Stochastic Adaptive Control

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Assignment available: 10:00 Wednesday, March 12th, 2025

The deadline for handing in the project report is **23:59 Tuesday, April 8th, 2025**. The assignment focuses on the prediction, control, and estimation for ARX models. Each student must submit an individual project report, but it is allowed to collaborate with other students in the course. The report must fulfill the following requirements.

1. The length of the report must not exceed 10 pages.
2. The front page of the report must contain the names of any collaborators (make sure they also have your name on their front page).
3. Each question must be answered with a discussion/argumentation, as well as the equations and any central Matlab commands that you use, e.g., Matlab's idare and how you call it. It is not sufficient to only provide the numerical result. You must demonstrate that you understand how to solve the exercise.

Assignment 2

Part I

Consider the discrete-time system from Assignment 1,

$$y_t - 2.438 y_{t-1} + 1.942 y_{t-2} - 0.502 y_{t-3} = 1.446 u_{t-1} - 0.158 u_{t-2} - 1.093 u_{t-3} + e_t, \tag{1}$$

where $e_t \sim N(0, 10^{-2})$.

Question 1.1

Solve the Diophantine equation, and design an MV_0 controller. What is the value of the prediction horizon, m? Next, perform a closed-loop simulation for a square wave setpoint.

Question 1.2

Combine the MV_0 controller from Question 1.1 with a recursive least-squares (RLS) estimator into a stochastic adaptive control algorithm. How do you implement closed-loop simulation of an adaptive control algorithm?

Test the controller using the true parameters and small covariances as initial estimates in the RLS estimator. Furthermore, use a square wave setpoint and plot J_r, J_u, and J_e together with their theoretical values.

Question 1.3

Repeat the test from the previous question where the initial estimates of the parameters are all ones and use a large value (e.g., 10) times an identity matrix as the initial estimate of the covariance matrix.

Part II

Consider the discrete-time system

$$\begin{aligned} y_t - 2.438y_{t-1} + 1.942y_{t-2} - 0.502y_{t-3} = 1.446u_{t-1} - 0.158u_{t-2} - 1.093u_{t-3} \\ + e_t - 1.441e_{t-1} + 0.296e_{t-2} + 0.167e_{t-3}, \end{aligned} \tag{2}$$

where $e_t \sim N(0, 10^{-2})$.

Question 2.1

Write the system (2) in the form

$$A(q^{-1})y_t = B(q^{-1})u_t + C(q^{-1})e_t, \tag{3}$$

What are the coefficients of C?

Question 2.2

Make a routine for simulating ARMAX models in the form (3), and test it by simulating for a PRBS input sequence. The initial inputs and outputs should be zero.

Question 2.3

Solve the Diophantine equation necessary for developing general minimum variance (GMV) controllers. What should the prediction horizon be?

Question 2.4

Make a routine that determines the polynomials, Q, R, and S in the control law

$$R(q^{-1})u_t = Q(q^{-1})w_t - S(q^{-1})y_t, \tag{4}$$

for a generalized minimum variance strategy, and use it to determine the polynomials for an MV_{1a} controller. Test the controller for a square wave setpoint.

Question 2.5

Write up the system in the form

$$y_t = \phi_t^T \theta + e_t \tag{5}$$

What are ϕ_t and θ?

Question 2.6

Design a recursive extended least-squares (RELS) estimator. Use the routine from Question 2.2 to create simulated measurements for a PRBS sequence of manipulated inputs. Are you able to estimate the parameters or do you need to consider some of the heuristics covered in class, e.g., forgetting factors or a model estimator?

Question 2.7

Regardless of how the RELS estimator performed in the previous question, combine it with the MV_{1a} controller designed in Question 2.4, and test it on the system in (2). Use the true parameters and small covariances as initial estimates in the RELS estimator, and use the same setpoint as in Question 2.4. Plot J_r, J_u, and J_e together with their theoretical values.

Question 2.8

Repeat the test from the previous question where the initial estimates of the parameters are all ones and use a large value (e.g., 10) times an identity matrix as the initial estimate of the covariance matrix.