Multiscale Numerical Modeling of Levee Breach Processes

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FEMTEC 2011
South Lake Tahoe
May 9 – 13, 2011

Acknowledgments

- Lea Jenkins (Clemson University)
- John Chrispell (Indiana University of Pennsylvania)
- Tim Kelley (North Carolina State University)
- Clint Dawson, Steve Mattis, Tim Povich (University of Texas at Austin)
- Stacy Howington, Jerry Ballard, Maureen Corcoran, John Peters (ERDC)

Outline

- Overview
- Subsurface Processes
- Surface Processes
- Open Issues

Why do we need to simulate Levees?

- Levees are structures intended to protect large areas from high water and are constructed primarily of porous materials.
- They typically run along one or both sides of a waterway for very large distances and, therefore, employ cheaper construction than dams.
- Many are very old and all require continuous maintenance.
- We need to model their performance in order to identify potential failures and to design new and/or improved levees.

How do levees function (or fail)?

- Earthen materials don't hold back water like a concrete dam.
- The free surface (air/water interface) is continuous as it passes from the waterway into the levee.
- As water flows through porous media it loses energy to the medium through friction, which causes a drop in water pressure.
- The result is a free surface that slopes down away from the flood side of the levee until it either forms a "seepage face" on the land side of the levee or (preferably) stays under ground.
- Failure can result from large scale stresses arising from pore water pressure and gravity, from erosion due to seepage, and from erosion due to overtopping.

Darcy's law and single phase flow in porous media

- The water momentum balance in the porous medium takes the special form of Darcy's law, which assumes that microscopic momentum loss to the soil is linear in the velocity and that macroscopic inertial and viscous terms are negligible:

$$\nabla p - \rho \mathbf{g} + \boldsymbol{k}^{-1} \mathbf{v} = 0 \tag{1}$$

- Combined with fluid mass conservation we usually write the flow equation as

$$\nabla \cdot -\boldsymbol{K} \nabla \phi = 0 \tag{2}$$

where $\phi = \frac{p}{\rho \|g\|} + z = \psi + z$ is known as the hydraulic head and ψ is known as the pressure head.

Modeling seepage

- The seepage problem can be modeled as a free boundary problem for single phase flow or as two-phase flow in porous media.
- Boundary conditions along a portion of the levee and land surface are Signiorini boundary conditions:

$$\psi(\mathbf{v} \cdot \mathbf{n}) = 0 \quad \mathbf{v} \cdot \mathbf{n} \geq 0 \quad \psi \leq 0 \; \mathbf{x} \in \Gamma^S \tag{3}$$

- We will focus on the two-phase approach, more specifically Richards' equation, which replaces K with a nonlinear relative conductivity $K(\psi)$

$$\frac{\partial m(\psi)}{\partial t} + \nabla \cdot -K(\psi)\nabla\phi(\psi) = 0 \tag{4}$$

Finite element formulation

- The quantity of interest in seepage modeling is the pressure so we use standard Galerkin finite elements on tetrahedra with nodal quadrature and element-based material properties.
- We use weak enforcement of boundary conditions, for global conservation and easier enforcement of the Signiorini condition.
- The nonlinear system is solved with Newton's method and a simple linear search and various parallel solvers/preconditioners from PETSc.

Finite element formulation, ctd.

