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Two-dimensional XY magnets with annealed non-magnetic impurities[a]

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The global phase diagram of a vectorial generalization of the Blume-Emery-Griffiths model is obtained using Migdal's approximate renormalization procedure. Classical two-component spins and non-magnetic impurities populate a triangular lattice, with nearest-neighbor interactions. The phase diagram in thermodynamic field space is divided into magnetic and impurity-rich phases by a first-order surface of discontinuous impurity concentrations, terminating in an Ising-type critical line. The magnetic region is further divided into a high-temperature paramagnetic phase and a low-temperature Kosterlitz-Thouless phase. The exponent $\eta = 1/4$ of the pure system is preserved at the higher-order surface separating these two phases. This surface terminates in a line of critical end-points on the first-order surface, and, consequently, no tricritical point occurs for any values of the model parameters. However, the Ising critical line and the line of critical end-points approach each other in a certain limit, yielding an effective tricritical phase diagram. Within the Kosterlitz-Thouless phase, lines of constant η bunch together as the effective tricritical point is approached, in apparent agreement with tricritical scaling. This is also a model for superfluidity and phase separation in helium films.

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The two-dimensional XY model of magnetism, consisting of two-component spins with short-ranged interactions, has attracted much interest. Conventional long-range order, namely non-zero magnetization, is rigorously ruled out [1]. Nevertheless, low-temperature spin-wave theory [2] yields algebraic, rather than exponential, decay of correlations, and high-temperature series expansions [3] suggest a phase transition at finite temperature. Experiments now seem to indicate that a detailed theory due to Kosterlitz and Thouless [4] is substantially correct, as explained below. At all temperatures below the transition, the magnetization is zero, but the susceptibility is infinite. At the transition, a universal jump discontinuity in the spin-wave stiffness (and, hence, in the spin-wave velocity) is predicted [5]. Although verification of this result in magnets is lacking, the analogous [5] universal jump in the superfluid density of helium films was recently reported [6]. In this paper, the above picture is extended by allowing for annealed, non-magnetic impurities. We find no XY tricritical point, in contrast to the three-dimensional situation.

We studied a vectorial generalization of the Blume-Emery-Griffiths (BEG) model [7], with Hamiltonian

$$-\frac{\mathcal{H}}{k_B T} = J\sum_{<ij>} \vec{s}_i \cdot \vec{s}_j + K\sum_{<ij>} |\vec{s}_i|^2 |\vec{s}_j|^2 - \Delta\sum_i |\vec{s}_i|^2 \quad . \qquad (1)$$

The spins $\vec{s}_i$ are two-component classical vectors of length unity or zero, located at the sites i of a triangular lattice. The first two sums in (1) are over all nearest-neighbor pairs of sites. The states $|\vec{s}_i|=1$ represent the XY spins, interacting with the coupling constant $k_B TJ$. The state $\vec{s}_i=\vec{0}$ allows for the occupation of site i by a non-magnetic impurity. The coupling $k_B TK$ measures the spin-impurity interaction, and $k_B T\Delta$ is the impurity chemical potential. The above is also a model for superfluidity and phase separation in helium films [8].

The diluted XY model (1) was treated [8,9] using a renormalization scheme due to Migdal [10] and Kadanoff [11]. This approximate procedure yields good

