Knowledge Representation and Reasoning

Exercise Sheet 12

Albert-Ludwigs-Universität Freiburg

Bernhard Nebel, Gregor Behnke, Thorsten Engesser, Rolf-David Bergdoll,
Leonardo Mieschendahl, Johannes Herrmann

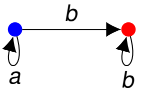
January 28th, 2022

Exercise 12.1 (LTL-Formulas and Büchi automata, 2+2)
Find equivalent Büchi automata for the following LTL formulas. No proof is required and you don't have to use the systematic construction from the lecture via alternating automata. Use at most 3 states for each automaton.

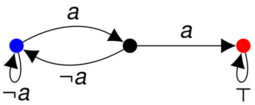
(a) $aUGb$

(b) $F(a \wedge Xa)$

(a) $aUGb$



(b) $F(a \wedge Xa)$



Exercise 12.2 (LTL-Formulas and alternating automata, 2+2)

Consider the following LTL formula:

$$\phi = G(a \to Fb)$$

(a) Use the construction from the lecture to construct an alternating automaton A_ϕ equivalent to ϕ.

(b) Depict the start of an accepting run for the path $\{a\}\{b\}(\{a\}\{b\})^\omega$. It should be clear that you can continue the run indefinitely.

The set of states is $Q = \{\top, \bot, q_{\neg a}, q_b, q_{Fb}, q_\phi\}$

The transition function δ is given as follows:

$$\begin{aligned}
\delta(q_{\neg a}, x) &= \Delta(\neg a, x) = a \notin x \\
\delta(q_b, x) &= \Delta(b, x) = b \in x \\
\delta(q_{Fb}, x) &= \Delta(Fb, x) \\
&= \Delta(b, x) \vee q_{Fb} \\
&= (b \in x) \vee q_{Fb}
\end{aligned}$$

$$\begin{aligned}
\delta(q_\phi, x) &= \Delta(\phi, x) \quad [\phi \equiv G(\neg a \vee Fb)] \\
&= \Delta(\neg a \vee Fb, x) \wedge q_\phi \\
&= (\Delta(\neg a, x) \vee \Delta(Fb, x)) \wedge q_\phi \\
&= ((a \notin x) \vee \Delta(b, x) \vee q_{Fb}) \wedge q_\phi \\
&= ((a \notin x) \vee (b \in x) \vee q_{Fb}) \wedge q_\phi
\end{aligned}$$

The initial states are $I = \{q_\phi\}$ and the goals are $F = \{\top, q_\phi\}$.

Exercise 12.2 (b)

LTL-Formulas and Alternating Automata

Given: $\phi = G(a \rightarrow Fb)$

The set of states is $Q = \{\top, \bot, q_{\neg a}, q_b, q_{Fb}, q_\phi\}$

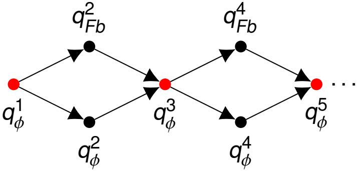
The transition function δ is given as follows:

$$\begin{aligned}
\delta(q_{\neg a}, x) &= \Delta(\neg a, x) = a \notin x \\
\delta(q_b, x) &= \Delta(b, x) = b \in x \\
\delta(q_{Fb}, x) &= \Delta(Fb, x) \\
&= \Delta(b, x) \vee q_{Fb} \\
&= (b \in x) \vee q_{Fb}
\end{aligned}$$

$$\begin{aligned}
\delta(q_\phi, x) &= \Delta(\phi, x) \quad [\phi \equiv G(\neg a \vee Fb)] \\
&= \Delta(\neg a \vee Fb, x) \wedge q_\phi \\
&= (\Delta(\neg a, x) \vee \Delta(Fb, x)) \wedge q_\phi \\
&= ((a \notin x) \vee \Delta(b, x) \vee q_{Fb}) \wedge q_\phi \\
&= ((a \notin x) \vee (b \in x) \vee q_{Fb}) \wedge q_\phi
\end{aligned}$$

The initial states are $I = \{q_\phi\}$ and the goals are $F = \{\top, q_\phi\}$.

