Knowledge Representation and Reasoning

Exercise Sheet 14

Albert-Ludwigs-Universität Freiburg

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Exercise 14.1 (LTL$_f$ and LDL$_f$, 2+2+2+2)

(a) Translate the LTL$_f$ formula $G(p \to W\neg p)$ to LDL$_f$ and try to simplify it as much as possible.

(b) Using the respective semantics, verify that the finite run $(p, \neg p, p)$ satisfies both formulas.

(c) Use the algorithm from the lecture to construct a DFA for the LDL$_f$ formula $[p](\text{end} \vee \langle \neg p \rangle\, tt)$.

(d) How can the DFA be changed to represent to the LDL$_f$ formula $[true^*; p](\text{end} \vee \langle \neg p \rangle\, tt)$?
Note: It is sufficient if you produce an equivalent DFA, we don't require you to adhere to the algorithm from the lecture for this part of the exercise.

14.1 (a) – LTL$_f$-Translation of $G(p \rightarrow W\neg p)$

We first do some equivalence transformations on the LTL$_f$ formula:

$$G(p \rightarrow W\neg p) \equiv G(p \rightarrow \neg Xp) \equiv \neg F(p \wedge Xp) \equiv \neg(\top \mathcal{U}(p \wedge Xp))$$

We then apply the translation to LDL$_f$:

$$\begin{aligned}
&tr(\neg(\top \mathcal{U}(p \wedge Xp))) \\
=&\neg tr(\top \mathcal{U}(p \wedge Xp)) \\
=&\neg \langle (tr(\top)?;\top)^* \rangle (tr(p \wedge Xp) \wedge \neg end) \\
=&\neg \langle (tr(\top)?;\top)^* \rangle (tr(p) \wedge tr(Xp) \wedge \neg end) \\
=&\neg \langle (tr(\top)?;\top)^* \rangle (tr(p) \wedge \langle \top \rangle tr(p) \wedge \neg end) \\
=&\neg \langle (\langle \top \rangle tt?;\top)^* \rangle (\langle p \rangle tt \wedge \langle \top \rangle \langle p \rangle tt \wedge \neg end)
\end{aligned}$$

14.1 (a) – LTL_f-Translation of $G(p \rightarrow W\neg p)$

Since $\langle p \rangle tt$ already implies $\neg end$ and $(\langle \top \rangle tt?; \top)^*$ characterizes the same paths as $\top^*$ (all paths), we can further simplify the formula:

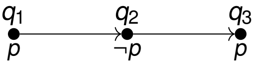
$$\begin{aligned} &\neg \langle (\langle \top \rangle tt?; \top)^* \rangle (\langle p \rangle tt \wedge \langle \top \rangle \langle p \rangle tt \wedge \neg end) \\ \equiv &\neg \langle \top^* \rangle (\langle p \rangle tt \wedge \langle \top \rangle \langle p \rangle tt) \end{aligned}$$

If we want, we can use the equivalences $[\rho]\phi \equiv \neg \langle \rho \rangle \neg \phi$ and $\neg(\phi \wedge \psi) \equiv \phi \rightarrow \neg \psi$ to further transform the formula:

$$\begin{aligned} &\neg \langle \top^* \rangle (\langle p \rangle tt \wedge \langle \top \rangle \langle p \rangle tt) \\ \equiv &[\top^*](\langle p \rangle tt \rightarrow \neg \langle \top \rangle \langle p \rangle tt) \\ \equiv &[\top^*](\langle p \rangle tt \rightarrow [\top] \neg \langle p \rangle tt) \end{aligned}$$

13.1 (b) – Checking $G(p \rightarrow W\neg p)$

We consider $\langle (q_1, q_2, q_3), \tau \rangle$ with $\tau(q_1) = \tau(q_3) = \{p\}$ and $\tau(q_2) = \emptyset$.



- $\langle (q_2, q_3), \tau \rangle \models Xp$, but $\langle (q_1, q_2, q_3), \tau \rangle \not\models Xp$ and $\langle (q_3), \tau \rangle \not\models Xp$
- Thus, $\neg Xp$ is satisfied from q_1 and q_3 but not from q_2.
- Subsequently, $p \rightarrow \neg Xp$ is satisfied from q_1, q_2, q_3.
- This means also $G(p \rightarrow \neg Xp)$ is satisfied from q_1, q_2, q_3.
- In particuar, $\langle (q_1, q_2, q_3), \tau \rangle \models G(p \rightarrow \neg Xp)$.

