Name:

Algorithms and Data Structures
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Quiz 5
given: 2017-04-20

You have 20 minutes.

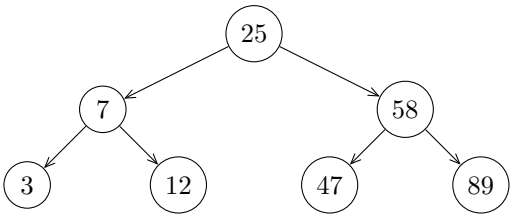
Problem 1

Points: (1+1)+(3+3)

1. Give the Θ-class of the worst-case time complexity of $contains(x : Set[A], a : A) : bool$ (in terms of $|x|$) if x is represented as a
   (a) bit vector
   (b) list set
2. Consider the set $\{3, 7, 12, 25, 47, 58, 89\} \in Set[\mathbb{N}]$.
   (a) Assume it is represented as a hash set using the hash function
   $$hash : x \mapsto (\text{sum of digits of } x) \bmod 10$$
   For every bucket, say which elements it contains.
   (b) Assume it is represented as a binary search tree using the ordering $\leq$. Give the binary search tree for the case where the tree is optimally balanced.

Solution:

1. (a) $\Theta(1)$
   (b) $\Theta(|x|)$
2. Consider the set $\{3, 7, 12, 25, 47, 58, 89\} \in Set[\mathbb{N}]$.
   (a) ten buckets, one each for hash values $0, \ldots 9$. Bucket 1: 47, bucket 3: 3, 12, 58, bucket 7: 7, 25, 89, other buckets empty.
   (b)

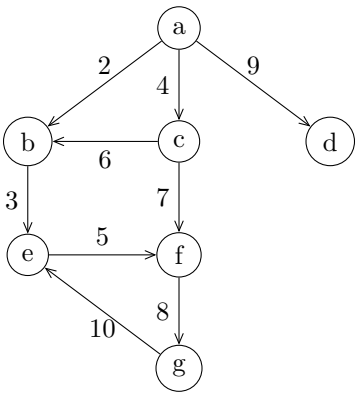


Problem 2

Points: 1+1+2+2+2+4

Consider the following graph:

Name:



1. Give the out-degree of the node a.
2. Give a cycle in this graph.
3. Give all nodes that are reachable from b.
4. Interpreting the weights as costs, give the cheapest path from a to g.
5. Interpreting the weights as capacities, give the capacity of the greatest flow from a to g.
6. Give the result of applying Kruskal's algorithm.

Solution:

1. 3
2. The path $[e, f, g, e]$ (or $[f, g, e, f]$ or $[g, e, f, g]$).
3. b, e, f, g
4. $[a, b, e, f, g]$ (with cost 18)
5. 6 (we flow 2 along $[a, b, e, f, g]$ and 4 along $[a, c, f, g]$; the paths overlap at $[f, g]$ but that edge has enough capacity for both)
6.