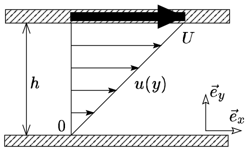
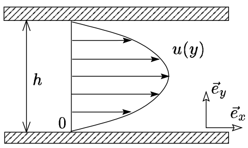
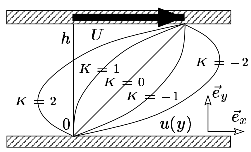
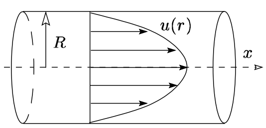
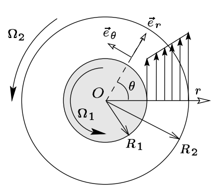
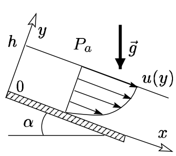
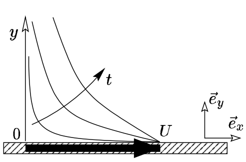
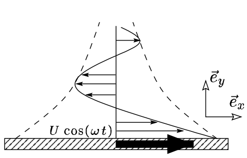
(Quelques) solutions exactes des équations de Navier-Stokes

Schéma	Caractéristiques	Solution	Nom de l'écoulement
	$\frac{\partial \vec{u}}{\partial t} = \vec{0}$ $\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial z} = \vec{0}$	$\vec{u} = u(y)\,\vec{e}_x \quad \text{avec} \quad u(y) = \frac{Uy}{h}$ $p = C^{te}$	Couette plan sans gradient de pression ($\partial p/\partial x = 0$)
	$\frac{\partial \vec{u}}{\partial t} = \vec{0}$ $\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial z} = \vec{0}$	$\frac{dp}{dx} = C^{te} \quad \text{et} \quad \vec{u} = u(y)\,\vec{e}_x$ $\text{avec} \quad u(y) = \frac{1}{2\mu}\frac{dp}{dx}y(h-y)$	Poiseuille plan
	$\frac{\partial \vec{u}}{\partial t} = \vec{0}$ $\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial z} = \vec{0}$	$\frac{dp}{dx} = C^{te} \quad \text{et} \quad \vec{u} = u(y)\,\vec{e}_x$ $\text{avec} \quad u(y) = U\left[\frac{y}{h} - K\frac{y}{h}\left(1 - \frac{y}{h}\right)\right]$ $\text{où} \quad K = \frac{h^2}{2\mu U}\frac{dp}{dx}$	Couette plan avec gradient de pression ($\partial p/\partial x \neq 0$) = “Couette-Poiseuille”
	$\frac{\partial \vec{u}}{\partial t} = \vec{0}$ $\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial \theta} = \vec{0}$	$\frac{dp}{dx} = C^{te} \quad \text{et} \quad \vec{u} = u(r)\,\vec{e}_x$ $\text{avec} \quad u(r) = -\frac{1}{4\mu}\frac{dp}{dx}\left(R^2 - r^2\right)$	Poiseuille cylindrique
	$\frac{\partial \vec{u}}{\partial t} = \vec{0}$ $\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial \theta} = \vec{0}$ $\frac{\partial p}{\partial x} = 0$	$\vec{u} = v(r)\vec{e}_\theta \quad \text{avec} \quad v(r) = Ar + \frac{B}{r}$ $\text{où} \quad A = \frac{\Omega_2 R_2^2 - \Omega_1 R_1^2}{R_2^2 - R_1^2}$ $\text{et} \quad B = \frac{(\Omega_1 - \Omega_2)\,R_1^2 R_2^2}{R_2^2 - R_1^2}$	Couette cylindrique = “Couette-Taylor”
	$\frac{\partial \vec{u}}{\partial t} = \vec{0}$ $\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial z} = \vec{0}$	$\vec{u} = u(y)\vec{e}_x$ $\text{avec} \quad u(y) = \frac{g\sin\alpha}{2\nu}y(2h-y)$ $p(y) = \rho g(h-y)\cos\alpha + P_a$	Film tombant
	$\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial z} = \vec{0}$ $\frac{\partial p}{\partial x} = 0$	$\vec{u} = u(y,t)\vec{e}_x$ $\text{avec} \quad u(y,t) = U\left[1 - \text{erf}\left(\frac{y}{2\sqrt{\nu\, t}}\right)\right]$ $p = C^{te}$	Premier problème de Stokes
	$\frac{\partial \vec{u}}{\partial x} = \frac{\partial \vec{u}}{\partial z} = \vec{0}$ $\frac{\partial p}{\partial x} = 0$	$\vec{u} = u(y,t)\vec{e}_x$ $\text{avec} \quad u(y,t) = Ue^{-y/\delta}\cos(\omega t - \frac{y}{\delta})$ $\text{où} \quad \delta = \sqrt{\frac{2\nu}{\omega}} \quad \text{et} \quad p = C^{te}$	Second problème de Stokes