Ferromagnetic and Spin-Glass Order but no Antiferromagetic Order in the d=1 Ising Model with Long-Range Power-Law Interactions

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The $d = 1$ Ising spin glass with long-range power-law interactions $J\,r^{-a}$ is studied for all interaction range exponents a by a renormalization-group transformation that simultaneously projects local ferromagnetism and antiferromagnetism. In the ferromagnetic case, $J > 0$, a finite-temperature ferromagnetic phase occurs for interaction range $0.74 < a < 2$. The transition temperature monotonically decreases between these two limits. At $a = 2$, the phase-transition temperature discontinuously drops to zero and for $a > 2$ there is no ordered phase above zero temperature, as predicted by rigorous results. On approaching $a = 0.74$ from above, namely increasing the range of the interaction, the phase-transition temperature diverges to infinity, meaning that, at all non-infinite temperatures, the system is ferromagnetically ordered. Thus, the equivalent-neighbor interactions regime is entered before ($a \neq 0$) the neighbors become equivalent, namely before the interactions become equal for all separations. The critical exponents $\alpha, \beta, \delta, \eta, \nu$ are calculated, from a large recursion matrix, as a function of a. Magnetization curves, as a function of temperature and as a function of magnetic field, are calculated and show sharp critical behavior. For the antiferromagnetic case, $J < 0$, all triplets of spins at all ranges have competing interactions and this highly frustrated system does not have an ordered phase. In the spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability p), a finite-temperatures spin-glass phase is obtained in the absence of antiferromagnetic phase. A truly unusual phase diagram is obtained. In the spin-glass phase, the signature chaotic occurs. Transient chaos is found within the ferromagnetic phase.

I. ORDERING IN ONE-DIMENSION: LONG-RANGE INTERACTIONS

Whereas systems with finite-range interactions do not order in one dimension, certain systems with long-range interactions do order.[1–5] The archetypical example are the Ising models with power-law interactions, $J\,r^{-a}$, which show varigated ordering behavior for ferromagnetic interactions. For antiferromagnetic interactions, the system incorporates saturated frustration and spin-glass phenomena, in the absence of quenched randomness. All of these behaviors are seen in our renormalization-group study.

The model that we study is defined by the Hamiltonian

$$-\beta\mathcal{H} = \sum_{r_1 \neq r_2} J\,|r_1 - r_2|^{-a} s_{r_1} s_{r_2} \,+\, H \sum_{r_1} s_{r_1} \qquad (1)$$

where $\beta = 1/k_B T$ is the inverse temperature, r_1 and r_2 designate the sites on the one-dimensional system, at each site there is an Ising spin $s_{r_i} = \pm 1$, and the sums are over all sites in the system. For ferromagnetic and antiferromagnetic systems, the two-spin interactions J are $J = |J| > 0$ and $J = -|J| < 0$, respectively. For the spin-glass system, for each two spins at any range, their interaction is randomly ferromagnetic (with probability $1 - p$) or antiferromagnetic (with probability p). The second term is the magnetic-field H term.

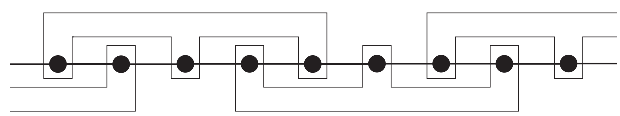

FIG. 1. Renormalization-group cells for $d = 1$. This cell structure projects both local ferromagnetism and antiferromagnetism, and therefore also spin-glass order.

II. METHOD: LONG-RANGE RENORMALIZATION GROUP

We solve this system with Niemeyer and van Leeuwen's two-cell cluster approximation.[6, 7] The renormalization-group transformation is constructed by first choosing cells on the $d = 1$, as shown in Fig. 1. Each of our cells has three spins. This cell structure projects both local ferromagnetism and antiferromagnetism, and therefore also spin-glass order. Secondly, for each cell, a cell-spin is defined as the sign of the sum of the three spins in the cell,

$$s'_{r'} = signum(s_{r-2} + s_{r-1} + s_r) \qquad (2)$$

where the signum function returns the sign of its argument, primes denote the renormalized system, and $r' = r/b$, where $b = 3$ is the length-rescaling factor. The renormalized interactions are obtained from the conservation of the partition function Z,

$$Z = \sum_{\{s\}} e^{-\beta\mathcal{H}(\{s\})} = \sum_{\{s'\}}\sum_{\{\sigma\}} e^{-\beta\mathcal{H}(\{s'\},\{\sigma\})} = \sum_{\{s'\}} e^{-\beta\mathcal{H}'(\{s'\})} = Z' \,, \quad (3)$$

where the summed variable σ represents, for each cell, the four states that give the same cell-spin value. Thus, the renormalized interactions are obtained from

$$e^{-\beta\mathcal{H}'(\{s'\})} = \sum_{\{\sigma\}} e^{-\beta\mathcal{H}(\{s'\},\{\sigma\})}. \quad (4)$$

