# Frustration, Chaos, and Order in the d=1 Ising Spin Glass with Long-Range Power-Law Interactions

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The $d = 1$ Ising spin glass with long-range power-law interactions $J\,r^{-a}$ is studied for all $a$ by a renormalization-group transformation that simultaneously projects local ferromagnetism and antiferromagnetism. In the ferromagnetic case, $J > 0$, a finite-temperature ferromagnetic phase occurs for $0.75 < a < 2$. The transition temperature monotonically decreases between these two limits. At $a = 2$, the phase-transition temperature discontinuously drops to zero and for $a > 2$ there is no ordered phase above zero temperature, as predicted by rigorous results. On approaching $a = 0.75$ from above, the phase-transition temperature diverges to infinity, meaning that, at all temperatures above zero, the system is ferromagnetically ordered. Thus, the equivalent-neighbor interactions regime is entered before ($a \neq 0$) the neighbors become equivalent, namely before the ineractions become equal for all separations. The critical exponents are calculated, from a large recursion matrix, as a function of $a$. For the antiferromagnetic case, $J < 0$, all triplets of spins have competing interactions and this highly frustrated system does not have an ordered phase. In the spin-glass phase, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability $p$), a finite-temperatures phase diagram is obtained with order in the presence of frustration. Transient chaos is found within the ferromagnetic phase.

## I. ORDERING IN ONE-DIMENSION: LONG-RANGE INTERACTIONS

Whereas systems with finite-range interactions do not order in one dimension, certain systems with long-range interactions do order. The archetypical example are the Ising models with power-law interactions, $J\,r^{-a}$, which show varigated ordering behavior for ferromagnetic interactions. For antiferromagnetic interactions, the system incorporates saturated frustration and spin-glass phenomena, in the absence of quenched randomness. All of these behaviors are seen in our renormalization-group study.

The model that we study is defined by the Hamiltonian

$$-\beta\mathcal{H} = \sum_{r_1 \neq r_2} J\,|r_1 - r_2|^{-a} s_{r_1} s_{r_2} \tag{1}$$

where $\beta = 1/k_B T$ is the inverse temperature, $r_1$ and $r_2$ designate the sites on the one-dimensional system, at each site there is an Ising spin $s_i = \pm 1$ and the sums are over all sites in the system. For ferromagnetic and antiferromagnetic systems, the two-spin interactions $J$ are $J = |J| > 0$ and $J = -|J| > 0$, respectively. For the spin-glass system, for each two spins, their interaction is randomly ferromagnetic (with probability $1 - p$) or antiferromagnetic (with probability $p$).

## II. METHOD: LONG-RANGE RENORMALIZATION GROUP

We solve this system with Niemeyer and van Leeuwen's two-cell cluster approximation. The renormalization-group transformation is constructed by first choosing cells on the $d = 1$, as shown in Fig. 1. Each of our cells has three spins. Secondly, for each cell, a cell-spin is defined as the sign of the sum of the three spins in the cell,

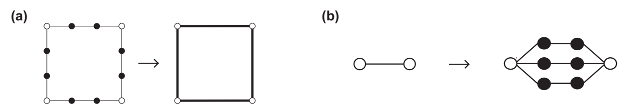


FIG. 1. Renormalization-group cells for $d = 1$. This cell structure projects both local ferromagnetism and antiferromagnetism, and therefore also spin-glass order.

$$s'_{r'} = signum(s_{r-2} + s_{r-1} + s_r) \tag{2}$$

where the signum function returns the sign of its argument, primes denote the renormalized system, and $r' = r/b$, where $b = 3$ is the length-rescaling factor. The renormalized interactions are obtained from the conservation of the partition function $Z$,

$$Z = \sum_{\{s\}} e^{-\beta\mathcal{H}(\{s\})} = \sum_{\{s'\}} \sum_{\{\sigma\}} e^{-\beta\mathcal{H}(\{s'\},\{\sigma\})} = \sum_{\{s'\}} e^{-\beta\mathcal{H}'(\{s'\})} = Z' , \tag{3}$$

where the summed variable $\sigma$ represents, for each cell, the four states that give the same cell-spin value. Thus, the renormalized interactions are obtained from

$$e^{-\beta\mathcal{H}'(\{s'\})} = \sum_{\{\sigma\}} e^{-\beta\mathcal{H}(\{s'\},\{\sigma\})}. \tag{4}$$

The two-cell cluster approximation of Niemeyer and van Leeuwen consists in carrying our this transformation for two cells, including the 6 intracell interactions and the 9 intercell interactions. A recursion relation is obtained for each renormalized interaction,

$$J'_{r'} = (1/2)\ln[R(s'_0 = +1, s'_{r'} = +1)/R(s'_0 = +1, s'_{r'} = -1)]\,, \tag{5}$$

where

$$R(s'_0, s'_{r'}) = \sum_{\sigma_0,\sigma_{r'}} e^{-\beta \mathcal{H}_{0r'}}, \tag{6}$$

where the unrenormalized two-cell Hamiltonian contains the 6 intracell interactions and the 9 intercell interactions between the 6 spins in cells 0 and $r'$.