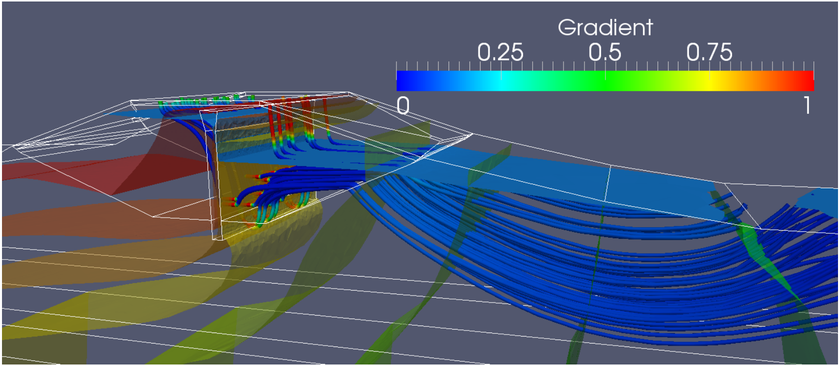
Find $\psi_h \in V_h(\Omega)$ such that

$$
\begin{aligned}
&(m(\psi_h), w_h)_{L_2(\Omega)} + (K(\psi_h)\nabla\phi_h, \nabla w_h)_{L_2(\Omega)} + && && \quad (5)\\
&(-K(\psi_h^-)\nabla\phi_h^- \cdot \mathbf{n} + \gamma(\psi_h^- - \psi^D), w_h^-)_{L_2(\Gamma^{D*})} &=& \; 0 \quad \forall w_h \in W_h(\Omega)
\end{aligned}
$$

where Γ^{D*} is the actual portion of $\partial\Omega$ with Dirichlet conditions AND the portion of the Signiorini boundary with

$$-K(\psi_h^-)\nabla\phi_h^- \cdot \mathbf{n} + \gamma(\psi_h^- - \psi^D) > 0 \qquad (6)$$

That is, when flow is out of the domain on the Signiorini boundary, weakly enforce ψ^D


Gradient
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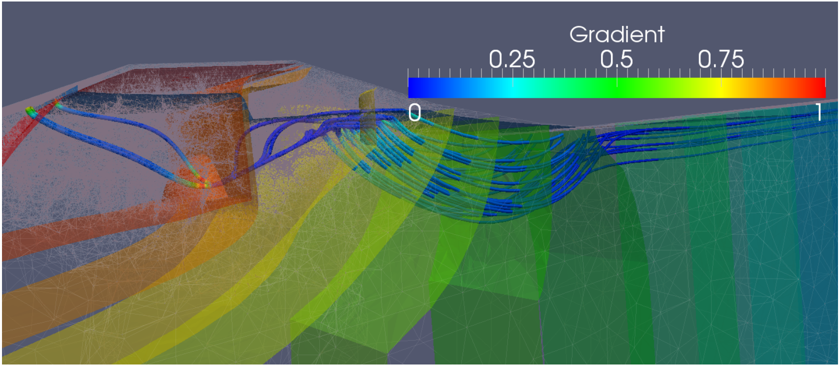
Gradient
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Elasto-Plastic Soil Mechanics

- As with the flow, we model the deformation of the levee by treating soil/fluid mixture as a continuum.

$$-\nabla \cdot \boldsymbol{\sigma} + \mathbf{f} = 0 \tag{7}$$

  where $\boldsymbol{\sigma}$ is the effective stress tensor, which includes the pressure head from the seepage calculation.
- The most significant difficulty is the fact that we have to solve a local, nonlinear differential equation involving stress, strain, and local "thermodynamic" variables (especially near slope failures) because the soil undergoes *plastic* strains.

Stress/Strain rates

A general elastic-plastic material response with isotropic hardening given in terms of the Jaumann rate of effective stress is

$$\begin{aligned}
\overset{\circ}{\boldsymbol{\sigma}} &= \boldsymbol{D}^e(\dot{\epsilon} - \dot{\epsilon}^p) && (8)\\
\dot{\epsilon}^p &= \dot{\lambda}\boldsymbol{r}(\sigma, q) && (9)\\
\dot{q} &= \dot{\lambda}h(\sigma, q) && (10)\\
\dot{\lambda} &\geq 0 && (11)\\
f(\sigma, q) &\leq 0 && (12)
\end{aligned}$$

Integration of the Stress rate

Writing the rates as backward differences and eliminating the time scale since the equation is homogeneous gives an algebraic system at each quadrature point:

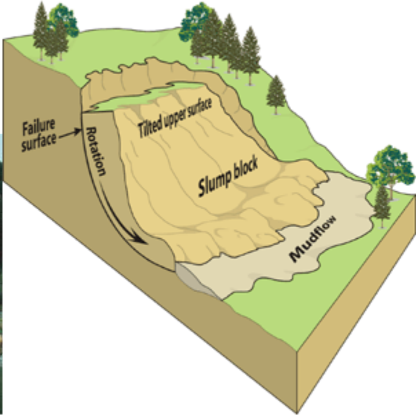
$$
\begin{aligned}
\Delta\sigma &= \boldsymbol{D}^e(\Delta\epsilon - \Delta\epsilon^p) && (13)\\
\Delta\epsilon &= \Delta\lambda \boldsymbol{r}(\sigma^{n+1}, q^{n+1}) && (14)\\
\Delta q &= \Delta\lambda h(\sigma^{n+1}, q^{n+1}) && (15)\\
\Delta\lambda &\geq 0 && (16)\\
f(\sigma^{n+1}, q^{n+1}) &\leq 0 && (17)
\end{aligned}
$$

where $\sigma^{n+1} = \sigma^n + \Delta\sigma$.

