Differential privacy - Basic notions and methods

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 - ϵ - δ -differential privacy and its properties
- Methods to archive differential privacy
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- So the probability for any individual in the database to have a property should barely differ from the base rate
- Then, analyzing the database an attacker can't reliably learn anything new about any individual in the database, no matter how much additional information he has

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- Latanya Sweeney from Carnegie Mellon University linked the anonymized Massachusetts Group Insurance Commission (GIC) medical encounter database with voter's registration records identifying the medical records of the Governor of Massachusetts.

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- We want to be able to still use surveys and statistical studies, without compromising the privacy of our subjects.
 - → A standard example are medical records, having an obvious use. However, many people want their medical data to be safe.
- If we don't give people a proof of their privacy, they might not submit the surveys or lie
 - → This destroys the reliability of the obtained results.

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Caveat: Depending on the query, the result of the query after the modification of the database, might not be very useful

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 - \hookrightarrow If the survey shows that any human a certain property, then we also have that property (example: smoking Mary)
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- - This is the key idea of differential privacy

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Definition (ϵ - δ -differential privacy)

Now \mathcal{A} is called ϵ - δ -differentially private if $\forall \mathcal{S} \subset \operatorname{Range}(\mathcal{A})$:

$$\forall D' \colon \mathrm{dist}(D,D') \leq 1 \Rightarrow \mathsf{Pr}\left[\mathcal{A}(D) \in \mathcal{S}\right] \leq \mathsf{e}^{\epsilon} \cdot \mathsf{Pr}\left[\mathcal{A}(D') \in \mathcal{S}\right] + \delta$$



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- Firstly, let us assume that $\delta = 0$.
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 - → Then, given the result of the survey, an attacker cannot learn any new property about us with a significant probability

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- If result is tails, report truthfully
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This way, participants are guaranteed plausible deniability, Even if participant has property P and reports it, this is not incriminating.

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 - \hookrightarrow Hence, this method is ln(3)-differentially private
 - \hookrightarrow Since, the ϵ 's for different sub-surveys add up, a survey of m such questions is $m \cdot \ln(3)$ -differentially private

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- The *l*₁-sensitivity intuitively tells us how much a single individual's data can affect the result of our query.
 - → This, gives upper bound, for amount of randomness we need to add to gain differential privacy

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The probability density function of the Laplace-Distribution is defined as the function

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Remark

We could also use the Gaussian-Distribution instead, but the Laplace-Distribution is a bit handier.

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Definition (Laplace mechanism)

The Laplace mechanism $\mathcal{M}_{L,f,\epsilon}(x)$ for f and a given ϵ is defined as:

$$\mathcal{M}_{L,f,\epsilon}(x) := f(x) + (\mathcal{Y}_1,\mathcal{Y}_2,\ldots,\mathcal{Y}_k),$$

where the \mathcal{Y}_j are random variables drawn from the Laplace-Distribution Lap $(\frac{\triangle f}{\epsilon})$.

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Theorem

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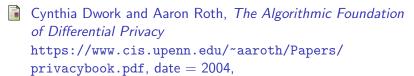
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Theorem

The Laplace mechanism is ϵ -differentially private.

Proving this theorem is beyond the scope of this talk.

References



Wang Yuxiang Differential Privacy: a short tutorial, https://www.cs.cmu.edu/~yuxiangw/docs/Differential%20Privacy.pdf, 2012

Thank you for your attention