Nr. 1

	a	b	c
a	1/3	1/2	
b	1/3		1/2
c	1/3	1/2	1/2

$$= M = \begin{pmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{pmatrix}$$

$$r^{(0)} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

$$\Rightarrow r^{(1)} = M r^{(0)} = \begin{pmatrix} 5/18 \\ 5/18 \\ 4/9 \end{pmatrix}$$

$$|r^{(1)} - r^{(0)}| = 0{,}22 \quad > \frac{1}{12}$$

$$\Rightarrow r^{(2)} = M r^{(1)} = \begin{pmatrix} 25/108 \\ 17/54 \\ 49/108 \end{pmatrix}$$

$$|r^{(2)} - r^{(1)}| = 0{,}093 > \frac{1}{12}$$

$$\Rightarrow r^{(3)} = M r^{(2)} = \begin{pmatrix} 19/81 \\ 197/648 \\ 299/648 \end{pmatrix} = \begin{pmatrix} 0{,}234 \\ 0{,}304 \\ 0{,}461 \end{pmatrix}$$

$$|r^{(3)} - r^{(2)}| = 0{,}022 < \frac{1}{12}$$

✓

b) As supposed in the hint, we calculate the eigenvector of M with respect to Eigenvalue 1

$$\Rightarrow \quad (M - I)\,x = 0$$

$$\Leftrightarrow \begin{pmatrix} -2/3 & 1/2 & 0 \\ 1/3 & -1 & 1/2 \\ 1/3 & 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 1/2 & 0 & 0 \\ 1/3 & -1 & 1/2 & 0 \\ 1/3 & 1/2 & -1/2 & 0 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 1/2 & 0 & 0 \\ 2/3 & -2 & 1 & 0 \\ 2/3 & 1 & -1 & 0 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 1/2 & 0 & 0 \\ 0 & -3/2 & 1 & 0 \\ 0 & 3/2 & -1 & 0 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 1/2 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 3/2 & -1 & 0 \end{array}\right)$$

$$\Rightarrow \quad x_3 = \tfrac{3}{2} x_2$$

$$\Rightarrow \quad x_2 = \frac{2}{3} x_3$$

$$-\frac{2}{3} x_1 = -\frac{1}{3} x_3$$

$$x_1 = \frac{1}{2} x_3$$

$\Rightarrow$ stationäre Verteilung $\quad r^* =$

Lösung des Gleichungssystems:

$$r^* = a \cdot \begin{pmatrix} 1/2 \\ 2/3 \\ 1 \end{pmatrix}$$

$$\Rightarrow r = r^* / \|r^*\|_1 \quad \text{mit } a =, \text{ also } a = \frac{1}{\left\| \begin{pmatrix} 1/2 \\ 2/3 \\ 1 \end{pmatrix} \right\|_1}$$

Stationäre Lösung

$$\Rightarrow r = \begin{pmatrix} 1/2 \\ 2/3 \\ 1 \end{pmatrix} / \left(1/2 + 2/3 + 1\right)$$

$$= \begin{pmatrix} 1/2 \\ 2/3 \\ 1 \end{pmatrix} / \left(13/6\right)$$

$$= \begin{pmatrix} 0{,}231 \\ 0{,}308 \\ 0{,}462 \end{pmatrix}$$

c)

$$M_T = \underset{\substack{\| \\ 4/5}}{0{,}8} \cdot \begin{pmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{pmatrix} + \underset{\substack{\| \\ 1/5}}{0{,}2} \cdot \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

$$= \begin{pmatrix} 1/3 & 7/15 & 1/15 \\ 1/3 & 1/15 & 7/15 \\ 1/3 & 7/15 & 7/15 \end{pmatrix}$$

$$\Rightarrow r^{(0)} = \begin{pmatrix} 1/3 \\ 1/3 \\ 1/3 \end{pmatrix}$$

$$r^{(1)} = = M_T r^{(0)} = \begin{pmatrix} 13/45 \\ 13/45 \\ 19/45 \end{pmatrix}$$

$$|r^{(1)} - r^{(0)}| = 0{,}178 \quad > \quad 1/12$$

$$\Rightarrow r^{(2)} = M_T r^{(1)} = \begin{pmatrix} 0,259 \\ 0,313 \\ 0,428 \end{pmatrix}$$

$$|r^{(2)} - r^{(1)}| = 0,059 < 1/12$$

d) Calculate the eigenvector with eigenvalue 1

$$\Rightarrow (M_T - I)x = 0$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 7/15 & 1/15 & 0 \\ 1/3 & -14/15 & 7/15 & 0 \\ 1/3 & 7/15 & -8/15 & 0 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 7/15 & 1/15 & 0 \\ 2/3 & -28/15 & 14/15 & 0 \\ 2/3 & 14/15 & -16/15 & 0 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 7/15 & 1/15 & 0 \\ 0 & -21/15 & 1 & 0 \\ 0 & 21/15 & -1 & 0 \end{array}\right)$$

$$\Leftrightarrow \left(\begin{array}{ccc|c} -2/3 & 7/15 & 1/15 & 0 \\ 0 & -21/15 & 1 & 0 \\ 0 & -21/15 & 1 & 0 \end{array}\right)$$

$$\Rightarrow x_3 = 21/15\, x_2 \Leftrightarrow x_2 = 5/7\, x_3$$
$$= 7/5\, x_2$$

$$\cancel{\Rightarrow -2/3\, x_1 + 5/7\, x_3 + 1/15\, x_3 = 0}$$
$$\cancel{\Leftrightarrow 2/3\, x_1 = 82/105\, x_3}$$

$\Rightarrow \cancel{M} -\frac{2}{3}x_1 + \frac{7}{15}x_2 + \frac{1}{15}x_3 = 0$

$\Leftrightarrow -\frac{2}{3}x_1 + \frac{7}{15} \cdot \frac{5}{7}x_3 + \frac{1}{15}x_3 = 0$

$\Leftrightarrow \frac{2}{3}x_1 = \frac{6}{15}x_3$

$x_1 = \frac{3}{5}x_3$

$\cancel{\Leftrightarrow x_3 = \frac{5}{3}x_1}$

$\Rightarrow$ Lösung des LGS

$$r^* = a \cdot \begin{pmatrix} 3/5 \\ 5/7 \\ 1 \end{pmatrix}$$

$\Rightarrow$ Stationäre Verteilung

$$r = \frac{r^*}{\|r^*\|_1} = \begin{pmatrix} 3/5 \\ 5/7 \\ 1 \end{pmatrix} / 2{,}314$$

$$= \begin{pmatrix} 0{,}259 \\ 0{,}309 \\ 0{,}432 \end{pmatrix}$$