13.1 (b) – Checking $[\top^*](\langle p\rangle tt \rightarrow \neg\langle\top\rangle\langle p\rangle tt)$

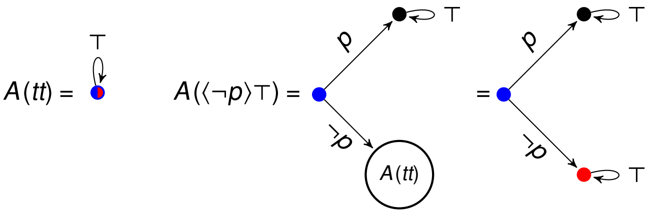
We consider $\langle (q_1, q_2, q_3), \tau\rangle$ with $\tau(q_1) = \tau(q_3) = \{p\}$ and $\tau(q_2) = \emptyset$.

$$\begin{tikzcd} \overset{q_1}{\underset{p}{\bullet}} \arrow[r] & \overset{q_2}{\underset{\neg p}{\bullet}} \arrow[r] & \overset{q_3}{\underset{p}{\bullet}} \end{tikzcd}$$

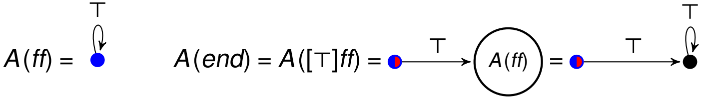
- $\langle (q_1, q_2, q_3), \tau\rangle \models \langle p\rangle tt$; $\langle (q_2, q_3), \tau\rangle \not\models \langle p\rangle tt$; and $\langle (q_3), \tau\rangle \models \langle p\rangle tt$
- Furthermore, $\langle\top\rangle\langle p\rangle tt$ is satisfied from q_2 but not from q_1 or q_3.
- Thus, $\neg\langle\top\rangle\langle p\rangle tt$ is satisfied from q_1 and q_3 but not from q_2.
- Thus, $\langle p\rangle tt \rightarrow \neg\langle\top\rangle\langle p\rangle tt$ is satisfied from q_1, q_2 and q_3.
- Thus, $[\top^*](\langle p\rangle tt \rightarrow \neg\langle\top\rangle\langle p\rangle tt)$ is satisfied from q_1, q_2 and q_3.
- In particular, $\langle (q_1, q_2, q_3), \tau\rangle \models [\top^*](\langle p\rangle tt \rightarrow \neg\langle\top\rangle\langle p\rangle tt)$.

14.1 (c) – DFA for $[p](end \vee \langle \neg p \rangle tt)$

We first construct $A(tt)$ and $A(\langle \neg p \rangle tt)$:



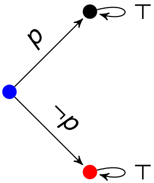
We also construct $A(end)$ (remember that $end = [\top]ff$):



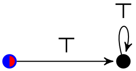
14.1 (c) – DFA for $[p](end \vee \langle \neg p \rangle tt)$

Remeber: We have constructed $A(\langle \neg p \rangle \top)$ and $A(end)$ as follows:

$A(\langle \neg p \rangle \top) =$

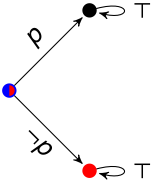


$A(end) =$



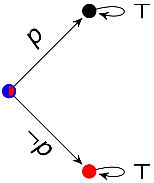
By computing their union and determinizing/minimizing, we obtain:

$A(end \vee \langle \neg p \rangle \top) =$

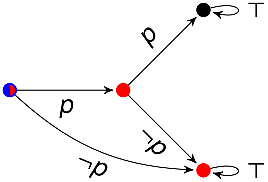


14.1 (c) – DFA for $[p](end \vee \langle \neg p \rangle tt)$

Remeber: We have constructed $A(end \vee \langle \neg p \rangle \top)$ as follows:

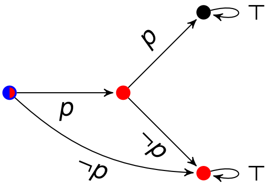
$A(end \vee \langle \neg p \rangle \top) =$ 

We thus obtain $A([p](end \vee \langle \neg p \rangle tt))$ as follows:

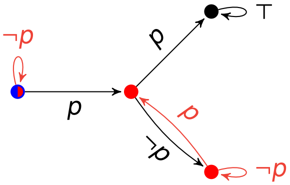
$A([p](end \vee \langle \neg p \rangle tt)) =$ 

14.1 (d) – DFA for $[\top^*; p](end \vee \langle\neg p\rangle\, tt)$

Remember: We constructed $A([p](end \vee \langle\neg p\rangle\, tt))$ as follows:

$A([p](end \vee \langle\neg p\rangle\, tt)) =$ 

For $A([\top^*; p](end \vee \langle\neg p\rangle\, tt))$ we change a few edges and minimize:

$A(\ldots) = \min($ 

$) =$