Finite element formulation

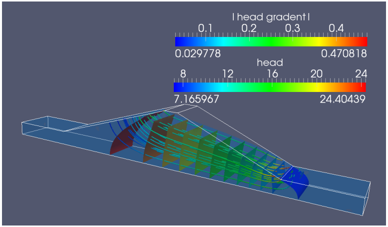
- Standard linear tetrahedral elements are too “stiff” and make the structure appear more stable than it is.
- In this work we use quadratic and linear tetrahedral elements.
- Structural failure is defined as the point when no stable equilibrium solution can be found or when the ratio of plastic work to total work becomes large enough.
- There is typically a spherical failure surface for steep slopes.

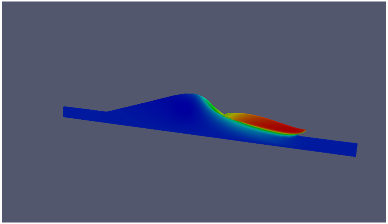
Slope failures



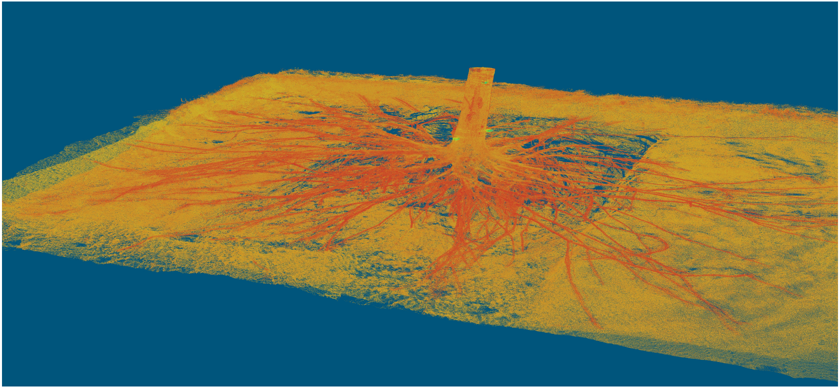
Slope stability analysis

Coupled Seepage and Stability

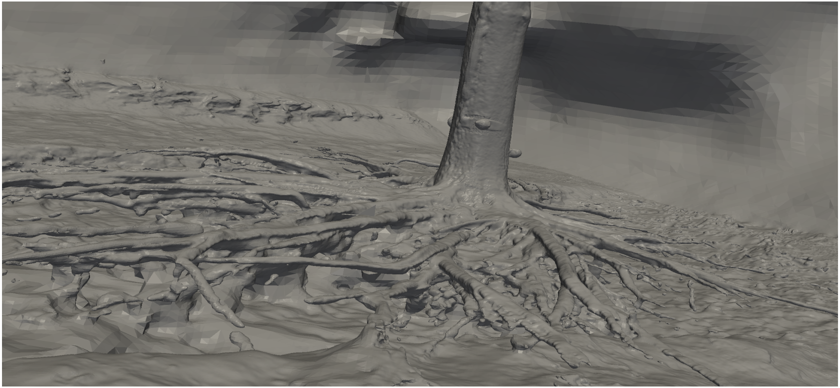




LiDAR root scans



Triangulated surface reconstruction



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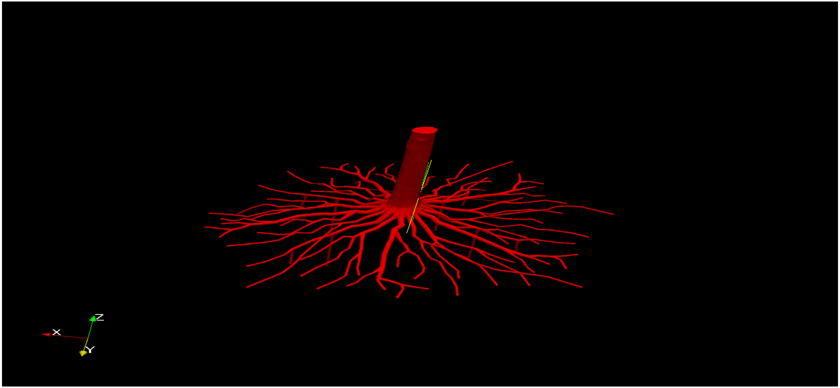
Hand-generated surface reconstruction



