MT Coursera Week 01

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§1 Lecture 1

§1.1 Use of Langauge in Mathematics

There are 4 general forms in using language in mathematics.

1. **Object A** has **Property P**. *3 is a prime number.*
2. Every **Object A** of **Type T** has **Property P**. *Every polynomial equation has a complex root.*
3. There is an **Object T** having **Property P**. *There is a prime number between 20 and 25.*
4. If **Statement A**, then **Statement B**. *If p is a prime of the form $4n + 1$, then p is the sum of 2 squares.*

§1.2 Mathematical Symbols

Some logical linguistics commonly used in mathematics are

1. and $\wedge$
2. or $\vee$
3. not $\neg$
4. implies $\implies$
5. for all $\forall$
6. there exists $\exists$

§2 Lecture 2

§2.1 "and" operator

The "and" operator is defined by $\wedge$.

Example 2.1

Consider the statement pi is bigger than 3 and less than 3.2.

This can be represented by

$$(\pi > 3) \wedge (\pi < 3.2)\,.$$

More generally, ϕ and ψ can be could be represented by $\phi \wedge \psi$. This means that both ϕ and ψ are true, and the statement $\phi \wedge \psi$ is a conjunction. Similarily, ϕ and ψ are conjuncts.

Notice that $\phi \wedge \psi = \psi \wedge \phi$, demonstrating their communitive property.

Another way to visualize how the "and" operator works is through a truth table, shown below.

ϕ	ψ	$\phi \wedge \psi$
T	T	T
T	F	F
F	T	F
F	F	F

§2.2 "or" operator

The "or" operator is represented by $\vee$.

Consider the following statement.

$$a > 0 \text{ \underline{or} the equation } x^2 + a = 0 \text{ has a real root.}$$

In this case, **either** one **or** the other condition could be fulfilled, not both at the same time. This sort of "or" is called **exclusive-or**.

Next, consider this statement.

$$ab = 0 \text{ if } a = 0 \text{ \underline{or} } b = 0.$$

In this case, either $a = 0$ or $b = 0$ could be fulfilled, but what differentiates it from the former is that **both** statements could be fulfilled, and the statement could still be true. This is called **inclusive-or**.

In mathematics, "or" means **inclusive or**. For two statements ϕ and ψ, $\underline{\phi \vee \psi}$ is called disjunction, and similarily, ϕ and ψ are called disjuncts.

The truth table representation for this would be

ϕ	ψ	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

§2.3 "not" operator

The "not" operator is represented by $\neg$ (or, less frequently, $\sim$).

Once again, consider the statements ϕ and ψ. $\neg\psi$ is called the negation of ψ.

Example 2.2

If ψ is false, then $\neg\psi$ is true.
If ψ is false, $\neg\psi$ is true.

Sometimes, we use $x \neq y$ instead of $\neg(x = y)$, but that could be very misleading. For example, the statement

$$\neg(a < x \leq b)$$

is a lot clearer than

$$a \nless x \nleq b.$$

The truth table for this case is simple, as T would simply become F's, and vice versa. To easily determine whether a given statement is the negation of another, find the truth value of the initial statement and the truth value of the given statement. If their truth values are the same (for example, one is true and the other is also true), then that means the given sentence cannot **negate** the initial one.