Knowledge Representation and Reasoning

Exercise Sheet 6

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Exercise 6.1 (Modeling and Unfolding, $3 + 3$)

(a) Extend the family TBox from the lecture (slide 8) by defining the following concepts:

- `Bachelor` (Unmarried man)
- `Only-child` (Person whose parents have no other children)
- `Mother-in-law` (Mother of a married person)

Besides the roles and concepts used in the lecture, you may use the atomic role `married-to`.

(b) Specify the unfolding of the concepts `Parent`, `Only-child`, and `Mother-in-law`. Determine the primitive components used in your definitions. Provide an initial interpretation using the ABox given in the lecture (slide 8). Finally, specify the full interpretation of these three concepts as induced by your initial interpretation.

Exercise 6.1(a)

Define the concepts Bachelor, Only-child and Mother-in-law

$$\texttt{Bachelor} \doteq \texttt{Man} \sqcap \neg\exists \texttt{married-to}$$

$$\texttt{Only-child} \doteq \texttt{Human} \sqcap \forall \texttt{has-child}^{-1}.(\leq 1\ \texttt{has-child}.\texttt{Human})$$

$$\texttt{Mother-in-law} \doteq \texttt{Woman} \sqcap \exists\, (\texttt{has-child} \circ \texttt{married-to})$$

Exercise 6.1(b)

Unfold Parent, Only-child and Mother-in-law

For unfolding we first have to normalize the TBox: Replace $\texttt{Human} \sqsubseteq \texttt{Living\_entity}$ by $\texttt{Human} \doteq \texttt{Human}^* \sqcap \texttt{Living\_entity}$

$$\texttt{Parent} \doteq \texttt{Father} \sqcup \texttt{Mother}$$

$$\texttt{Parent} \doteq (\texttt{Man} \sqcap \exists \texttt{has-child}.\texttt{Human}) \sqcup (\texttt{Woman} \sqcap \exists \texttt{has-child}.\texttt{Human})$$

$$\texttt{Parent} \doteq (\texttt{Human} \sqcap \texttt{Male} \sqcap \exists \texttt{has-child}.\texttt{Human}) \sqcup (\texttt{Human} \sqcap \texttt{Female} \sqcap \exists \texttt{has-child}.\texttt{Human})$$

$$\begin{aligned}\texttt{Parent} \doteq\ & (\texttt{Human}^* \sqcap \texttt{Living\_entity} \sqcap \neg\texttt{Female} \sqcap \exists \texttt{has-child}.(\texttt{Human}^* \sqcap \texttt{Living\_entity})) \\ & \sqcup (\texttt{Human}^* \sqcap \texttt{Living\_entity} \sqcap \texttt{Female} \sqcap \exists \texttt{has-child}.(\texttt{Human}^* \sqcap \texttt{Living\_entity}))\end{aligned}$$

$$\texttt{Only-child} \doteq \texttt{Human} \sqcap \forall \texttt{has-child}^{-1}.(\leq 1\ \texttt{has-child}.\texttt{Human})$$

$$\begin{aligned}\texttt{Only-child} \doteq\ & \texttt{Human}^* \sqcap \texttt{Living\_entity} \\ & \sqcap \forall \texttt{has-child}^{-1}.(\leq 1\ \texttt{has-child}.(\texttt{Human}^* \sqcap \texttt{Living\_entity}))\end{aligned}$$

$$\texttt{Mother-in-law} \doteq \texttt{Woman} \sqcap \exists\,(\texttt{has-child} \circ \texttt{married-to})$$

$$\texttt{Mother-in-law} \doteq \texttt{Human} \sqcap \texttt{Female} \sqcap \exists\,(\texttt{has-child} \circ \texttt{married-to})$$

$$\texttt{Mother-in-law} \doteq \texttt{Human}^* \sqcap \texttt{Living\_entity} \sqcap \texttt{Female} \sqcap \exists\,(\texttt{has-child} \circ \texttt{married-to})$$

Exercise 6.1(b)

Determine primitive components and construct an initial interpretation

The primitive components of those definitions are the concepts `Human*`, `Living_entity` and `Female` and the roles `has-child` and `married-to`. Based on the given ABox, we can construct the initial interpretation $\mathcal{J}$:

$$
\begin{aligned}
\texttt{Human*}^{\mathcal{J}} &= \{\texttt{DIANA, ELIZABETH, CHARLES, EDWARD, ANDREW, WILLIAM}\} \\
\texttt{Living\_entity}^{\mathcal{J}} &= \{\texttt{DIANA, ELIZABETH, CHARLES, EDWARD, ANDREW, WILLIAM}\} \\
\texttt{Female}^{\mathcal{J}} &= \{\texttt{DIANA, ELIZABETH}\} \\
\texttt{has-child}^{\mathcal{J}} &= \{(\texttt{ELIZABETH, CHARLES}), (\texttt{ELIZABETH, EDWARD}), \\
&\quad (\texttt{ELIZABETH, ANDREW}), (\texttt{DIANA, WILLIAM}), \\
&\quad (\texttt{CHARLES, WILLIAM})\} \\
\texttt{married-to}^{\mathcal{J}} &= \emptyset
\end{aligned}
$$

Note that our ABox never actually states that `WILLIAM` is a `Human`. However, since `DIANA` is a `Mother-without-daughter`, which implies that she has a human child, we either need to classify `WILLIAM` as `Human` or give `DIANA` an additional child.