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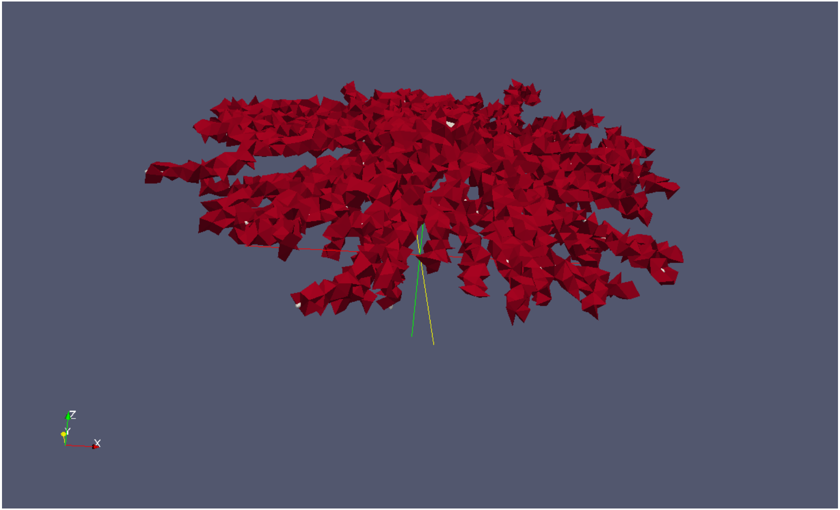
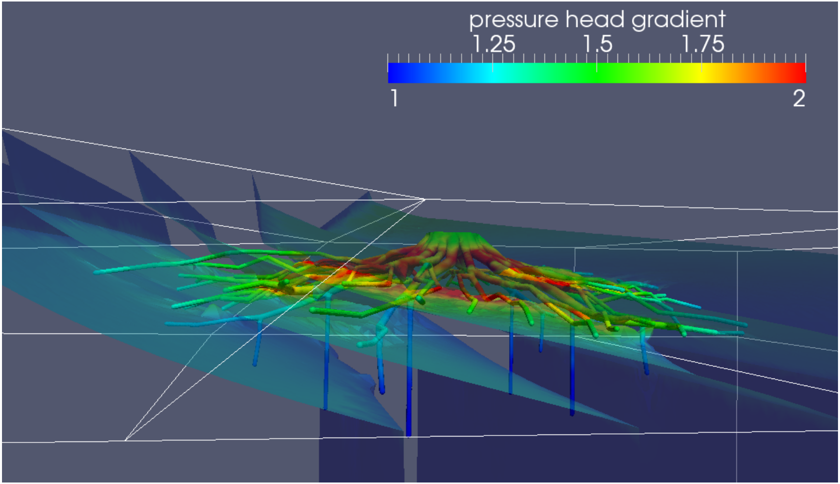
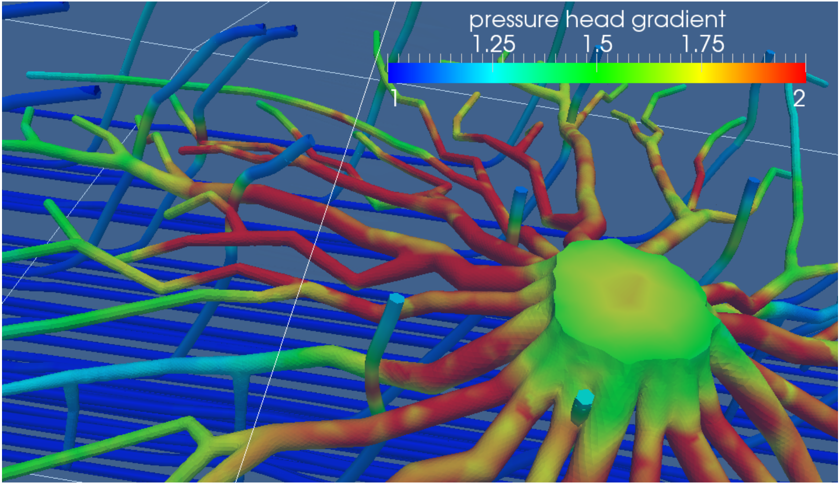
Material reclassification approach