We get the following accepting run for $\{a\}\{b\}(\{a\}\{b\})^\omega$



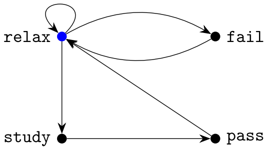
$$\begin{aligned}
\delta(q_\phi, \{a\}) &= (\bot \vee \bot \vee q_{Fb}) \wedge q_\phi \equiv q_{Fb} \wedge q_\phi \\
\delta(q_\phi, \{b\}) &= (\top \vee \top \vee q_{Fb}) \wedge q_\phi \equiv q_\phi \\
\delta(q_{Fb}, \{b\}) &= \top \vee q_{Fb} \equiv \top
\end{aligned}$$

Exercise 12.3

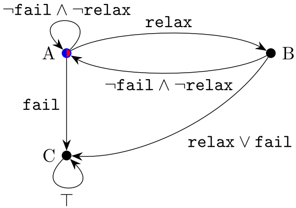
LTL-Formulas and Alternating automata

Exercise 12.3 (Büchi automata and transition systems, 2+3+1)
Consider the transition system $\mathcal{T}$ and the LTL formula $\phi = G(\neg\texttt{fail} \wedge (\texttt{relax} \rightarrow X\neg\texttt{relax}))$ which has the equivalent Büchi automaton A^B_ϕ:

Transition system $\mathcal{T}$ with $\forall q : \tau(q) = \{q\}$:



Büchi automaton A^B_ϕ:



(a) Construct the Büchi automaton $A^B_\mathcal{T}$ equivalent to $\mathcal{T}$.

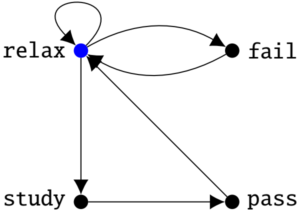
(b) Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$. You may omit unreachable states.

(c) Find an accepting run for $A^B_{\phi\mathcal{T}}$ (no proof required).

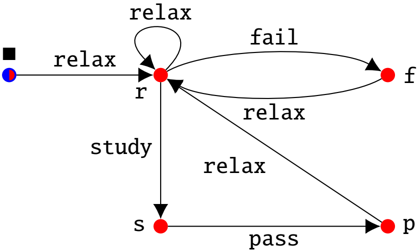
Exercise 12.3 (a)

Construct the Büchi automaton $A^B_{\mathcal{T}}$ equivalent to $\mathcal{T}$

Transition system $\mathcal{T}$:



Büchi automaton $A^B_{\mathcal{T}}$:



Construction rules for $A^B_{\mathcal{T}} = (Q^{\mathcal{T}}, \mathcal{A}, \delta^{\mathcal{T}}, I^{\mathcal{T}}, F^{\mathcal{T}})$:

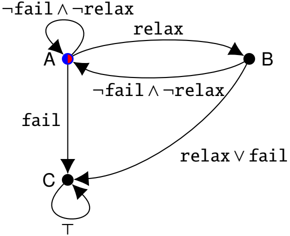
$$Q^{\mathcal{T}} = \{\blacksquare\} \cup Q \qquad \delta^{\mathcal{T}}(\blacksquare, \tau(q_i)) = \{q_i\}$$

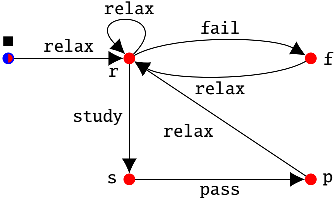
$$I^{\mathcal{T}} = \blacksquare \qquad \delta^{\mathcal{T}}(q, a) = \{q' \mid (q, q') \in R, \tau(q') = a\} \quad \forall q \in Q$$

$$F^{\mathcal{T}} = Q^{\mathcal{T}}$$

Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$

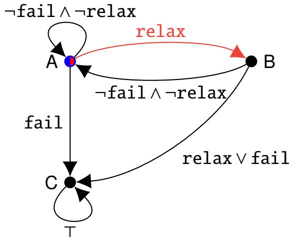


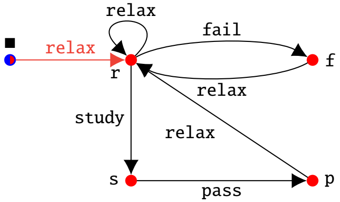


Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:

Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$

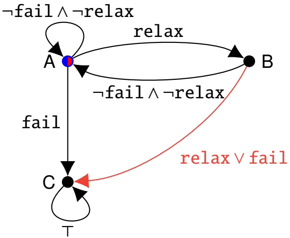


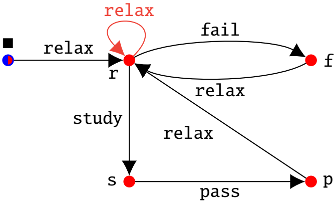


Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:

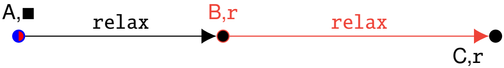
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



