一 四则运算文法

1.1 文法

$$
\begin{aligned}
G[E]: & E \to E+T|E-T|T \\
& T \to T \times F|T \div F|F \\
& F \to (E)|n
\end{aligned}
\quad \overset{\text{拓广文法}}{\Rightarrow} \quad
\begin{aligned}
G[\mathrm{S}']: & S' \to E \\
& E \to E+T|E-T|T \\
& T \to T \times F|T \div F|F \\
& F \to (E)|n
\end{aligned}
$$

1.2 非终结符 FIRST 集

$\mathrm{FIRST}(E) = \{(, n\}$
$\mathrm{FIRST}(T) = \{(, n\}$
$\mathrm{FIRST}(F) = \{(, n\}$

1.3 项目集规范族与预测分析表

S_0 : $S' \to \cdot E$, # $E \to \cdot E+T$, #+− $E \to \cdot E-T$, #+− $E \to \cdot T$, #+− $T \to \cdot T \times F$, #+−×÷ $T \to \cdot T \div F$, #+−×÷ $T \to \cdot F$, #+−×÷ $F \to \cdot (E)$, #+−×÷ $F \to \cdot n$, #+−×÷	S_1 : $S' \to E \cdot$, # $E \to E \cdot +T$, #+− $E \to E \cdot -T$, #+−	S_2 : $E \to T \cdot$, #+− $T \to T \cdot \times F$, #+−×÷ $T \to T \cdot \div F$, #+−×÷
S_3 : $T \to F \cdot$, #+−×÷	S_4 : $F \to (\cdot E)$, #+−×÷ $E \to \cdot E+T$,)+− $E \to \cdot E-T$,)+− $E \to \cdot T$,)+− $T \to \cdot T \times F$,)+−×÷ $T \to \cdot T \div F$,)+−×÷ $T \to \cdot F$,)+−×÷ $F \to \cdot (E)$,)+−×÷ $F \to \cdot n$,)+−×÷	S_5 : $F \to n \cdot$, #+−×÷
S_6 : $F \to (E \cdot)$, #+−×÷ $E \to E \cdot +T$,)+− $E \to E \cdot -T$,)+−	S_7 : $E \to T \cdot$,)+− $T \to T \cdot \times F$,)+−×÷ $T \to T \cdot \div F$,)+−×÷	S_8 : $T \to F \cdot$,)+−×÷

S_9 : $F \to (\cdot E)$,)+−×÷ $E \to \cdot E + T$,)+− $E \to \cdot E - T$,)+− $E \to \cdot T$,)+− $T \to \cdot T \times F$,)+−×÷ $T \to \cdot T \div F$,)+−×÷ $T \to \cdot F$,)+−×÷ $F \to \cdot (E)$,)+−×÷ $F \to \cdot n$,)+−×÷	S_{10} : $F \to n \cdot$,)+−×÷	S_{11} : $F \to (E \cdot)$,)+−×÷ $E \to E \cdot + T$,)+− $E \to E \cdot - T$,)+−
S_{12} : $F \to (E) \cdot$,)+−×÷	S_{13} : $E \to E + \cdot T$,)+− $T \to \cdot T \times F$,)+−×÷ $T \to \cdot T \div F$,)+−×÷ $T \to \cdot F$,)+−×÷ $F \to \cdot (E)$,)+−×÷ $F \to \cdot n$,)+−×÷	S_{14} : $E \to E - \cdot T$,)+− $T \to \cdot T \times F$,)+−×÷ $T \to \cdot T \div F$,)+−×÷ $T \to \cdot F$,)+−×÷ $F \to \cdot (E)$,)+−×÷ $F \to \cdot n$,)+−×÷
S_{15} : $E \to E - T \cdot$,)+− $T \to T \cdot \times F$,)+−×÷ $T \to T \cdot \div F$,)+−×÷	S_{16} : $T \to T \times \cdot F$,)+−×÷ $F \to \cdot (E)$,)+−×÷ $F \to \cdot n$,)+−×÷	S_{17} : $T \to T \div \cdot F$,)+−×÷ $F \to \cdot (E)$,)+−×÷ $F \to \cdot n$,)+−×÷
S_{18} : $T \to T \div F \cdot$,)+−×÷	S_{19} : $T \to T \times F \cdot$,)+−×÷	S_{20} : $E \to E + T \cdot$,)+− $T \to T \cdot \times F$,)+−×÷ $T \to T \cdot \div F$,)+−×÷
S_{21} : $F \to (E) \cdot$,#+−×÷	S_{22} : $T \to T \times \cdot F$,#+−×÷ $F \to \cdot (E)$,#+−×÷ $F \to \cdot n$,#+−×÷	S_{23} : $T \to T \div \cdot F$,#+−×÷ $F \to \cdot (E)$,#+−×÷ $F \to \cdot n$,#+−×÷
S_{24} : $T \to T \div F \cdot$,#+−×÷	S_{25} : $T \to T \times F \cdot$,#+−×÷	S_{26} : $E \to E + \cdot T$,#+− $T \to \cdot T \times F$,#+−×÷ $T \to \cdot T \div F$,#+−×÷ $T \to \cdot F$,#+−×÷ $F \to \cdot (E)$,#+−×÷ $F \to \cdot n$,#+−×÷
S_{27} : $E \to E - \cdot T$,#+− $T \to \cdot T \times F$,#+−×÷ $T \to \cdot T \div F$,#+−×÷ $T \to \cdot F$,#+−×÷ $F \to \cdot (E)$,#+−×÷ $F \to \cdot n$,#+−×÷	S_{28} : $E \to E - T \cdot$,#+− $T \to T \cdot \times F$,#+−×÷ $T \to T \cdot \div F$,#+−×÷	S_{29} : $E \to E + T \cdot$,#+− $T \to T \cdot \times F$,#+−×÷ $T \to T \cdot \div F$,#+−×÷

