

Name: _____

Algorithms and Data Structures
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Quiz 7
given: 2017-05-16

You have 20 minutes.

Problem 1

Points: 5+2+3

Consider the following recursive function:

```
fun product( $x : \mathbb{Z}$ ) :  $\mathbb{Z}$  =  
  if  $x == 0$   
    1  
  else  
     $x \cdot \text{product}(x - 1)$ 
```

1. Convert it to a dynamic program by completing the blanks below.
2. Give the Θ -class of the space complexity of the resulting dynamic program.
3. Explain in which situations a dynamic program performs better than the recursive variant. Does it matter in this case?

```
fun product( $x : \mathbb{Z}$ ) :  $\mathbb{Z}$  =  
   $results := \text{new Array}[\mathbb{Z}](x)$   
  for  $i$  in  $0, \dots, x$   
  
    _____  
  
   $results[_____]$ 
```

Problem 2

Points: 3+2+5

Assume a fixed list D of denominations of coins, sorted decreasingly. We want to find the smallest set of coins whose denominations add up to n .

We use the following greedy-style algorithm:

```
fun greedyCoinChange( $D : \text{List}[\mathbb{Z}], n : \mathbb{Z}$ ) :  $\text{List}[\mathbb{Z}]$  =  
   $solution := Nil$   
  for  $x$  in  $D$   
    while  $x + \text{sumList}(solution) \leq n$   
       $solution := \text{prepend}(x, solution)$   
   $solution$   
  
fun sumList( $l : \text{List}[\mathbb{Z}]$ ) :  $\mathbb{Z}$  =  
  return the sum of all elements in  $l$ 
```

precondition: D is sorted by \geq

1. Give the result for running $\text{greedyCoinChange}([50, 20, 10, 5, 2, 1], 14)$.
2. For fixed D , the algorithm's run time is $\Theta(n^2)$. Give a simple improvement that makes the run time $\Theta(n)$.
3. The algorithm is not correct in general. Give an example where $\text{greedyCoinChange}(D, n)$ returns a non-optimal result.