Knowledge Representation and Reasoning

Exercise Sheet 5

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Exercise 5.1 – Inheritance networks with negation

Exercise 5.1 (Inheritance networks with negation, 1+1)

Simple inheritance networks can be extended by allowing negated concepts, i.e., concept terms of the form **not** C, where C is an atomic concept. The logical semantics is **not** $C \mapsto \neg C(x)$. For example:

$$\texttt{Student}\ \textbf{isa not}\ \texttt{Professor} \mapsto \forall x.\texttt{Student}(x) \rightarrow \neg\texttt{Professor}(x).$$

(a) Show that a simple inheritance network with negation Θ can be inconsistent, i.e. $\Theta \models \bot$.

(b) Show that the simple inheritance networks without negation from the lecture cannot be inconsistent.

Exercise 5.1 (a)

Show that inheritance networks with negation can be inconsistent

Consider the inheritance network

$$C \textbf{ isa not } C$$

$$\textbf{not } C \textbf{ isa } C$$

which has the logical interpretation

$$\begin{aligned} & \forall x.(C(x) \rightarrow \neg C(x)) \land (\neg C(x) \rightarrow C(x)) \\ \models \quad & \forall x.\neg C(x) \land C(x) \\ \models \quad & \bot \end{aligned}$$

$\leadsto$ Inheritance networks with negation can be inconsistent.

Exercise 5.1 (b)

Show that inheritance networks without negation cannot be inconsistent

- Without negations, each **isa** formula corresponds to a logical formula $\forall x.C(x) \rightarrow D(x)$.
- Such a formula can easily be satisfied by an interpretation $\mathcal{I}$, which assigns no variables in the domain to C, i.e. $C^{\mathcal{I}} = \emptyset$.
- Therefore, for an arbitrary inheritance network (without negation), the interpretation $\mathcal{I}$, with $C^{\mathcal{I}} = \emptyset$ for all concepts C, is a model.

$\rightsquigarrow$ Inheritance networks without negation always have a model and are thereby always consistent.

Exercise 5.3 – Description Logics

Exercise 5.3 (DESCRIPTION LOGICS, 1+1+1)

Consider the following pairs of TBoxes $\mathcal{T}$ and concept inclusions $C \sqsubseteq D$. For which of the pairs does $\mathcal{T} \models C \sqsubseteq D$ hold? Explain your answers and provide a counterexample in case the concept inclusion does not hold.

(a) $\mathcal{T} = \{A \sqsubseteq B\}$ $\quad\quad$ $\forall r.A \sqsubseteq \exists r.B$

(b) $\mathcal{T} = \{A \doteq B \sqcap \exists r.B\}$ $\quad\quad$ $\exists(r \circ r).B \sqsubseteq A \sqcup \exists r.A$

(c) $\mathcal{T} = \{A \doteq \forall(r \sqcap s).B\}$ $\quad\quad$ $\exists r.\neg B \sqcap \neg\exists s \sqsubseteq A$

Exercise 5.3 (a)

Does $\mathcal{T} = \{A \sqsubseteq B\}$ imply $\forall r.A \sqsubseteq \exists r.B$?

We show, that $\forall r.A \sqsubseteq \exists r.B$ is not implied by $\mathcal{T}$. Counterexample:

$$\mathcal{D} = \{d_1\}$$
$$A^{\mathcal{I}} = \emptyset$$
$$B^{\mathcal{I}} = \emptyset$$
$$r^{\mathcal{I}} = \emptyset$$

- $\mathcal{I} \models \{A \sqsubseteq B\}$ since both $A^{\mathcal{I}}$ and $B^{\mathcal{I}}$ are empty.
- $(\forall r.A)^{\mathcal{I}} = \{d_1\}$ while $(\exists r.B)^{\mathcal{I}} = \emptyset$.
- Therefore, $(\forall r.A)^{\mathcal{I}} \not\subseteq (\exists r.B)^{\mathcal{I}}$ which contradicts $\forall r.A \sqsubseteq \exists r.B$.

Exercise 5.3 (a)

Real world example

We substitute A with $\mathtt{Drummer}$, B with $\mathtt{Musician}$ and r with $\mathtt{has\text{-}child}$.

- $\mathcal{T} = \{\mathtt{Drummer} \sqsubseteq \mathtt{Musician}\}$
  The TBox states that every drummer is a musician.
- $\forall \mathtt{has\text{-}child}.\mathtt{Drummer} \sqsubseteq \exists \mathtt{has\text{-}child}.\mathtt{Musician}$
  This concept inclusion states, that if all your children are drummers, you have a child, which is a musician.
- This is not the case if you do not have any children: It technically holds true, that all of your children are drummers (and also that all your children are musicians), but you do not actually have a child which is a musician.