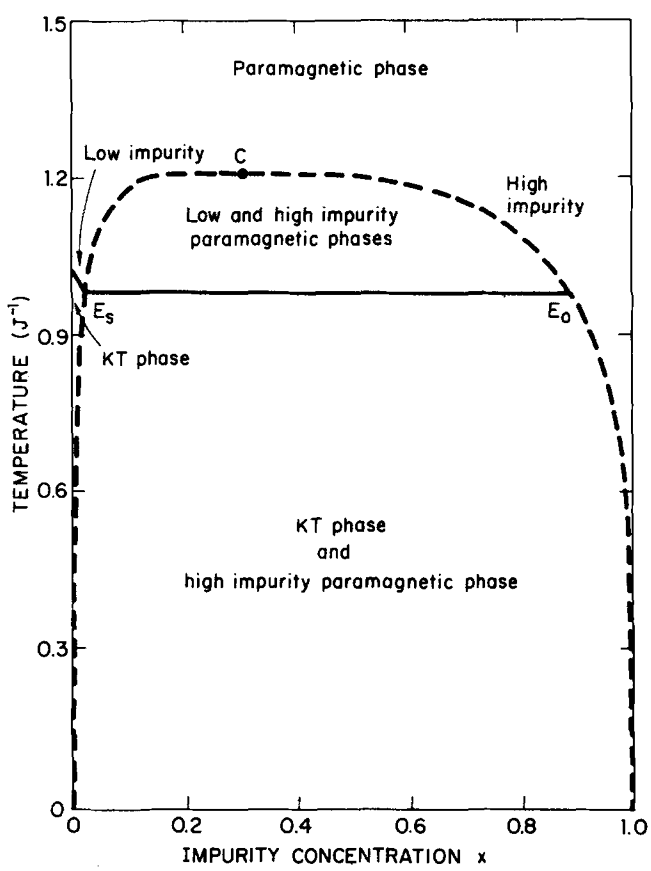

Fig. 1. End-point phase diagram of the diluted XY model at K/J = 1. The full lines indicate higher-order transitions. The boundary of the coexistence regions is shown with the dashed line. The Kosterlitz-Thouless phase is labeled KT.

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agreement [12] with the Kosterlitz-Thouless theory [4] of the pure system, obtained in the limit $\Delta \to -\infty$ in (1).

Our results are exhibited by phase diagrams in the variables temperature (J^{-1}) and impurity concentration

$$x = 1 - \langle |\vec{s}_i|^2 \rangle \quad , \qquad (2)$$

at given K/J. The phase diagram for K/J=1 is shown in Fig. 1. The line of XY transitions drops to lower temperature with increasing impurity concentration, until it terminates on the phase-separation boundary, at a critical end-point E_s at x = 0.02. As K/J is increased, the phase-separation (Ising-type) critical point C moves to temperatures higher and higher than the XY transitions, the end-point E_s moves closer to x=0, and, thus, phase-separation dominates the phase diagram more and more. As K/J is decreased, the end-point slides up the phase-separation boundary, but never actually reaches the tip. Thus, there is never a tricritical point. This is in contrast to the renormalization-group treatments [13] of the two-dimensional scalar BEG model [7], and, indeed, to one's expectations for the Hamiltonian (1) in three dimensions. (In the latter case, the tricritical phase diagram observed experimentally in bulk ^{3}He-^{4}He mixtures [14] is expected.) On the other hand, the end-point and the phase-separation critical point do come very close together at small K/J. For K/J = 0, shown in Fig. 2, they are respectively at J^{-1} = 0.6581 and J^{-1} = 0.6615. Thus, an effective tricritical phase diagram is obtained. In all cases, the ordered (Kosterlitz-Thouless) phase accomodates a small (compared to similar situations [14] in three dimensions) amount of impurities before phase separation occurs. The upper-limit concentration is $x(E_s)$ = 0.12, for K/J = 0.

Also of interest is the critical exponent η, defined by the algebraic decay of correlations in the Kosterlitz-Thouless phase,

$$\langle \vec{s}(\vec{r}) \cdot \vec{s}(\vec{0}) \rangle \sim 1/r^{\eta} \quad . \qquad (3)$$

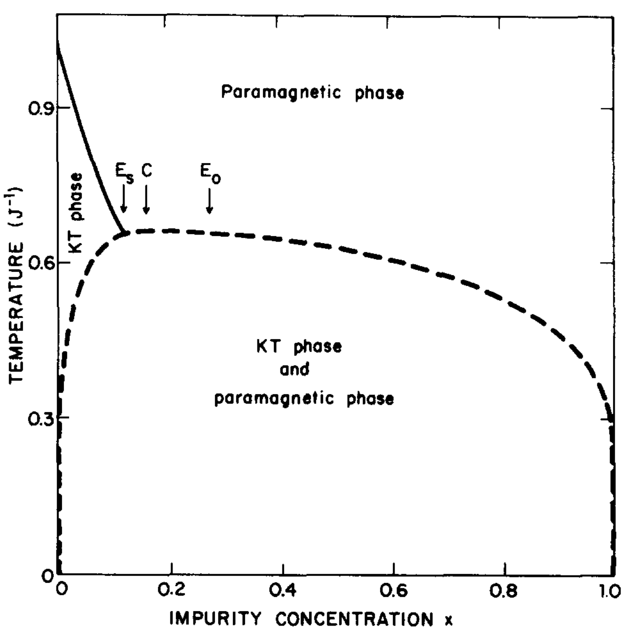

Fig. 2. Effective tricritical phase diagram of the diluted XY model at K/J = 0. The full lines indicate higher-order transitions. The boundary of the coexistence region is shown with the dashed line.

