Coordinate Transformation Matrix

Let $F=\{f^1, f^2, f^3, .. f^n\}$ and $M=\{m^1, m^2, m^3, .. m^n\}$ be coordinate frames of R^n with F being an orthonormal frame. Then for each point p in R^n,

$$\boxed{[p]^F = A\,[p]^M}$$

where A is an nxn matrix defined by $A_{kj} = f^k . m^j$ for $1 \le k, j \le n$

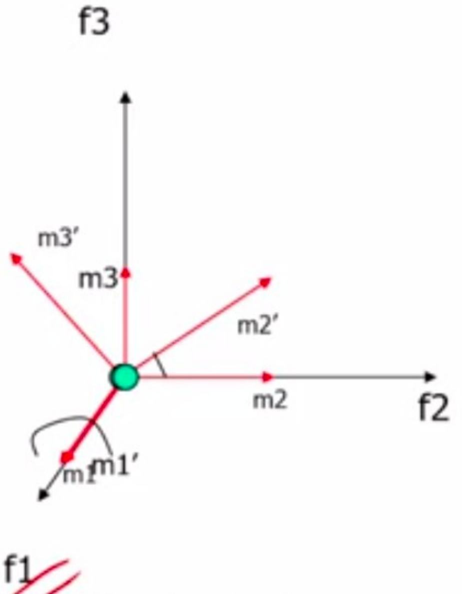
Fundamental rotations

$$R_1(\phi) \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix}$$

$$R_2(\phi) \Rightarrow \begin{bmatrix} \cos(\phi) & 0 & \sin(\phi) \\ 0 & 1 & 0 \\ -\sin(\phi) & 0 & \cos(\phi) \end{bmatrix}$$

$$R_3(\phi) \Rightarrow \begin{bmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Pattern



The k^{th} row and the k^{th} column of $R_k(\varphi)$ are identical to the k th row and the k th column of identity matrix. In the remaining 2x2 matrix, the diagonal terms are $\cos(\phi)$ while the off diagonal terms are $\pm\sin(\phi)$. The sign of the off diagonal term above the diagonal is $(-1)^k$.

Homogeneous Transformation matrix

If a physical point in three-dimensional space is expressed in terms of its homogeneous coordinates and we want to change from one coordinate frame to another, we use a 4x4 homogeneous transformation matrix.

In general T is

$$T = \left[\begin{array}{c|c} R & p \\ \hline \eta^T & \sigma \end{array}\right]$$

The 3x3 matrix R is a rotation matrix

P is a 3x1 translation vector

η is a perspective vector, set to zero

3 x 3

4x4

Inverse Homogeneous Transformation

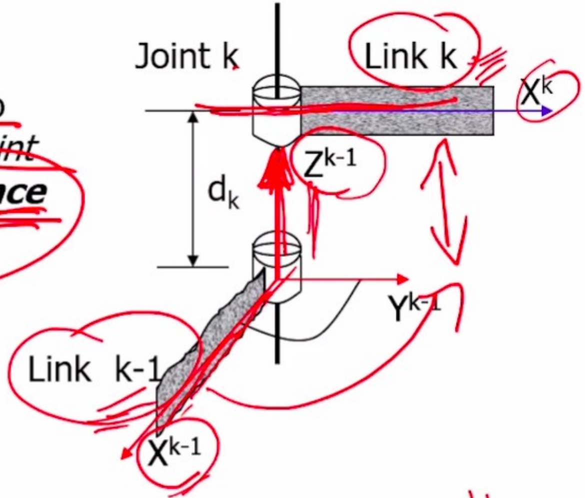
If T be a homogeneous transformation matrix with rotation R and translation p between two orthonormal coordinate frames and if $\eta=0, \sigma=1$, then the inverse transformation is:

$$T^{-1} = \left[\begin{array}{ccc|c} & R^T & & -R^T p \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Joint Parameters

The relative position and orientation of two successive links can be specified by two *joint parameters*, **joint angle** *and* ***joint distance***

Joint k connects link k-1 to link k. The parameters associated with joint k are defined w.r.t z^{k-1}, which is aligned with the axis of joint k.

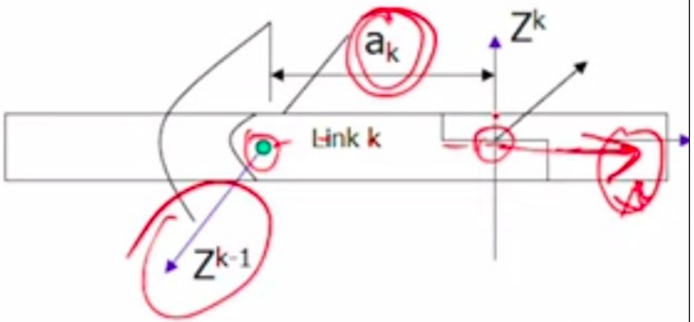


The joint angle, θ_k, is the rotation about z^{k-1} needed to make axis x^{k-1} parallel with axis x^k.