表 1　LR(1) 项目集表

状态	$ACTION$								$GOTO$		
	$+$	$-$	$\times$	$\div$	$($	$)$	n	$\#$	E	T	F
0					S_4		S_5		1	2	3
1	S_{26}	S_{27}						ACC			
2	$R_{E\text{-}2}$	$R_{E\text{-}2}$	S_{22}	S_{23}				$R_{E\text{-}2}$			
3	$R_{T\text{-}2}$	$R_{T\text{-}2}$	$R_{T\text{-}2}$	$R_{T\text{-}2}$				$R_{T\text{-}2}$			
4					S_9		S_{10}		6	7	8
5	$R_{F\text{-}1}$	$R_{F\text{-}1}$	$R_{F\text{-}1}$	$R_{F\text{-}1}$				$R_{F\text{-}1}$			
6	S_{13}	S_{14}				S_{21}					
7	$R_{E\text{-}2}$	$R_{E\text{-}2}$	S_{16}	S_{17}		$R_{E\text{-}2}$					
8	$R_{T\text{-}2}$	$R_{T\text{-}2}$	$R_{T\text{-}2}$	$R_{T\text{-}2}$		$R_{T\text{-}2}$					
9					S_9		S_{10}		11	7	8
10	$R_{F\text{-}1}$	$R_{F\text{-}1}$	$R_{F\text{-}1}$	$R_{F\text{-}1}$		$R_{F\text{-}1}$					
11	S_{13}	S_{14}				S_{12}					
12	$R_{F\text{-}0}$	$R_{F\text{-}0}$	$R_{F\text{-}0}$	$R_{F\text{-}0}$		$R_{F\text{-}0}$					
13					S_9		S_{10}			20	8
14					S_9		S_{10}			15	8
15	$R_{E\text{-}1}$	$R_{E\text{-}1}$	S_{16}	S_{17}		$R_{E\text{-}1}$					
16					S_9		S_{10}				19
17					S_9		S_{10}				18
18	$R_{T\text{-}1}$	$R_{T\text{-}1}$	$R_{T\text{-}1}$	$R_{T\text{-}1}$		$R_{T\text{-}1}$					
19	$R_{T\text{-}0}$	$R_{T\text{-}0}$	$R_{T\text{-}0}$	$R_{T\text{-}0}$		$R_{T\text{-}0}$					
20	$R_{E\text{-}0}$	$R_{E\text{-}0}$	S_{16}	S_{17}		$R_{E\text{-}0}$					
21	$R_{F\text{-}0}$	$R_{F\text{-}0}$	$R_{F\text{-}0}$	$R_{F\text{-}0}$				$R_{F\text{-}0}$			
22					S_4		S_5				25
23					S_4		S_5				24
24	$R_{T\text{-}1}$	$R_{T\text{-}1}$	$R_{T\text{-}1}$	$R_{T\text{-}1}$				$R_{T\text{-}1}$			
25	$R_{T\text{-}0}$	$R_{T\text{-}0}$	$R_{T\text{-}0}$	$R_{T\text{-}0}$				$R_{T\text{-}0}$			
26					S_4		S_5			29	3
27					S_4		S_5			28	3
28	$R_{E\text{-}1}$	$R_{E\text{-}1}$	S_{22}	S_{23}				$R_{E\text{-}1}$			
29	$R_{E\text{-}0}$	$R_{E\text{-}0}$	S_{22}	S_{23}				$R_{E\text{-}0}$			