## III. RESULTS: FINITE-TEMPERATURE FERROMAGNETIC PHASE IN $d=1$

Calculated phase diagram of the $d = 1$ long-range ferromagnetic Ising model with interactions $J\,r^{-a}$ , is shown in Fig. 2, in terms of temperature $1/J$ and interaction range $a$. A finite-temperature ferromagnetic phase occurs for $0.75 < a < 2$. The transition temperature monotonically decreases between these two limits. At $a = 2$, the phase-transition temperature discontinuously drops to zero and for $a > 2$ there is no ordered phase above zero temperature, as predicted by rigorous results. On approaching $a = 0.75$ from above, the phase-transition temperature diverges to infinity, meaning that, at all temperatures above zero, the system is ferromagnetically ordered. Thus, the equivalent-neighbor interactions regime is entered before ($a \neq 0$) the neighbors become equivalent, namely before the ineractions become equal for all separations.

Specific heat critical exponent $\alpha$ and correlation-length critical exponent $\nu$, continuously varying as a function of interaction range $a$, for the finite-temperature ferromagnetic phase transition are also shown in Fig. 2. These critical exponents are calculated from the recursion relations $J'_1, ..., J'_n = funct(J_1, ..., J_n)$. Convergence is obtained by calculation up to $n = 3$. The largest eigenvalue, $\lambda = b^y$ of the derivative matrix of these recursion relations at the critical point gives the correlation-length critical exponent $\nu = 1/y$ and the specific heat critical exponent $\alpha = 2 - d/y = 2 - 1/y$.

The antiferromagnetic, overly frustrated without randomness, system does not have a finite-temperature phase transition, but the spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability $p$), does have finite-temperature spin-glass phase transitions and chaos inside the spin-glass phase, as seen in Fig. 3.

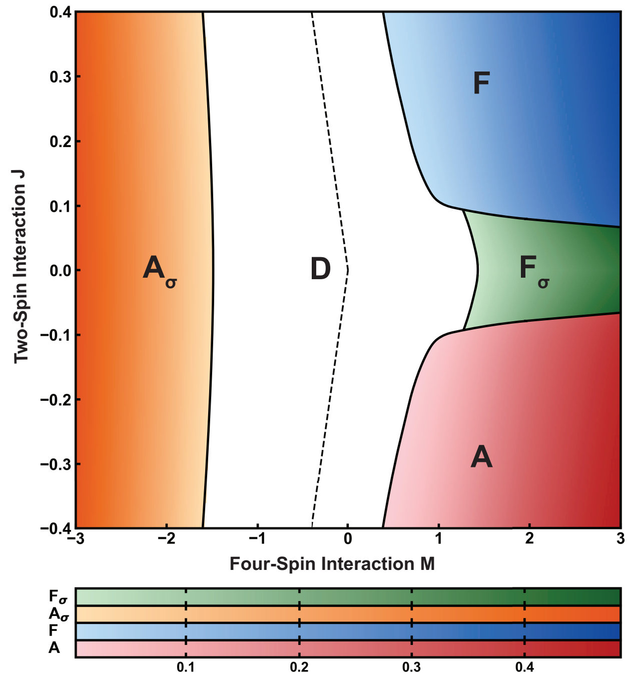


FIG. 2. Calculated phase diagram of the $d = 1$ long-range ferromagnetic Ising model with interactions $J\,r^{-a}$. Top panel: Phase diagram in temperature $1/J$ and interaction range $a$. A finite-temperature ferromagnetic phase occurs for $0.75 < a < 2$. The transition temperature monotonically decreases between these two limits. At $a = 2$, the phase-transition temperature discontinuously drops to zero and for $a > 2$ there is no ordered phase above zero temperature, as predicted by rigorous results. On approaching $a = 0.75$ from above, the phase-transition temperature diverges to infinity, meaning that, at all temperatures above zero, the system is ferromagnetically ordered. Thus, the equivalent-neighbor interactions regime is entered before ($a \neq 0$) the neighbors become equivalent, namely before the ineractions become equal for all separations. Bottom panel: Specific heat critical exponent $\alpha$ and correlation-length critical exponent $\nu$ as a function of interaction range $a$ for the finite-temperature ferromagnetic phase transition. The antiferromagnetic, overly frustrated without randomness, system does not have a finite-temperature phase transition, but the spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic, does have finite-temperature spin-glass phase transitions and chaos inside the spin-glass phase, as seen in Fig. 3.

## IV. RESULTS: FINITE-TEMPERATURE SPIN-GLASS PHASE IN $d=1$

The spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability $p$), does have finite-temperature spin-glass phase transitions and chaos inside the spin-glass phase, as seen in Fig. 3.

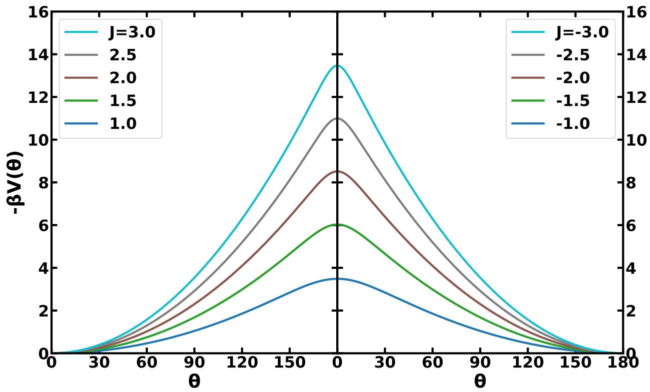


FIG. 3. Calculated finite temperature phase diagram of the $d = 1$ long-range Ising spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability $p$). Bottom panel: Chaos inside the spin-glass phase in $d = 1$.

## V. CONCLUSION

## ACKNOWLEDGMENTS

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