu t}}\right)\right]$ $\color{red}{\tau_p = \mu\left.\dfrac{\partial u}{\partial x}\right|_{x=0} = -\rho U\sqrt{\dfrac{\nu}{\pi t}}}$ | Premier problème de Stokes |
|  | $\dfrac{\partial \vec{u}}{\partial x} = \dfrac{\partial \vec{u}}{\partial z} = \vec{0}$ $\dfrac{\partial p}{\partial x} = 0$ | $\vec{u} = u(y,t)\vec{e}_x$ $u(y,t) = Ue^{-y/\delta}\cos(\omega t - \dfrac{y}{\delta}); \; \delta = \sqrt{\dfrac{2\nu}{\omega}}$ $\color{red}{\tau_p = \mu\left.\dfrac{\partial u}{\partial x}\right|_{x=0} = -\dfrac{\mu U \sqrt{2}}{\delta}\cos(\omega t - \pi/4)}$ | Second problème de Stokes |

**(Quelques) solutions exactes des équations de Navier-Stokes**