Homework 7

You have to submit your solutions as announced in the lecture.

Unless mentioned otherwise, all problems are due 2017-03-30, 11:00.

There will be no deadline extensions unless mentioned otherwise in the lecture.

Problem 7.1 Heap-Backed Priority Queue

Points: 8

Points: 3+6+6

Homework 7

given: 2017-03-21

Implement $Heap[int, \geq]$ in any programming language.

Use it to implement PriorityQueue[int] where the priority of each element is the element itself. Depending on your programming language, this may look like

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 \begin{array}{lll} \textbf{class } \textit{MaxHeap}() \\ \textbf{private } \textit{elements} := & \text{the underlying data structure backing the heap, e.g., a binary tree} \\ \textbf{fun } \textit{insert}(x:\textit{int}) : \textit{unit} = & & & & & & \\ & \dots & & & & & & \\ \textbf{fun } \textit{extract}() : \textit{Option}[\textit{int}] = & & & & & \\ & \dots & & & & & \\ \textbf{fun } \textit{find}() : \textit{Option}[A] = & & & & & \\ & \dots & & & & & \\ \textbf{class } \textit{IntPriorityQueue}() & & & & & \\ \textbf{private } \textit{elements} := \textbf{new } \textit{MaxHeap}() & & & & \\ \textbf{fun } \textit{enqueue}(x:\textit{int}) : \textit{unit} = & & & \\ & & & & & & \\ \textit{elements.insert}(x) & & & & \\ \textbf{fun } \textit{dequeue}() : \textit{Option}[\textit{int}] = & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & \\ & & \\ & & \\ & \\ & & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ & \\ &
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Write a test program that

- creates a new priority queue
- enqueues some values into it
- dequeues all values and prints them

Problem 7.2 Trees, DFS/BFS

Implement a data structure for Tree[A].

Implement two functions $Tree[A] \to Iterator[A]$ that return the nodes of a tree in

- 1. DFS order
- 2. BFS order

One way to do this is to use the functions from the lecture notes to build the list of all nodes in the tree and then to turn the list into an iterator. That is bad because it forces traversing the entire tree. The right wy to do it is to build the iterator in such a way that the next node is only visited when *next* is called on the iterator. The functions given in the lecture can be adapted in this way.

Problem 7.3 Points: 5

Prove the theorem in the lecture notes about the number of nodes in a perfect binary tree