Knowledge Representation and Reasoning

Exercise Sheet 9

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Exercise 9.1

Extensions in Default Logic

Exercise 9.1 (EXTENSIONS IN DEFAULT LOGIC, 2)

Consider the propositional default theory $\Delta = \langle D, W \rangle$ with

$$D = \{\frac{\top : m}{m}, \frac{\top : i}{i}, \frac{m : \neg s}{\neg s}, \frac{m : b}{b}, \frac{i : s \wedge \neg b}{s \wedge \neg b}\}, W = \{\neg(m \wedge i)\}$$

Determine all extensions of Δ. Which of the propositions s, b, $s \vee b$, $s \wedge b$ are entailed by Δ using credulous reasoning? Which of them are entailed using skeptical reasoning?

Observation: Since $\neg(m \wedge i) \in E$ for any extension E, only one of the defaults $\frac{\top : m}{m}$ and $\frac{\top : i}{i}$ can be applied (with the other being inconsistent to E). Also, one of these defaults must be applied since no extension can be constructed (using only the consequences of the defaults) where both are inconsistent.

$$\begin{aligned} E &= Th(\{\neg(m \wedge i), m, \ldots\}) \\ E_0 &= \{\neg(m \wedge i)\} \\ E_1 &= Th(E_0) \cup \{m\} \\ &\ldots \end{aligned}$$

$$\begin{aligned} E' &= Th(\{\neg(m \wedge i), i, \ldots\}) \\ E'_0 &= \{\neg(m \wedge i)\} \\ E'_1 &= Th(E'_0) \cup \{i\} \\ &\ldots \end{aligned}$$

Exercise 9.1

Extensions in Default Logic

Given: $\Delta = \langle D, W \rangle$ with $D = \{\frac{\top : m}{m}, \frac{\top : i}{i}, \frac{m : \neg s}{\neg s}, \frac{m : b}{b}, \frac{i : s \wedge \neg b}{s \wedge \neg b}\}$, $W = \{\neg(m \wedge i)\}$

Since $m \in E$, we can neither have $s \in E$ nor $\neg b \in E$ (bc. this could only come from $\frac{i : s \wedge \neg b}{s \wedge \neg b}$). Thus, both defaults $\frac{m : \neg s}{\neg s}$ and $\frac{m : b}{b}$ must be applied. Similarly, since $i \in E'$, we can neither have $\neg s \in E'$ nor $b \in E'$ (bc. this could only come from $\frac{m : \neg s}{\neg s}$ and $\frac{m : b}{b}$). Thus the default $\frac{i : s \wedge \neg b}{s \wedge \neg b}$ must be applied. Since no defaults will be applicable afterwards, we now know the only Es we could have started out with to obtain successful proofs.

$$E = Th(\{\neg(m \wedge i), m, \neg s, b\})$$

$$E_0 = \{\neg(m \wedge i)\}$$

$$E_1 = Th(E_0) \cup \{m\}$$

$$E_2 = Th(E_1) \cup \{\neg s, b\}$$

$$E_3 = Th(E_2) \cup \emptyset = E$$

$$E' = Th(\{\neg(m \wedge i), i, s \wedge \neg b\})$$

$$E'_0 = \{\neg(m \wedge i)\}$$

$$E'_1 = Th(E'_0) \cup \{i\}$$

$$E'_2 = Th(E'_1) \cup \{s \wedge \neg b\}$$

$$E'_3 = Th(E'_2) \cup \emptyset = E'$$

Exercise 9.1

Extensions in Default Logic

We have obtained the following two extensions:

$$E = Th(\{\neg(m \land i), m, \neg s, b\})$$
$$E' = Th(\{\neg(m \land i), i, s \land \neg b\})$$

Which of s, b, $s \lor b$, $s \land b$ are entailed skeptically/credulously?

- s follows credulously, as it is only contained in E'
- b follows credulously, as it is only contained in E
- $s \lor b$ follows skeptically (and credulously), as it is contained in E and E'
- $s \land b$ follows neither skeptically nor credulously, as it is neither contained in E nor in E'

Exercise 9.2

Exercise 9.2 (Knowledge Representation And Reasoning in Default Logic, 2+2)
Translate into first-order default logic and check whether the given conclusions follow credulously and/or skeptically.