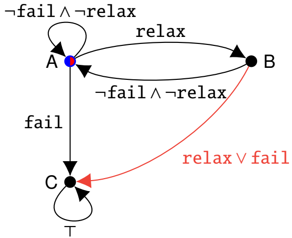


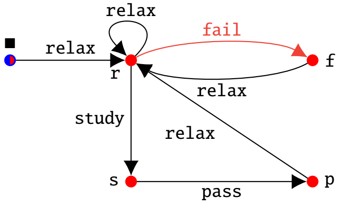
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



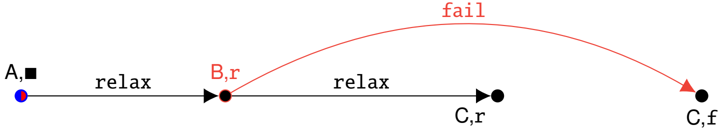
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



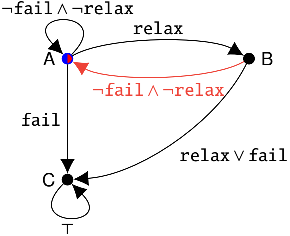


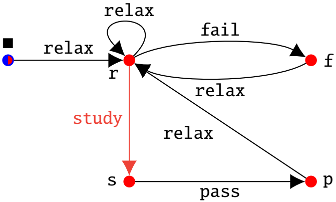
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



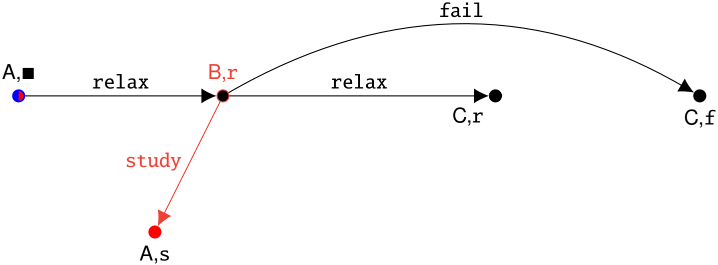
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$

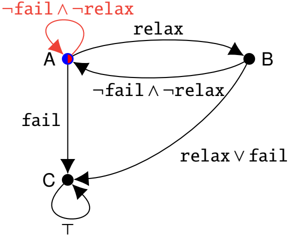


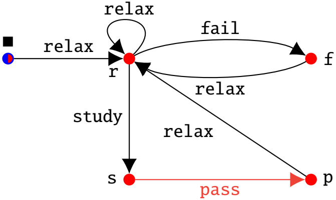


Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:

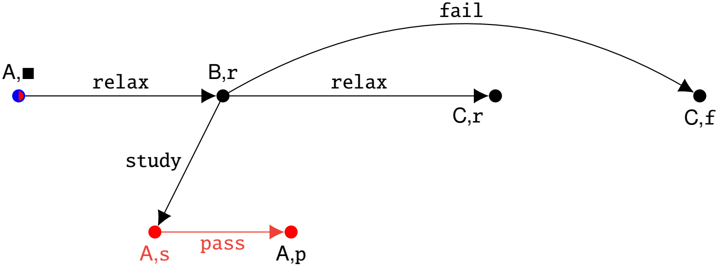


Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



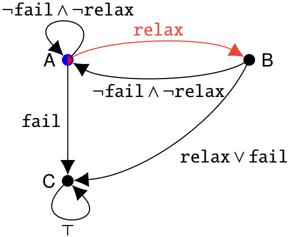


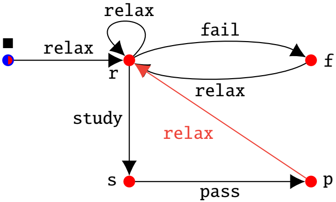
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



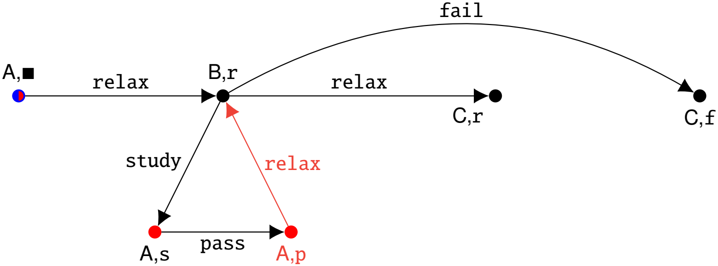
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



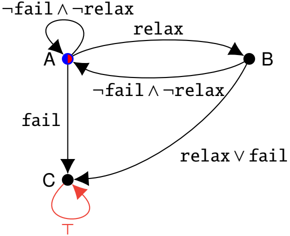


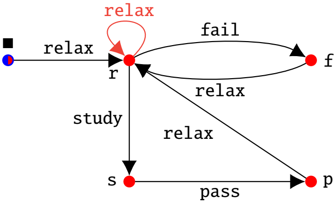
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