Seepage if root-soil is low-permeability



Seepage if root-soil is low-permeability (zoom)



Stability with root-soil strengthening marginal

Stability with root-soil strengthening x1000

Failure by Overtopping

- High-velocity flow forms on land-side slope
- Shear stress generates rapid erosion
- Head cut evolves up slope to generate failure [4].

Process regimes/scales

- Sediment-laden mixtures
- Supercritical and turbulent flow
- Free surface deformations proportional to depth
- Fully three-dimensional deformations of free surface and sediment layer (no height function)

Existing approaches

- Depth-averaged and partly time-averaged (waves)
- Surface and layer height are unknowns
- Constitutive relations for morphological processes posed at space/time-integrated scale.

St. Venant/Exner System

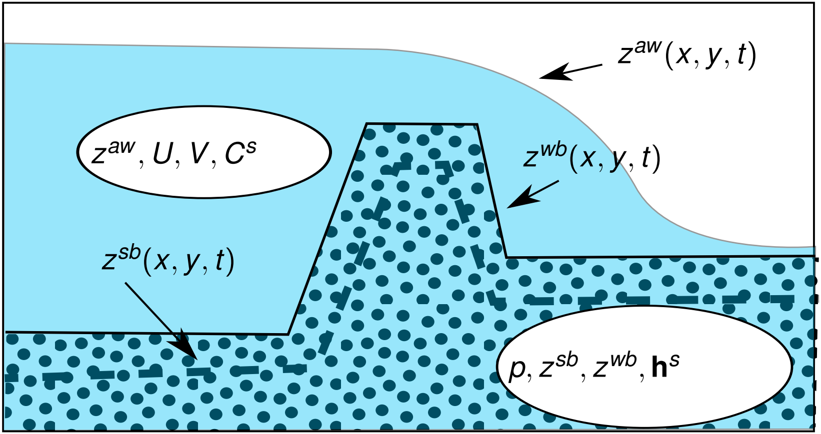
$$\frac{\partial H}{\partial t} + \nabla^{x,y} \cdot \mathbf{U} = 0 \tag{18}$$

$$\frac{\partial H\mathbf{U}}{\partial t} + \nabla^{x,y} \cdot (H\mathbf{U} \otimes \mathbf{U} - \boldsymbol{\sigma}) = gH\nabla z^{wb} + S^{w} \tag{19}$$

$$\frac{\partial (HC)}{\partial t} + \nabla^{x,y} \cdot \mathbf{q}^{s} = E - D \tag{20}$$

$$\frac{\partial \epsilon^{s} z^{wb}}{\partial t} + \nabla^{x,y} \cdot \left(\mathbf{q}^{s} + \mathbf{q}^{b}\right) = -\frac{\partial (HC)}{\partial t} \tag{21}$$