The two-cell cluster approximation of Niemeyer and van Leeuwen consists in carrying our this transformation for two cells, including the 6 intracell interactions and the 9 intercell interactions. A recursion relation is obtained for each renormalized interaction,

$$J'_{r'} = \frac{1}{4}\ln\frac{R_{r'}(+1,+1)R_{r'}(-1,-1)}{R_{r'}(+1,-1)R_{r'}(-1,+1)}\,, \qquad H' = \frac{1}{4}\ln\frac{R_1(+1,+1)}{R_1(-1,-1)}\,, \quad (5)$$

where

$$R_{r'}(s'_0, s'_{r'}) = \sum_{\sigma_0,\sigma_{r'}} e^{-\beta\mathcal{H}_{0r'}}\,, \quad (6)$$

where the unrenormalized two-cell Hamiltonian contains the 6 intracell interactions and the 9 intercell interactions between the 6 spins in cells 0 and r'.

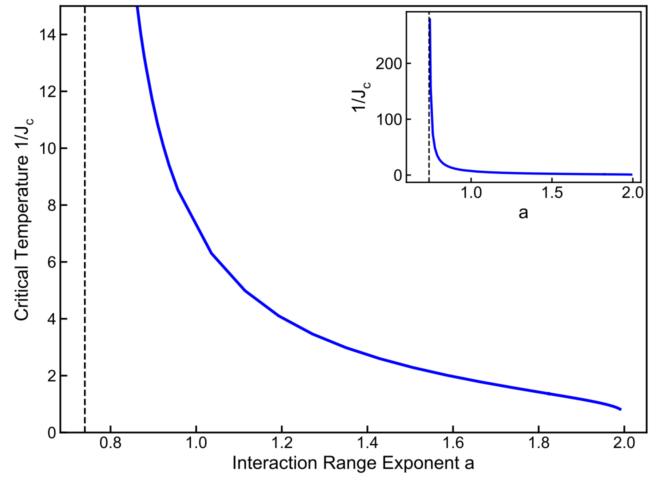

FIG. 2. Calculated phase diagram of the $d = 1$ long-range ferromagnetic Ising model with interactions $J\,r^{-a}$. A finite-temperature ferromagnetic phase occurs for $0.75 < a < 2$. The transition temperature monotonically decreases between these two limits. At $a = 2$, the phase-transition temperature discontinuously drops to zero and for $a > 2$ there is no ordered phase above zero temperature, as predicted by rigorous results. On approaching $a = 0.74$ from above, the phase-transition temperature diverges to infinity, meaning that, at all non-infinite temperatures, the system is ferromagnetically ordered. Thus, the equivalent-neighbor interactions regime is entered before ($a \neq 0$) the neighbors become equivalent, namely before the ineractions become equal for all separations.

III. RESULTS: FINITE-TEMPERATURE FERROMAGNETIC PHASE IN $d = 1$

The calculated phase diagram of the $d = 1$ long-range ferromagnetic Ising model, with interactions $J\,r^{-a}$, is shown in Fig. 2, in terms of temperature $1/J$ and interaction range a. A finite-temperature ferromagnetic phase occurs for $0.74 < a < 2$. The transition temperature monotonically decreases between these two limits. At $a = 2$, the phase-transition temperature discontinuously drops to zero and for $a > 2$ there is no ordered phase above zero temperature, as predicted by rigorous results.[1–5] On approaching $a = 0.74$ from above, the phase-transition temperature diverges to infinity, meaning that, at all non-infinite temperatures, the system is ferromagnetically ordered. Thus, the equivalent-neighbor interactions regime is entered before ($a \neq 0$) the neighbors become equivalent, namely before the ineractions become equal for all separations.

The correlation-length critical exponent ν, correlation-function critical exponent η, specific heat critical exponent α, magnetization critical exponents β and δ, continuously varying as a function of interaction range a for the finite-temperature ferromagnetic phase transition, are shown in Fig. 3. These critical exponents are calculated, with $H = H' = 0$, from the recursion relations $J'_1, ..., J'_n = funct(J_1, ..., J_n)$. Convergence is obtained by calculation up to $n = 5$. The largest (and, as expected, only relevant, namely greater than 1) eigenvalue $\lambda_T = b^{y_T}$ of the derivative matrix of these recursion relations at the critical point gives the correlation-length critical exponent $\nu = 1/y_T$ and the specific heat critical exponent $\alpha = 2 - d/y_T = 2 - 1/y_T$. The magnetization critical exponent $\delta = y_T/(d - y_H)$ and the correlation function critical exponent $\eta = 2 - d - y_H = 1 - y_H$ are calculated, at the critical point, with $H = H' = 0$, from $\partial H'/\partial H = b^{y_H}$.[13]

The antiferromagnetic, overly frustrated without randomness, system does not have a finite-temperature phase transition, but the spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability p), does have finite-temperature spin-glass phase transitions and chaos inside the spin-glass phase, as seen in Fig. 4.