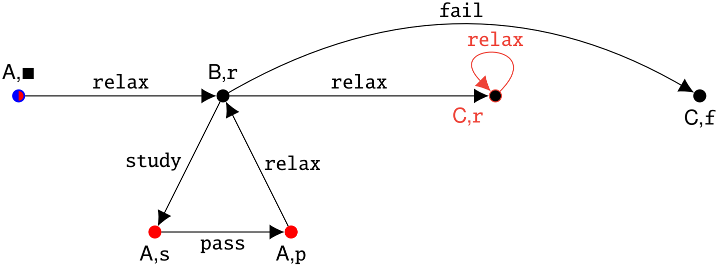
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



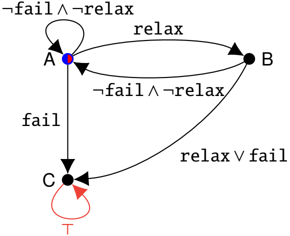


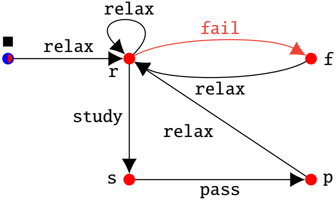
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



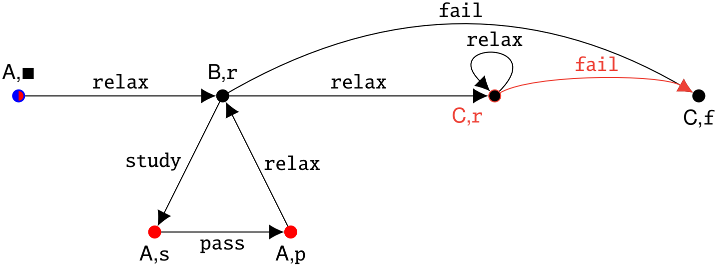
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



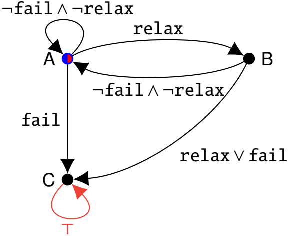


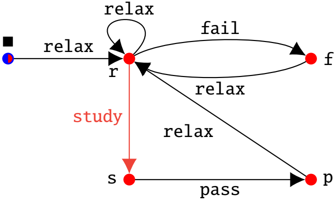
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



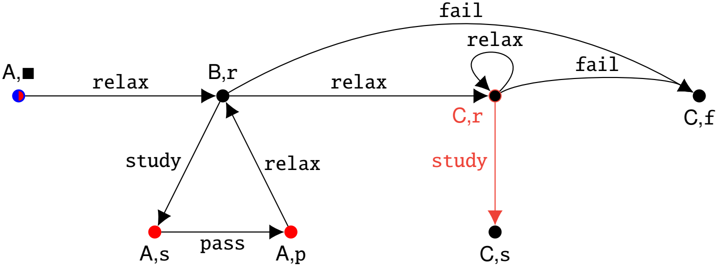
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



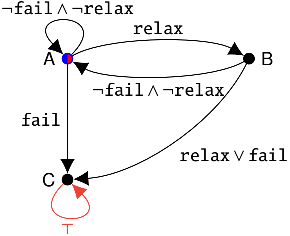


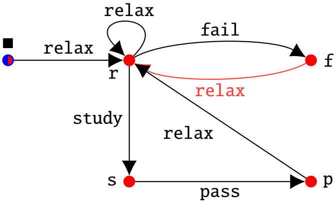
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



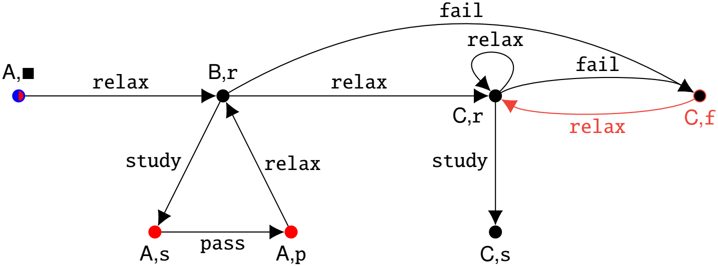
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