Exercise 6.1(b)

Extend the interpretation to the concepts in question

Extending the initial interpretation $\mathcal{J}$ to a full interpretation $\mathcal{I}$ yields:

$$\begin{aligned} \texttt{Parent}^{\mathcal{I}} &= \{\texttt{DIANA, ELIZABETH, CHARLES}\} \\ \texttt{Only-child}^{\mathcal{I}} &= \{\texttt{DIANA, ELIZABETH, WILLIAM}\} \\ \texttt{Mother-in-law}^{\mathcal{I}} &= \emptyset \end{aligned}$$

Exercise 6.2 – Model Extension

Exercise 6.2 (Model Extension, 2)

Condsider the interpretation $\mathcal{I}$ with $\Delta^{\mathcal{I}} = \{a, b, c, d, e, f, g\}$, $A^{\mathcal{I}} = \{a, c, e, g\}$, $B^{\mathcal{I}} = \{a, c, d\}$, $r^{\mathcal{I}} = \{(a, b), (b, c), (e, c), (c, f)\}$, $s^{\mathcal{I}} = \{(a, b), (d, b), (d, c), (d, e), (f, g)\}$, and $t^{\mathcal{I}} = \{(a, b), (d, d), (c, g)\}$. Extend this interpretation for the following concept C, in other words determine $C^{\mathcal{I}}$:

$$C \doteq \exists r \circ s^{-1}.(\geq 2s.(\forall r.((r \circ s \sqsubseteq t) \sqcap (A \sqcup B))))$$

Exercise 6.2

We examine the term inside out and determine at each step which objects fit the partial concept description:

$C \doteq \exists r \circ s^{-1}.(\geq 2s.(\forall r.((r \circ s \sqsubseteq t) \sqcap (A \sqcup B))))$ $\{a, c, d, e, g\}$

$C \doteq \exists r \circ s^{-1}.(\geq 2s.(\forall r.((r \circ s \sqsubseteq t) \sqcap (A \sqcup B))))$ $\{a, b, c, d, e, f, g\}$

$C \doteq \exists r \circ s^{-1}.(\geq 2s.(\forall r.((r \circ s \sqsubseteq t) \sqcap (A \sqcup B))))$ $\{a, c, d, e, g\}$

$C \doteq \exists r \circ s^{-1}.(\geq 2s.(\forall r.((r \circ s \sqsubseteq t) \sqcap (A \sqcup B))))$ $\{b, d, e, f, g\}$

$C \doteq \exists r \circ s^{-1}.(\geq 2s.(\forall r.((r \circ s \sqsubseteq t) \sqcap (A \sqcup B))))$ $\{d\}$

$C \doteq \exists r \circ s^{-1}.(\geq 2s.(\forall r.((r \circ s \sqsubseteq t) \sqcap (A \sqcup B))))$ $\{a, b, e\}$

$\rightsquigarrow C^{\mathcal{I}} = \{a, b, e\}$

Exercise 6.3 – Model Extension

Exercise 6.3 (Reasoning with Protégé, 4)

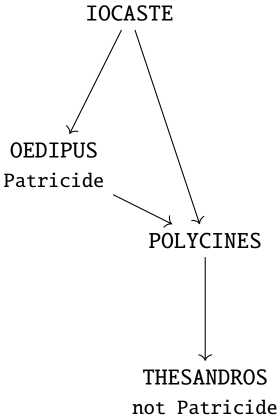
Familiarize yourself with Protégé (which you can download from `http://protege.stanford.edu/`) for which we implemented the example ontology from the lecture (which you can find on ILIAS). Extend the TBox (i.e. the classes) by the basic concept `Patricide`. Next, assert the Oedipus ABox as given below:

`hasChild(IOCASTE, OEDIPUS)` , `hasChild(IOCASTE, POLYNICES)`
`hasChild(OEDIPUS, POLYNICES)` , `hasChild(POLYNICES, THERSANDROS)`
`Patricide(OEDIPUS)` , `not Patricide(THERSANDROS)`

Use Protégé's reasoner to check whether for the ABox the following holds: Iocaste has a child that is a patricide and that itself has a child that is no patricide. To do so, first define a new class which corresponds to the concept of having a child that is a patricide and that itself has a child that is no patricide. Then start the reasoner and check whether Iocaste is among the inferred instances. Try to explain the result in light of the fact that reasoning in description logics is based on the Open-World Assumption. Please save your ontology as an OWL document and submit it with your solution.

Exercise 6.3

Does Iocaste have a child that is a patricide and that itself has a child which is no patricide?



- At first glance, one might answer "No": Oedipus is a patricide, but we do not know whether his child is no patricide. Polycines has a child that is no patricide, but we do not know whether he is a patricide.
- However, Polycines must either be or not be a patricide. If he is one, he fits the description. If he is not, Oedipus does.
- Thus, one can indeed infer that Iocaste satisfies the concept without violating the open-world assumption: Whether or not Polycines is in fact a patricide, Iocaste has a child which is a patricide that has a child that is not.