Joint distance d_k, is the translation along z^{k-1} needed to make axis x^{k-1} intersect with axis x^k.

Thus joint angle is a rotation about axis of joint k, while joint distance is a translation along joint axis

Link Parameters



The relative position and orientation of the axes of two successive joints can be specified by two link *parameters*, ***link length*** *and* ***link twist angle***

Link k connects joint k-1 to joint k. The parameters associated with link k are defined w.r.t x^k, which is a common normal between the axes of joint k-1 and k.

Link length, a_k, is the translation along x^k needed to make axis z^{k-1} intersect z^k

Twist angle, α_k, is the rotation about x^k needed to make axis z^{k-1} parallel with axis z^k

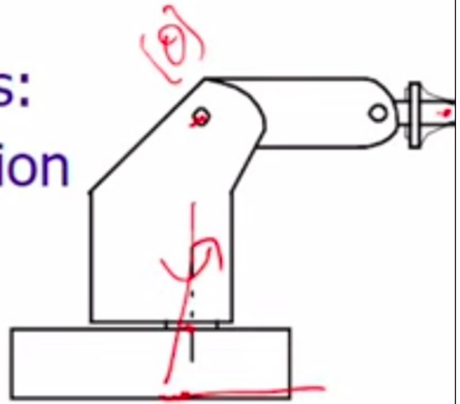
θ_k
d_k
a_k
α_k

Kinematic Parameters……

Arm Parameter	Symbol	Revolute Joint (R)	Prismatic Joint (P)
Joint Angle	θ	Variable	Fixed
Joint Distance	d	Fixed	Variable
Link Length	a	Fixed	Fixed
Link Twist angle	α	Fixed	Fixed

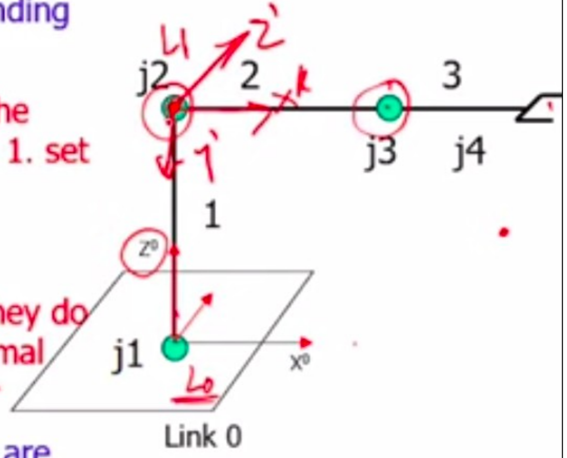
For an n-axis robot manipulator, the 4n kinematic parameters constitute the *minimal set needed to specify the kinematic configuration of the robot.*

Assignment of Coordinate frames: Denavit-Hartenberg Representation



DH Algorithm

1. Number the joints from 1 to n starting with the base and ending with the tool yaw, pitch and roll in that order.
2. Assign a right-handed orthonormal coordinate frame L_0 to the robot base, making sure that Z^0 aligns with the axis of joint 1. set k=1
3. Align Z^k with the axis of joint k+1
4. Locate origin of L_k at the intersection of the Z^k and Z^{k-1}. If they do not intersect, use the intersection of Z^k with a common normal between Z^k and Z^{k-1}.
5. Select X^k to be orthogonal to both Z^k and Z^{k-1}. If Z^k and Z^{k-1} are parallel, point X^k away from Z^{k-1}.



DH

- 8. Set the origin of L_n at the tool tip. Align Z^n with the approach vector, y^n with the sliding vector, and x^n with the normal vector of the tool. Set k=1
- 9. Locate point b_k at the intersection of x^k and z^{k-1} axes. If they do not intersect, use the intersection of x^k with a common normal between x^k and z^{k-1}.
- 10 Compute θ_k as the angle of rotation from x^{k-1} to x^k measured about z^{k-1}