表 2　LR(1) 预测分析表

1.4 分析过程

步骤	状态栈	符号栈	输入串	$ACTION$	$GOTO$
1	0	$\#$	$(n+n)\times n-n\div n\#$	S_4	
2	0,4	$\#($	$n+n)\times n-n\div n\#$	S_{10}	
3	0,4,10	$\#(n$	$+n)\times n-n\div n\#$	$R_{F\text{-}1}$	8
4	0,4,8	$\#(F$	$+n)\times n-n\div n\#$	$R_{T\text{-}2}$	7

步骤	状态栈	符号栈	输入串	$ACTION$	$GOTO$
5	0,4,7	$\#(T$	$+n)\times n-n\div n\#$	$R_{E\text{-}2}$	6
6	0,4,6	$\#(E$	$+n)\times n-n\div n\#$	S_{13}	
7	0,4,6,13	$\#(E+$	$n)\times n-n\div n\#$	S_{10}	
8	0,4,6,13,10	$\#(E+n$	$)\times n-n\div n\#$	$R_{F\text{-}1}$	8
9	0,4,6,13,8	$\#(E+F$	$)\times n-n\div n\#$	$R_{T\text{-}2}$	20
10	0,4,6,13,20	$\#(E+T$	$)\times n-n\div n\#$	$R_{E\text{-}0}$	6
11	0,4,6	$\#(E$	$)\times n-n\div n\#$	S_{21}	
12	0,4,6,21	$\#(E)$	$\times n-n\div n\#$	$R_{F\text{-}0}$	3
13	0,3	$\#F$	$\times n-n\div n\#$	$R_{T\text{-}2}$	2
14	0,2	$\#T$	$\times n-n\div n\#$	S_{22}	
15	0,2,22	$\#T\times$	$n-n\div n\#$	S_5	
16	0,2,22,5	$\#T\times n$	$-n\div n\#$	$R_{F\text{-}1}$	25
17	0,2,22,25	$\#T\times F$	$-n\div n\#$	$R_{T\text{-}0}$	2
18	0,2	$\#T$	$-n\div n\#$	$R_{E\text{-}2}$	1
19	0,1	$\#E$	$-n\div n\#$	S_{27}	
20	0,1,27	$\#E-$	$n\div n\#$	S_5	
21	0,1,27,5	$\#E-n$	$\div n\#$	$R_{F\text{-}1}$	3
22	0,1,27,3	$\#E-F$	$\div n\#$	$R_{T\text{-}2}$	28
23	0,1,27,28	$\#E-T$	$\div n\#$	S_{23}	
24	0,1,27,28,23	$\#E-T\div$	$n\#$	S_5	
25	0,1,27,28,23,5	$\#E-T\div n$	$\#$	$R_{F\text{-}1}$	24
26	0,1,27,28,23,24	$\#E-T\div F$	$\#$	$R_{T\text{-}1}$	28
27	0,1,27,28	$\#E-T$	$\#$	$R_{E\text{-}1}$	1
28	0,1	$\#E$	$\#$	ACC	