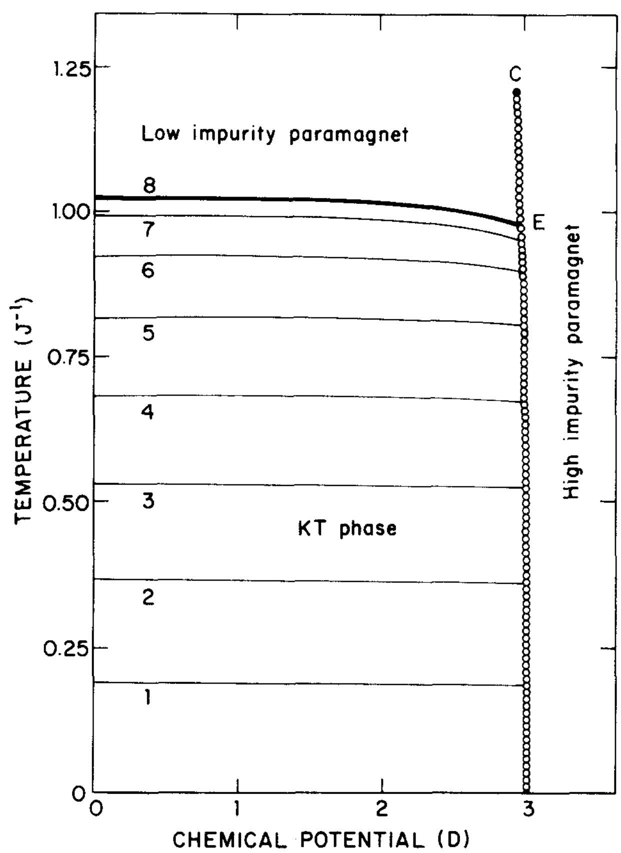

Fig. 3. End-point phase diagram of the diluted XY model at K/J = 1. The dark full line and the line of open circles (ooo) respectively indicate higher-order and first-order transitions. The dark circle (•) marks the isolated critical point, and E labels the end-point. Within the Kosterlitz-Thouless (KT) phase, constant η curves are drawn with light full lines and labeled by 32η. The chemical potential variable (see Ref. 8) is

$$D = \{\Delta + \ln\int_0^{2\pi} (d\phi/2\pi)\, \exp[4J(\cos\phi - 1)]\}/K \ .$$

At the transition temperature in the pure system, this exponent [4] equals ¼, a value intimately related to the size of the jump in the spin-wave stiffness [5]. Lines of constant η are shown in the temperature-chemical potential phase diagrams of Figs. 3 and 4. The pure system result $\eta = \frac{1}{4}$ is preserved along the line of XY transitions until phase separation occurs. In the K/J = 0 case of Fig. 4, several lines of constant η seem to converge onto the effective tricritical point T_{eff} formed by the closely situated end-point and critical point. The precise value of η at T_{eff} thus appears indeterminate, depending instead on the path of approach! This peculiar behavior is anticipated [8] from tricritical scaling theory [15] together with the invariance property of η under rescaling.

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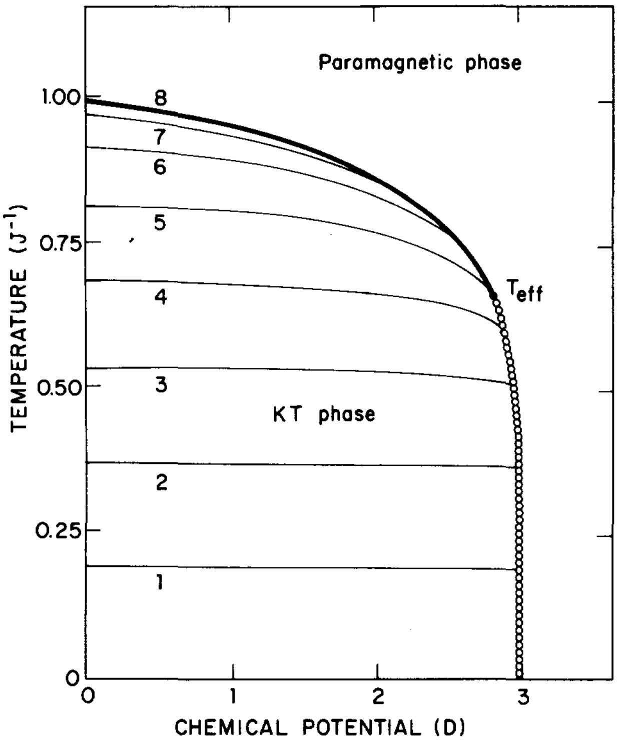

Fig. 4. Effective tricritical phase diagram of the diluted XY model at K/J = 0. The dark full line and the line of open circles (ooo) respectively indicate higher-order and first-order transitions. The actual end-point and critical point are both within the dark circle at the effective tricritical point T_{eff}. Within the Kosterlitz-Thouless (KT) phase, constant η curves are drawn with light full lines and labeled by 32η. As predicted by tricritical scaling theory, the $\eta=\frac{1}{4}$ to 5/32 curves apparently converge onto T_{eff}.

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