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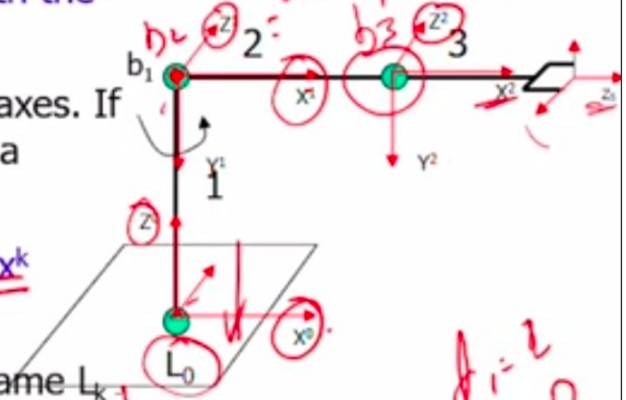
- 11 Compute d_k as the distance from the origin of frame L_{k-1} to point b_k measured along Z^{k-1}.
- 12 Compute a_k as the distance from point b_k to the origin of frame L_k measured along x^k.
- 13 Compute α_k as the angle of rotation from Z^{k-1} to Z^k measured about x^k
- 14 set k=k+1. If k≤n, go to step 9; else stop

$\alpha_n = 0$ $\theta_1 = 0$

$\theta_2 = 0$



$d_1 = 2$

$d_2 = 0$

$\alpha_1 = -90$

$a_1 = L_1 \to b_1 = 0$

DH Matrix

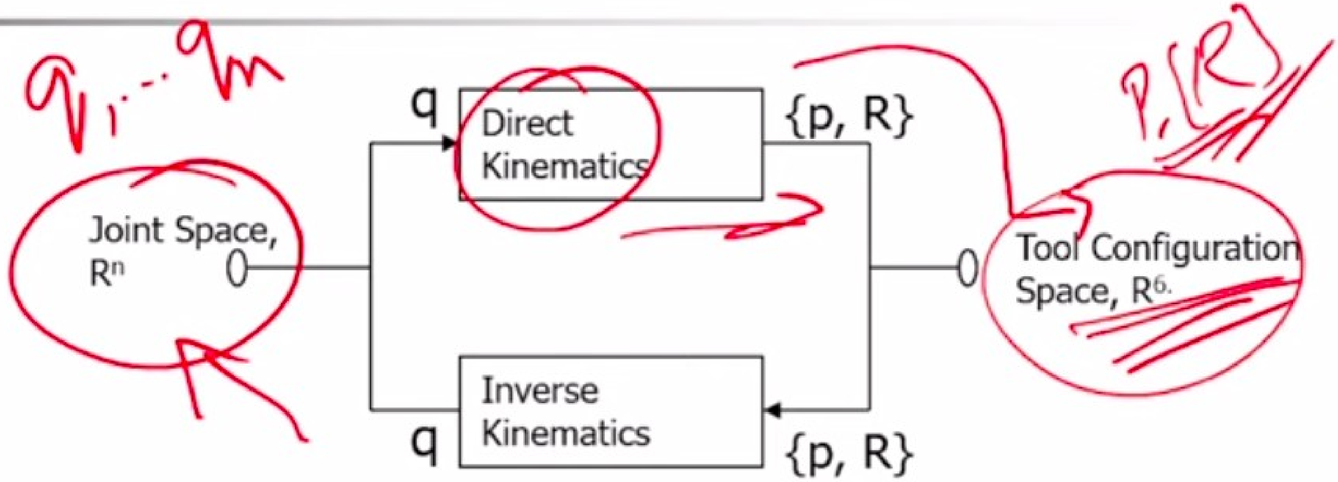
Link Coordinate Transformation

$$T_{k-1}^{k} = \begin{bmatrix} C\theta_k & -C\alpha_k S\theta_k & S\alpha_k S\theta_k & a_k C\theta_k \\ S\theta_k & C\alpha_k C\theta_k & -S\alpha_k C\theta_k & a_k S\theta_k \\ 0 & S\alpha_k & C\alpha_k & d_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Link Coordinate Transformation

$$T_{k}^{k-1} = \begin{bmatrix} C\theta_k & S\theta_k & 0 & -a_k \\ -C\alpha_k S\theta_k & C\alpha_k C\theta_k & S\alpha_k & -d_k S\alpha_k \\ S\alpha_k S\theta_k & -S\alpha_k C\theta_k & C\alpha_k & -d_k C\alpha_k \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R transpose n minus R transpose T you will



Solvability

$$\begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1C_{234}C_5 + S_1S_5 & -C_1C_{234}S_5 + S_1C_5 & -C_1S_{234} & C_1(177.8C_2 + 177.8C_{23} - 129.5S_{234}) \\ S_1C_{234}C_5 - C_1S_5 & -S_1C_{234}S_5 - C_1C_5 & -S_1S_{234} & S_1(177.8C_2 + 177.8C_{23} - 129.5S_{234}) \\ -S_{234}C_5 & S_{234}S_5 & -C_{234} & 215 - 177.8S_2 - 177.8S_{23} - 129.5C_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$