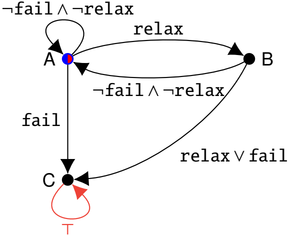


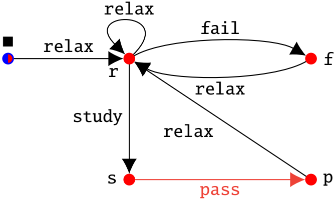
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



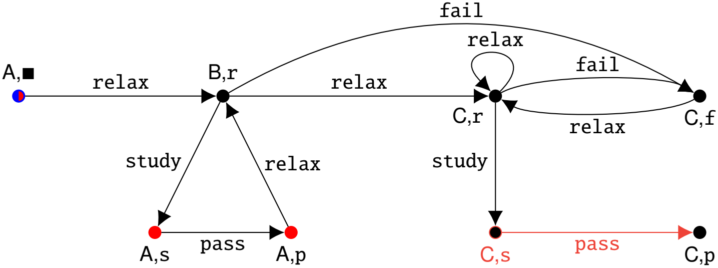
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



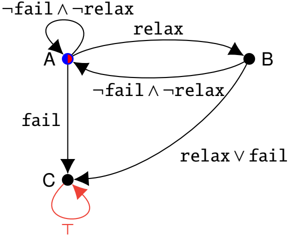


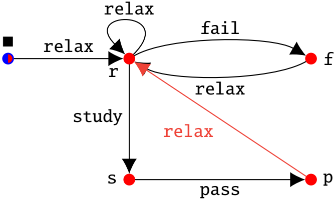
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



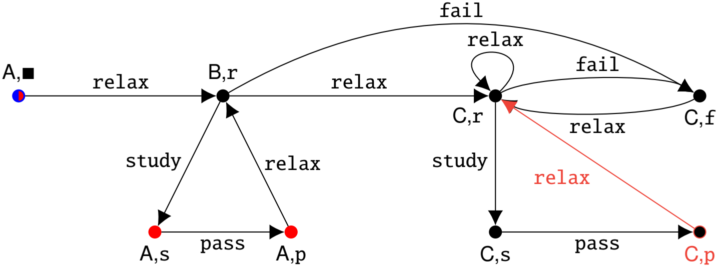
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



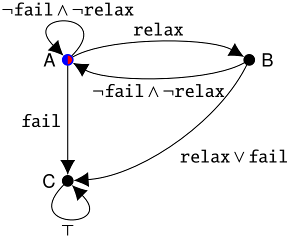


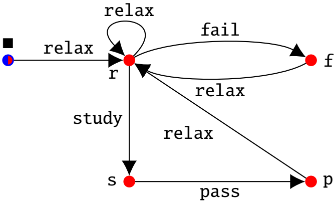
Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



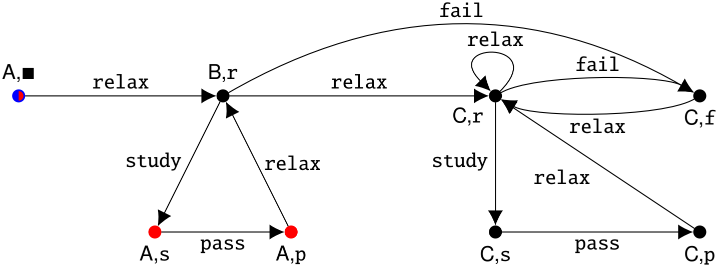
Exercise 12.3 (b)

Construct the product automaton $A^B_{\phi\mathcal{T}}$ of A^B_ϕ and $A^B_\mathcal{T}$



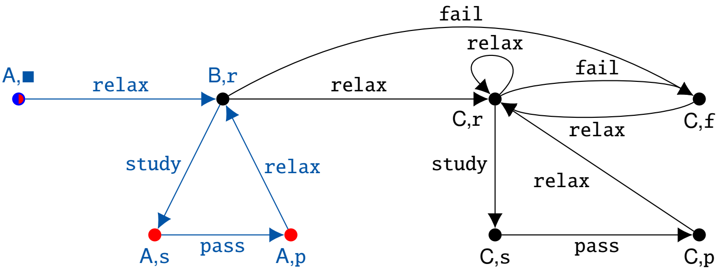


Iteratively construct product automaton $A^B_{\phi\mathcal{T}}$:



Exercise 12.3 (c)

Find an accepting run for $A^B_{\phi\mathcal{T}}$



Accepting run for $A^B_{\phi\mathcal{T}}$:

$(((A,\blacksquare),\texttt{relax}),\overline{((B,\texttt{r}),\texttt{study}),((A,\texttt{s}),\texttt{pass}),((A,\texttt{p}),\texttt{relax})})$