Stochastic Adaptive Control

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Assignment available: 10:00 Monday, April 21st, 2025

The deadline for handing in the project report is **23:59 Tuesday, May 6th, 2025**. The assignment focuses on the state and parameter estimation, feedback control, and adaptive control for linear state space models. Each student must submit an individual project report, but it is allowed to collaborate with other students in the course. The report must fulfill the following requirements.

1. The length of the report must not exceed 10 pages.
2. The front page of the report must contain the names of any collaborators (make sure they also have your name on their front page).
3. Each question must be answered with a discussion/argumentation, as well as the equations and any central Matlab commands that you use, e.g., Matlab's idare and how you call it. It is not sufficient to only provide the numerical result. You must demonstrate that you understand how to solve the exercise.

Assignment 3

Consider a system in the form

$$x_{t+1} = Ax_t + Bu_t + v_t, \qquad v_t \sim N(0, R_1), \tag{1a}$$

$$y_t = Cx_t + e_t, \qquad e_t \sim N(0, R_2), \tag{1b}$$

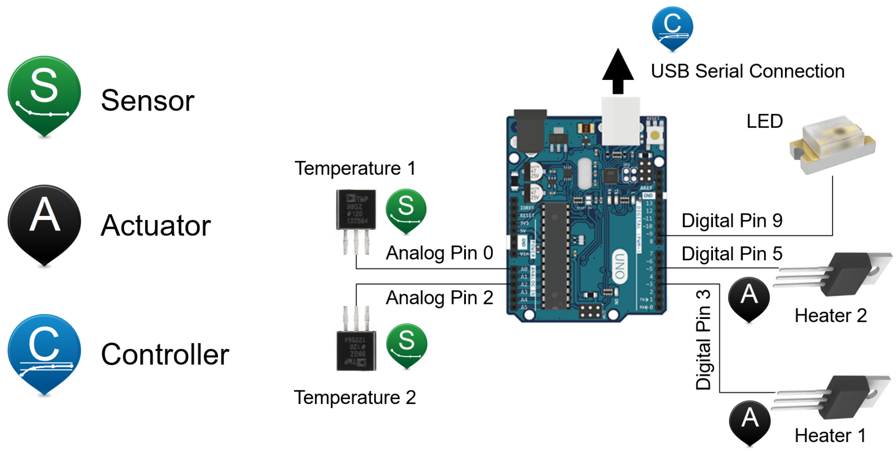

Figure 1: The temperature control lab (TCLab) device.

where the system matrices are

$$A = \begin{bmatrix} 0.7901 & 0.0238 & 0.1667 & 0.0081 \\ 0.0238 & 0.7901 & 0.0081 & 0.1667 \\ 0.1667 & 0.0081 & 0.7236 & 0.0014 \\ 0.0081 & 0.1667 & 0.0014 & 0.7236 \end{bmatrix}, \quad B = \begin{bmatrix} 1.4459 & 0.0622 \\ 0.0622 & 1.4459 \\ 13.3615 & 0.0077 \\ 0.0077 & 13.3615 \end{bmatrix}, \tag{2a}$$

$$R_1 = \begin{bmatrix} 8.0209 & 0.2260 & 1.5334 & 0.0812 \\ 0.2260 & 8.0209 & 0.0812 & 1.5334 \\ 1.5334 & 0.0812 & 7.4082 & 0.0172 \\ 0.0812 & 1.5334 & 0.0172 & 7.4082 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad R_2 = I \tag{2b}$$

This is a discretized model of the Temperature Control Lab device[1] shown in Fig. 1. It is modeled as shown in Fig. 2, and the state variables are the temperature deviations, T_i^r for $i = 1, \ldots, 4$, from the ambient temperature, T_a, for the four compartments. We can only measure the temperature deviations T_1^r and T_2^r and only every 10 seconds. The manipulated inputs represent the electrical heating of compartment 3 and 4.

Question 1.1

Design a stationary ordinary Kalman filter for the discrete-time system (1). What is the stationary Kalman filter gain, κ? Simulate the true system and use the measurements in the Kalman filter to estimate the states. Are you able to estimate the unmeasured states?

[1] See `tclab.readthedocs.io` for more information.

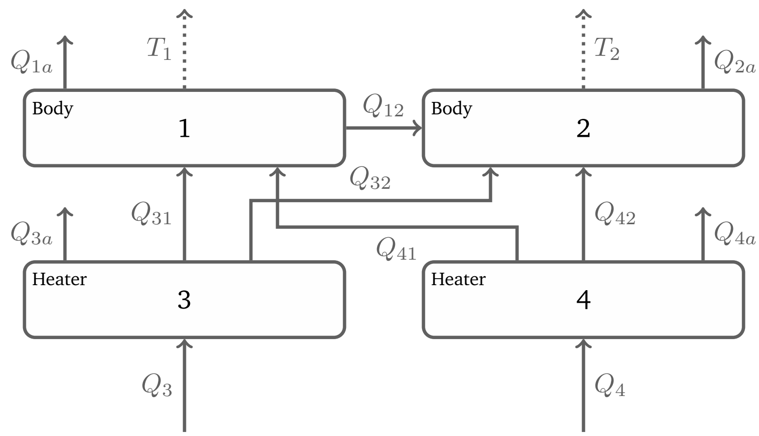

Figure 2: Four-compartment model of TCLab device.

Question 1.2

Design a general predictive control (GPC) algorithm and combine it with the stationary Kalman filter from Question 1.2. Test it using a square wave setpoint.

Question 1.3

In this question, we will design a disturbance estimation algorithm for the system

$$x_{t+1} = Ax_t + Bu_t + d + v_t, \qquad v_t \sim N(0, R_1), \tag{3a}$$

$$y_t = Cx_t + e_t, \qquad e_t \sim N(0, R_2). \tag{3b}$$

Note that the only difference from (1a) is the disturbance, d.

Step 1: Introduce the auxiliary model of the disturbance

$$d_{t+1} = \kappa d_t + \eta_t, \qquad \eta_t \sim N(0, \sigma_d^2 I), \tag{4}$$

and augment the state vector with the disturbance. Specifically, write the system in the form

$$\begin{bmatrix} x_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \times & \times \\ \times & \times \end{bmatrix} \begin{bmatrix} x_t \\ d_t \end{bmatrix} + \begin{bmatrix} \times \\ \times \end{bmatrix} u_t + \begin{bmatrix} \times & \times \\ \times & \times \end{bmatrix} \begin{bmatrix} v_t \\ \eta_t \end{bmatrix}. \tag{5}$$

Fill out the blanks.

Step 2: Implement a stationary Kalman filter for the augmented system (5) in order to estimate both x and d.

Test the algorithm by increasing the “true” value of d from zero to a positive value and see if the algorithm is able to track the change.

Hint: You need to choose values of $\kappa \in (0, 1)$ (i.e., strictly between 0 and 1) and σ_d^2. The closer you choose κ to 0, the quicker your estimate will decay to zero, and the higher you choose σ_d^2, the more aggressive the estimation of d will be.

Question 1.4

Extend the GPC algorithm from Question 1.2 to the augmented system (5) and combine it with the state and disturbance estimation algorithm from Question 1.3. Again, increase the “true” value of d and test the algorithm for a square wave setpoint. Is the controller able to both estimate d and control the system?