(a) *Typically, computer science students like computers. Female students who like computers are typically interested in cognitive science. Computer science students are typically female, as for example Anne; but Bob is an exception to this rule.* Conclusions: *Anne is interested in cognitive science. Bob is not interested in cognitive science.*

(b) *By default, students are not lazy. But computer science students are typically intelligent, and intelligent students are usually lazy. Jim and Mary study the humanities, Anne and Bob study computer science.* Conclusions: *Anne and Bob are lazy; Mary and Jim are not.*

Exercise 9.2 (a)

Typically, computer science students like computers. Female students who like computers are typically interested in cognitive science. Computer science students are typically female, as for example Anne; but Bob is an exception to this rule.

$$D = \left\{ \frac{cs(x) : lc(x)}{lc(x)}, \frac{lc(x) \wedge fem(x) : intcog(x)}{intcog(x)}, \frac{cs(x) : fem(x)}{fem(x)} \right\}$$

$$W = \{cs(\text{Anne}), fem(\text{Anne}), cs(\text{Bob}), \neg fem(\text{Bob})\}$$

Exercise 9.2 (a)

$$D = \left\{ \frac{cs(x) : lc(x)}{lc(x)}, \frac{lc(x) \wedge fem(x) : intcog(x)}{intcog(x)}, \frac{cs(x) : fem(x)}{fem(x)} \right\}$$

$$W = \{cs(\mathrm{Anne}), fem(\mathrm{Anne}), cs(\mathrm{Bob}), \neg fem(\mathrm{Bob})\}$$

Conclusions: *Anne is interested in cognitive science. Bob is not interested in cognitive science.*

We first show that $intcog(\mathrm{Anne})$ follows credulously:

$$\begin{aligned} E &= Th(W \cup \{lc(\mathrm{Anne}), lc(\mathrm{Bob}), intcog(\mathrm{Anne})\}) \\ E_0 &= W \\ E_1 &= Th(W) \cup \{lc(\mathrm{Anne}), lc(\mathrm{Bob})\}) \\ E_2 &= Th(E_1) \cup \{intcog(\mathrm{Anne})\} \\ E_3 &= Th(E_2) \cup \emptyset = E \end{aligned}$$

Exercise 9.2 (a)

$$D = \left\{ \frac{cs(x) : lc(x)}{lc(x)}, \frac{lc(x) \wedge fem(x) : intcog(x)}{intcog(x)}, \frac{cs(x) : fem(x)}{fem(x)} \right\}$$

$$W = \{cs(\text{Anne}), fem(\text{Anne}), cs(\text{Bob}), \neg fem(\text{Bob})\}$$

We now explain why $intcog(\text{Anne})$ also follows skeptically:

- $cs(\text{Anne})$ is contained in each extension because it is contained in W.
- There is no rule that derives $\neg lc(\text{Anne})$ and it cannot result from W together with any derivable atoms. It thus cannot be in any extension. Hence the first default rule is always applied, introducing $lc(\text{Anne})$.
- In conclusion, $lc(\text{Anne}) \wedge fem(\text{Anne})$ is true in each extension.
- As $\neg intcog(\text{Anne})$ cannot be added to an extension by building the deductive closure of W and any derivable atoms, the second default rule adds $intcog(\text{Anne})$ to each extension.

Exercise 9.2 (a)

$$D = \left\{ \frac{cs(x) : lc(x)}{lc(x)}, \frac{lc(x) \wedge fem(x) : intcog(x)}{intcog(x)}, \frac{cs(x) : fem(x)}{fem(x)} \right\}$$

$$W = \{cs(\text{Anne}), fem(\text{Anne}), cs(\text{Bob}), \neg fem(\text{Bob})\}$$

What about Bob?

- The second conclusion *Bob is not interested in cognitive science* ($\neg intcog(\text{Bob})$) is not derivable from any extension. Hence, it neither follows credulously nor skeptically.

Exercise 9.2 (b)

By default, students are not lazy. But computer science students are typically intelligent, and intelligent students are usually lazy. Jim and Mary study the humanities, Anne and Bob study computer science.

$$D = \left\{ \frac{stud(x) : \neg lazy(x)}{\neg lazy(x)}, \frac{cs(x) : int(x)}{int(x)}, \frac{int(x) \wedge stud(x) : lazy(x)}{lazy(x)} \right\}$$

$$W = \{stud(\text{Anne}), stud(\text{Bob}), stud(\text{Jim}), stud(\text{Mary}), cs(\text{Anne}), cs(\text{Bob})\}$$

Exercise 9.2 (b)

$$D = \left\{ \frac{stud(x) : \neg lazy(x)}{\neg lazy(x)}, \frac{cs(x) : int(x)}{int(x)}, \frac{int(x) \land stud(x) : lazy(x)}{lazy(x)} \right\}$$

$$W = \{stud(\text{Anne}), stud(\text{Bob}), stud(\text{Jim}), stud(\text{Mary}), cs(\text{Anne}), cs(\text{Bob})\}$$

Conclusions: *Anne and Bob are lazy; Mary and Jim are not.*

We can construct the following extensions:

- $Th(W \cup \{int(\text{Anne}), int(\text{Bob}), \neg lazy(\text{Anne}), \neg lazy(\text{Bob}), \neg lazy(\text{Jim}), \neg lazy(\text{Mary})\})$
- $Th(W \cup \{int(\text{Anne}), int(\text{Bob}), lazy(\text{Anne}), \neg lazy(\text{Bob}), \neg lazy(\text{Jim}), \neg lazy(\text{Mary})\})$
- $Th(W \cup \{int(\text{Anne}), int(\text{Bob}), \neg lazy(\text{Anne}), lazy(\text{Bob}), \neg lazy(\text{Jim}), \neg lazy(\text{Mary})\})$
- $Th(W \cup \{int(\text{Anne}), int(\text{Bob}), lazy(\text{Anne}), lazy(\text{Bob}), \neg lazy(\text{Jim}), \neg lazy(\text{Mary})\})$

$\Rightarrow$ $lazy(\text{Anne}) \land lazy(\text{Bob})$ follows credulously.

$\Rightarrow$ $\neg lazy(\text{Jim}) \land \neg lazy(\text{Mary})$ follows skeptically.

Exercise 9.3

Exercise 9.3 (Properties of Default Logic, 2+2+2)

Prove or disprove the following statements:

(a) Let $\langle D, W \rangle$ be a propositional default theory and let D' be a set of normal defaults with $D \subseteq D'$. If E is an extension of $\langle D, W \rangle$, then there exists an extension E' of $\langle D', W \rangle$ such that $E \subseteq E'$.

(b) Let $\langle D, W \rangle$ be a propositional default theory and ϕ be a formula that follows skeptically from $\langle D, W \rangle$. Then, each formula ψ follows skeptically from $\langle D, W \cup \{\phi\} \rangle$ if and only if ψ follows skeptically from $\langle D, W \rangle$.

(c) Let $\langle D, W \rangle$ be a propositional, semi-normal default theory that has a consistent extension E such that the justification (consistency condition) of each default rule in D is consistent with E (und thus with W). Then W must be consistent with the set of all justifications of the default rules in D.

Exercise 9.3 (a)

Theorem?

Let $\langle D, W\rangle$ be a propositional default theory and let D' be a set of normal defaults with $D \subseteq D'$.
If E is an extension of $\langle D, W\rangle$, then there exists an extension E' of $\langle D', W\rangle$ such that $E \subseteq E'$.

Proof

- $\langle D, W\rangle$ is a default theory, $D \subseteq D'$, with D' being normal defaults.
- Thus $\langle D, W\rangle$ is a normal default theory.
- Let E be an extension of $\langle D, W\rangle$.
- Consider then $\langle D', E\rangle$ which is also a normal default theory.
- Thus, there exists an extension E' of $\langle D', E\rangle$ and obviously $E = E'_0 \subseteq E'$.
- This extension E' is also an extension for $\langle D', W\rangle$.

Exercise 9.3 (b)

Theorem?

Let $\langle D, W\rangle$ be a propositional default theory and ϕ be a formula that follows skeptically from $\langle D, W\rangle$. Then, each formula ψ follows skeptically from $\langle D, W \cup \{\phi\}\rangle$ if and only if ψ follows skeptically from $\langle D, W\rangle$.

Counter-Example

- Let $D = \left\{\frac{:a}{a}, \frac{a \vee b : \neg a}{\neg a}\right\}, W = \emptyset$
- $\langle D, W\rangle$ has exactly one extension: $Th(\{a\})$.
- Thus $\phi = a \vee b$ follows skeptically from $\langle D, W\rangle$.
- $\langle D, \{a \vee b\}\rangle$ has two extensions : $Th(\{a\})$ and $Th(\{\neg a, b\})$.
- $\psi = a$ follows skeptically from $\langle D, W\rangle$, but not from $\langle D, W \cup \{\phi\}\rangle$.

Exercise 9.3 (c)

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Theorem?

Let $\langle D, W \rangle$ be a propositional, semi-normal default theory that has a consistent extension E such that the justification (consistency condition) of each default rule in D is consistent with E (and thus with W). Then W must be consistent with the set of all justifications of the default rules in D.

Counter-Example

- Let $D = \left\{ \frac{a:b \land c}{c}, \frac{a:\neg b \land d}{d} \right\}$ and $W = \{a\}$
- We have $E = Th(\{a, c, d\})$ as a consistent extension.
- Obviously, $E \cup \{b \land c\}$ is consistent and $E \cup \{\neg b \land d\}$ is consistent.
- But $W \cup \{b \land c, \neg b \land d\}$ is inconsistent.