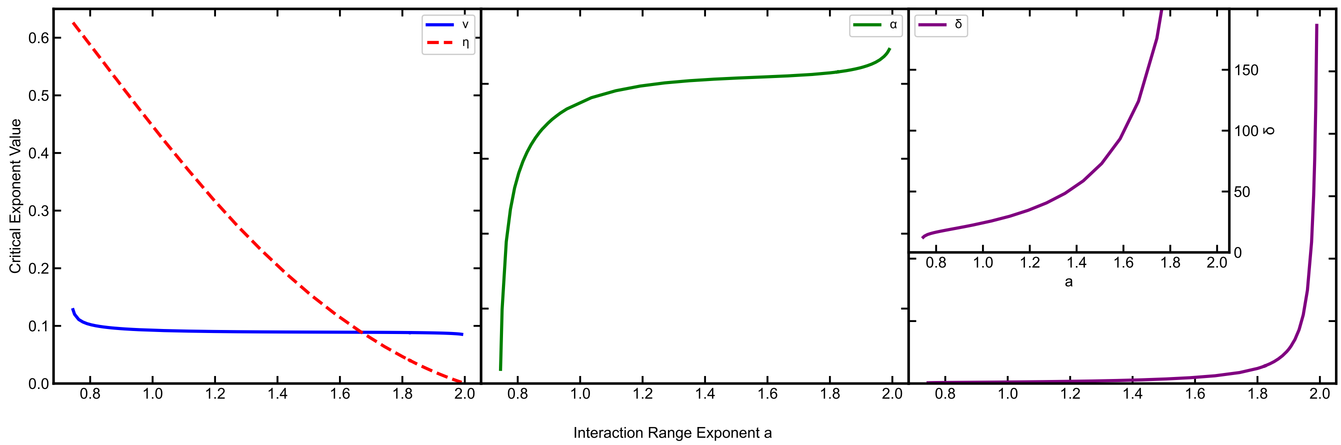

FIG. 3. Correlation-length critical exponent ν, correlation-function critical exponent η, specific heat critical exponent α, magnetization critical exponents β and δ, as a function of interaction range a for the finite-temperature ferromagnetic phase transition.

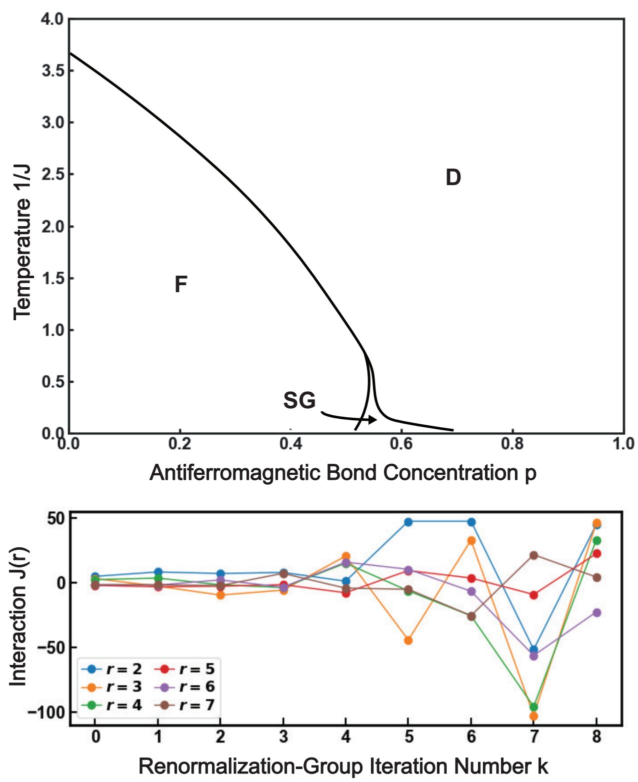

FIG. 4. Calculated finite-temperature phase diagram of the $d = 1$ long-range Ising spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability p). This truly unusual spin-glass phase diagram, actually does not have an antiferromagnetic phase but has a spin-glass phase. Bottom panel: Chaos inside the spin-glass phase in $d = 1$.

IV. RESULTS: FINITE-TEMPERATURE SPIN-GLASS PHASE IN $d = 1$

The spin-glass system, where all couplings for all separations are randomly ferromagnetic or antiferromagnetic (with probability p), does have finite-temperature spin-glass phase transitions and chaos inside the spin-glass phase, as seen in Fig. 4. This truly unusual spin-glass phase diagram, actually does not have an antiferromagnetic phase but has a spin-glass phase.

For a previous $d = 1$ Ising spin-glass study, with short-range interactions and a zero-temperature spin-glass phase, see [14].

V. CONCLUSION

ACKNOWLEDGMENTS

Support by the Academy of Sciences of Turkey (TÜBA) is gratefully acknowledged.

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