Exercise 2

Lets imagine the following example, since the input is 2-dimensional we have a x and a y parameter:

We choose the dimension so be very simple [d=2]

$o =\alpha_1x^2+\alpha_2x+\alpha_3$

The real calues for the alpha vector are as follows: $\alpha_1=1,\alpha_2=2,\alpha_3=3$

The initial values however are: $\alpha_1=1,\alpha_2=1,\alpha_3=1$

Calculate the error - for an example point (3, 12):

The current estimation returns: $o = 1^2 3+1 3+1 = 7$

The true value however should be: $o = 1^2 3+2 3+3 = 12$

Since we only have a single point the calculation of the mean squared error is very simple also:

$mse = \frac{\sum(true-fale)^2}{n} = \frac{\sum(12-7)^2}{1}=25$

Gradient Descent

The above formula to calculate the error now needs to be determined and derived for all parameters (alphas):

$\frac{\delta f}{\delta \alpha_1}=\frac{1}{n}\sum 2*-2x* (true -(\alpha_1x+\alpha_2y+\alpha_3))$

$\frac{\delta f}{\delta \alpha_2}=\frac{1}{n}\sum 2*-x* (true -(\alpha_1x+\alpha_2y+\alpha_3))$

$\frac{\delta f}{\delta \alpha_1}=\frac{1}{n}\sum 2*-1* (true -(\alpha_1x+\alpha_2y+\alpha_3))$

Now we can insert the example point to find the gradients(x=3,y=10):

$g_1 = 2*-6*5 = -60$

$g_2 = 2*-3*5 = -30$

$g_3 = 2*-5= -10$

With the learning rate we will receive the new estimation:

$\alpha_1 = \alpha_1 - 0.01*g_1 = 1,6$

$\alpha_2 = \alpha_2 - 0.01*g_2 = 1,3$

$\alpha_3 = \alpha_3 - 0.01*g_3 = 1,1$

Validation with example point (3,2)

We can validate that our solution improved as follows: $o = 1^2*3+1*3+1 = 12,68$

Threfore the new MSE error is: 0,4624