Exercise 5.3 (b)

Does $\mathcal{T} = \{A \doteq B \sqcap \exists r.B\}$ imply $\exists(r \circ r).B \sqsubseteq A \sqcup \exists r.A$?

We show, that $\exists r.B \sqsubseteq A$ is not implied by $\mathcal{T}$. Counterexample:

$$\mathcal{D} = \{d_1, d_2, d_3\}$$
$$A^{\mathcal{I}} = \emptyset$$
$$B^{\mathcal{I}} = \{d_3\}$$
$$r^{\mathcal{I}} = \{\langle d_1, d_2\rangle, \langle d_2, d_3\rangle\}$$

- $\mathcal{I} \models \{A \doteq B \sqcap \exists r.B\}$ since both $A^{\mathcal{I}}$ and $(B \sqcap \exists r.B)^{\mathcal{I}}$ are empty.
- While $(A \sqcup r.A)^{\mathcal{I}} = \emptyset$ is also empty, we get $(\exists(r \circ r).B)^{\mathcal{I}} = \{d_1\}$, since we have $\mathcal{I} \models r(d_1, d_2)$, $\mathcal{I} \models r(d_2, d_3)$ and $\mathcal{I} \models B(d_3)$.
- Therefore, $(\exists(r \circ r).B)^{\mathcal{I}} \not\subseteq (A \sqcup \exists r.A)^{\mathcal{I}}$ which contradicts $\exists(r \circ r).B \sqsubseteq A \sqcup \exists r.A$.

Exercise 5.3 (b)

Real world example

We substitute A with $\texttt{Mother-of-a-daughter}$, B with $\texttt{Female}$ and r with $\texttt{has-child}$.

- $\mathcal{T} = \{\texttt{Mother-of-a-daughter} \doteq \texttt{Female} \sqcap \exists \texttt{has-child}.\texttt{Female}\}$
  The TBox states that a mother of a daughter is a woman who has a child, which is also a woman.
- $\exists(\texttt{has-child} \circ \texttt{has-child}).\texttt{Female} \sqsubseteq \texttt{Mother-of-a-daughter} \sqcup \exists \texttt{has-child}.\texttt{Mother-of-a-daughter}$
  This concept inclusion states that everyone who has a granddaughter is a mother of a daughter or the parent of a mother of a daughter.
- This does not necessarily hold true for the father's father of the granddaughter in question: Assuming that this grandfather has no daughters he is neither the mother of a daughter nor the parent of a mother of a daughter.

Exercise 5.3 (c)

Does $\mathcal{T} = \{A \doteq \forall(r \sqcap s).B\}$ imply $\exists r.\neg B \sqcap \neg\exists s \sqsubseteq A$?

We show, that $\exists r.\neg B \sqcap \neg\exists s \sqsubseteq A$ is implied by $\mathcal{T}$:

- $\neg\exists s$ states that concept in question cannot have any s-successors, which already guarantees subsumption by A: If there are no s-successors the intersection of r- and s-successors is empty, too. Thus, any $\forall$-quantification over this set, including $A \doteq \forall(r \sqcap s).B$, is inherently satisfied. $\rightsquigarrow$ $\neg\exists s \sqsubseteq A$
- Since intersection can only remove objects from a set, $\exists r.\neg B$ does not really matter. We already established $\neg\exists s \sqsubseteq A$, therefore $C \sqcap \neg\exists s \sqsubseteq A$ holds for any arbitrary concept C.

$\Rightarrow$ $\exists r.\neg B \sqcap \neg\exists s \sqsubseteq A$

Exercise 5.3 (c)

Real world example

We substitute A with Good-teacher, B with Grade-A-student, r with teaches and s with listened-to-by.

- $\mathcal{T} = \{\texttt{Good-teacher} \doteq \forall(\texttt{teaches} \sqcap \texttt{listened-to-by}).\texttt{Grade-A-student}\}$
  The TBox states that you are a good teacher, if all the students you teach and that also listen to you get very good grades.
- $\exists\texttt{teaches}.\neg\texttt{Grade-A-student} \sqcap \neg\exists\texttt{listened-to-by} \sqsubseteq \texttt{Good-teacher}$
  This concept inclusion states, that if you teach a student who does not get good grades but also nobody listens to you, you are still a good teacher.
- This does indeed follow from the given TBox, since this definition of a good teacher only requires students who actually listen to you to get good grades. By this (admittedly flawed) definition, anyone who is not listened to by anybody is actually a good teacher, regardless of how they teach.

Note: This is just an example, not a proof of the statement!