Stochastic Adaptive Control

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Assignment available: 10:00 Tuesday, February 11th, 2025

The deadline for handing in the project report is **23:59 Tuesday, March 11th, 2025**. The assignment focuses on the prediction, control, and estimation for ARX models. Each student must submit an individual project report, but it is allowed to collaborate with other students in the course. The report must fulfill the following requirements.

1. The length of the report must not exceed 10 pages.
2. The front page of the report must contain the names of any collaborators (make sure they also have your name on their front page).
3. Each question must be answered with a discussion/argumentation, as well as the equations and any central Matlab commands that you use, e.g., Matlab's roots and how you call it. It is not sufficient to only provide the numerical result. You must demonstrate that you understand how to solve the exercise.

Assignment 1

Consider the discrete-time system

$$\begin{aligned} y_t - 3.88098254y_{t-1} + 5.64726905y_{t-2} - 3.6515443y_{t-3} + 0.88525793y_{t-4} \\ = 0.4453368u_t - 1.71110786u_{t-1} + 2.49838163u_{t-2} - 1.64272524u_{t-3} \\ + 0.41012833u_{t-4} + e_t, \end{aligned} \tag{1}$$

where $e_t \sim N(0, 10^{-10})$.

Hint: It might be tempting to truncate the values when typing them into Matlab, but you should make sure that it doesn't change your conclusions.

Question 1.1

Write the system (1) in the form

$$A(q^{-1})y_t = B(q^{-1})u_t + e_t, \tag{2}$$

What are the coefficients of A and B?

Question 1.2

Write the system (1) in the form

$$y_t = \phi^T\theta + e_t. \tag{3}$$

What are ϕ and θ?

Question 1.3

If you ignore the stochastic term, e_t, in (1), is the system stable? If you consider the system (1) with the stochastic term, does there exist a stationary distribution? Why?

Question 1.4

Make a routine for simulating ARX models in the general form (2) for given A, B, and initial values of u and y. Test it by simulating for the inputs (2^{nd} row) in the file Assignment1_ObservationsAndInputs.txt. The initial outputs should be zero.

Question 1.5

Solve the (simple) Diophantine equation necessary to make output predictions based on (1) for a specific value of m.

Question 1.6

Use the solution to the Diophantine equation from Question 1.5 to predict future outputs, y_{t+m}, assuming that the current and past outputs are known. For every point in time, compare the 100-step prediction with the observations (1^{st} row) in the file Assignment1_ObservationsAndInputs.txt.

Hint: Use the mean absolute error (MAE) for the comparison. It is given by

$$e = \frac{1}{N-m+1}\sum_{t=0}^{N-m} |\hat{y}_{t+m|t} - y_{t+m}|, \tag{4}$$

where e is the MAE, there are $N+1$ observations in total, m is the prediction horizon, $\hat{y}_{t+m|t}$ is the m-step prediction at time t, and y_{t+m} is the true observation.

Question 1.7

Use the solution to the Diophantine equation to design an MV_0 controller. What is the value of the prediction horizon, m? Next, perform a closed-loop simulation for a setpoint that is 1 for the first 100 s, 0 for the next 100 s, and 0.5 for the final 100 s. The initial output and manipulated input, y and u, are zero.

Question 1.8

Design a least-squares estimator for ARX models in the general form (3), and identify the parameters in an ARX model with the same orders as the system in (1) based on the data in the file Assignment1_ObservationsAndInputs.txt. Are the estimated parameters significant? What is the condition number of the variance matrix? Is there any pole-zero cancellation? Is there significant improvement in the objective function if you increase the model order?

Question 1.9

Design a recursive least-squares (RLS) estimator for ARX models in the general form (3), and repeat the estimation from Question 1.8. How do the parameter estimates compare to those you obtained in Question 1.8?