MT Coursera Week 02

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§1 Implication

In mathematics, we frequently occur the expressions $\phi \implies \psi$. Implication is the means by which we prove results in mathematics, starting with observations and axioms.

We say that ϕ implies ψ is the truth of ϕ follows from the truth of ψ.

Example 1.1

Let ϕ be the statement that $\sqrt{2}$ is rational.

Let ψ be the statement that $0 < 1$.

Is "ϕ implies ψ" true?

Although both statements are truth, this does not mean that $\phi \implies \psi$, but the truth of ϕ does not follow from the truth of ψ.

There is a certain complexity in this that we did not encounter previously, that being **implication involves causality**. Implication has a truth part, and a causation part. However, we only need to focus on the truth part, and not the causation part, as the truth part is enough for mathematics. We shall call the truth part, **the conditional** (or sometimes, the material conditional).

We will split implication into two parts: **conditional** ($\implies$), and the causation (which we shall leave for the philosophers).

For example, $\phi \implies \psi$ is the truth part of "ϕ implies ψ", where ϕ is the **antecedent**, and ψ is the **consequent**.

We will define the truth of $\phi \implies \psi$ in the terms of the truth or falsity of ψ or ϕ. When ϕ does imply ψ, $\phi \implies \psi$. The causation is always defined.

The truth table for this is going to be

ϕ	ψ	$\phi \implies \psi$
T	T	T
T	F	F
F	T	T
F	F	T

§2 Equivalence

Two statements are said to be equivalent, or **logically equivalent**, if one implies the other. We have to introduce an analogous version of the conditional for equivalence, called the **biconditional**.

Two statementes ϕ and ψ are said to be logically equivalent, if one implies the other.

Biconditional is represented by $\phi \iff \psi$, which is an abbreviation of $(\phi \implies \psi) \wedge (\psi \implies \phi)$. The previous is true of ϕ and ψ are both true or both false. One way to show that two statements Ψ and Φ are equivalent if they have the same truth values.

Example 2.1

$(\phi \wedge \psi) \vee (\neg)$ is equivalent to $\phi \implies \psi$, where the first sentence is Φ and the second is Ψ.

The following all means "ϕ implies ψ",

1. If ϕ, then ψ.
2. ϕ is sufficient for ψ.
3. ϕ only if ψ.
4. ψ if ϕ.
5. ψ whenever ϕ.
6. ψ is necessary for ϕ.

To understand this, an example would be to interpret ϕ as "I have a bicycle", and ψ as "I can ride in the tour de france".

The following all means "ϕ is equivalent to ψ",

1. ϕ is <u>necessary and sufficient</u> for ψ.
2. ϕ <u>if and only if</u> ψ (abbreviated **iff**).

Both shows how ubquious iff is, as both conditions must imply the other.