表 3　$(n+n)\times n-n\div n$ 的 LR(1) 分析过程

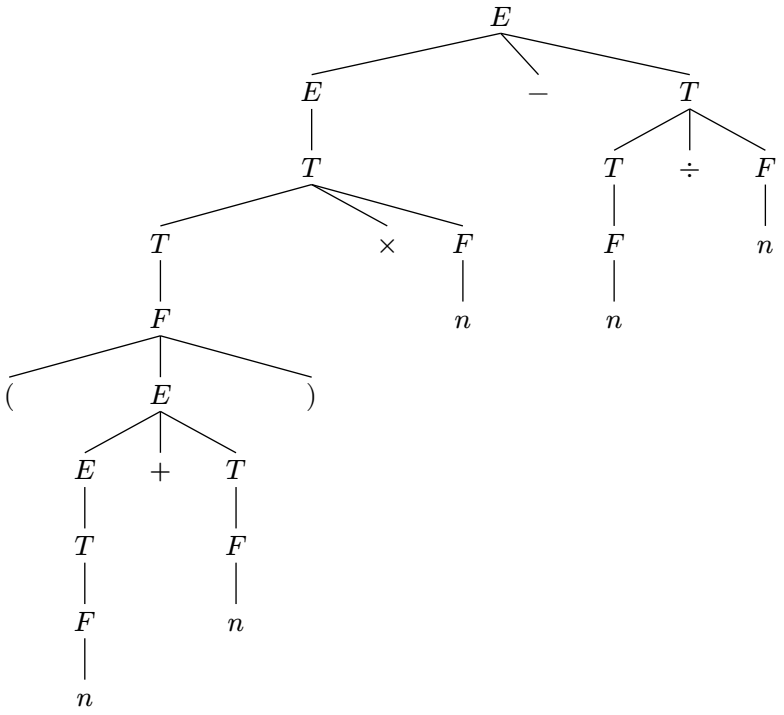

图 1　(n+n)×n−n÷n 的 LR(1) 语法树

二 非 LR1 文法

2.1 文法

$$
\begin{aligned}
G[S]: S &\to Aa|Bb \\
A &\to Ab|\varepsilon|b \\
B &\to Ba|\varepsilon
\end{aligned}
\quad \overset{\text{拓广文法}}{\Rightarrow} \quad
\begin{aligned}
G[\mathrm{S}']: S' &\to S \\
S &\to Aa|Bb \\
A &\to Ab|\varepsilon|b \\
B &\to Ba|\varepsilon
\end{aligned}
$$

2.2 非终结符 FIRST 集

$$\mathrm{FIRST}(S) = \{b, a\}$$
$$\mathrm{FIRST}(A) = \{\varepsilon, b\}$$
$$\mathrm{FIRST}(B) = \{\varepsilon, a\}$$

2.3 项目集规范族与预测分析表

S_0 : $S' \to \cdot S$, # $S \to \cdot Aa$, # $S \to \cdot Bb$, # $A \to \cdot Ab$, ab $A \to \cdot$, ab $A \to \cdot b$, ab $B \to \cdot Ba$, ba $B \to \cdot$, ba	S_1 : $S' \to S \cdot$, #	S_2 : $S \to A \cdot a$, # $A \to A \cdot b$, ab
S_3 : $S \to B \cdot b$, # $B \to B \cdot a$, ba	S_4 : $A \to b \cdot$, ab	S_5 : $S \to Bb \cdot$, #
S_6 : $B \to Ba \cdot$, ba	S_7 : $S \to Aa \cdot$, #	S_8 : $A \to Ab \cdot$, ab

表 4　LR(1) 项目集表

状态	$ACTION$			$GOTO$		
	a	b	#	S	A	B
0	$R_{A\text{-}1}$,$R_{B\text{-}1}$	S_4,$R_{A\text{-}1}$,$R_{B\text{-}1}$		1	2	3
1			ACC			
2	S_7	S_8				
3	S_6	S_5				
4	$R_{A\text{-}2}$	$R_{A\text{-}2}$				
5			$R_{S\text{-}1}$			
6	$R_{B\text{-}0}$	$R_{B\text{-}0}$				
7			$R_{S\text{-}0}$			
8	$R_{A\text{-}0}$	$R_{A\text{-}0}$				

表 5　LR(1) 预测分析表

!!! 该文法非 LR1 文法