Field variables in depth-averaged approach



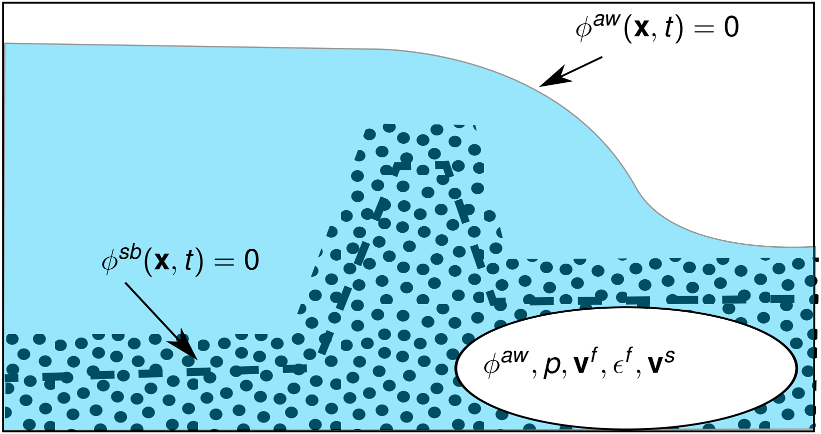
3D Multiphase Approach

- Use a fully three-dimensional approach
- Replace the height function for air/water interface using the level-set approach
- Replace the bed height function using mixture formulation for sediment/fluid mixture
- This results in two Navier-Stokes type systems plus some auxiliary equations: $\kappa - \epsilon$(turbulence), ϕ (level set)
- Sediment/fluid coupling formulated in terms of closure relations for fluid/sediment interaction force, f^{fs}, stress, and turbulence production.

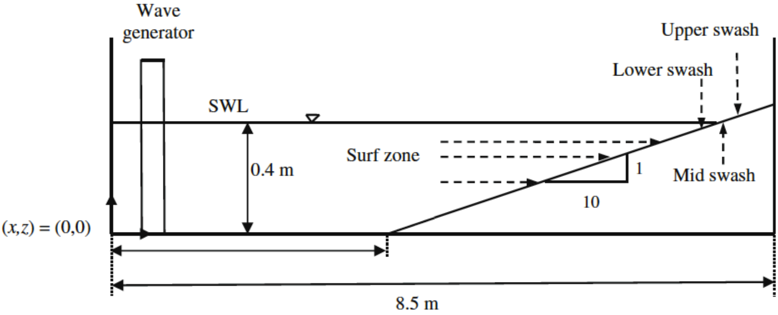
Basic 3D Multiphase Model

$$
\begin{aligned}
\frac{\partial \rho^f \epsilon^f}{\partial t} + \nabla \cdot \rho^f \epsilon^f \mathbf{v}^f &= 0 & (22)\\
\frac{\partial \rho^f \epsilon^f \mathbf{v}^f}{\partial t} + \nabla \cdot \left[\rho^f \epsilon^f \mathbf{v}^f \otimes \mathbf{v}^f - \boldsymbol{\sigma}^{\boldsymbol{f}}\right] &= \rho^f \epsilon^f \mathbf{g} - \mathbf{f}^{fs} & (23)\\
\frac{\partial \rho^s \epsilon^s}{\partial t} + \nabla \cdot \rho^s \epsilon^s \mathbf{v}^s &= 0 & (24)\\
\frac{\partial \rho^s \epsilon^s \mathbf{v}^s}{\partial t} + \nabla \cdot \left[\rho^s \epsilon^s \mathbf{v}^s \otimes \mathbf{v}^s - \boldsymbol{\sigma}^{\boldsymbol{s}}\right] &= \rho^s \epsilon^s \mathbf{g} + \mathbf{f}^{fs} & (25)\\
\frac{\partial \phi^{aw}}{\partial t} + \epsilon^f \mathbf{v}^f \cdot \nabla \phi^{aw} &= 0 & (26)
\end{aligned}
$$

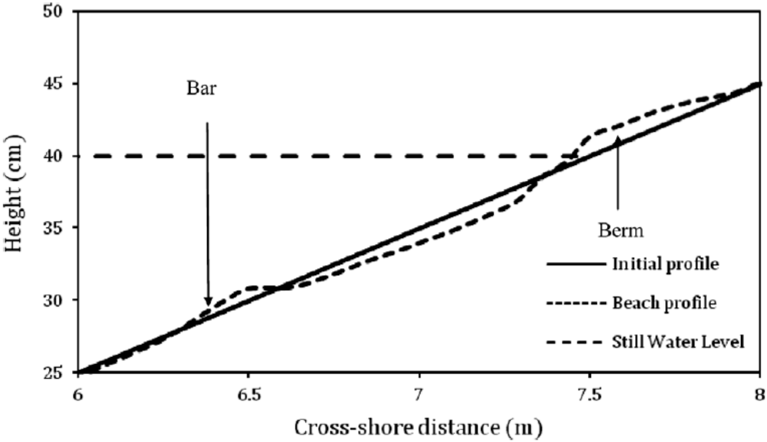
Field Variables in 3D Multiphase Model



Wavetank Test Problem



Beach Profile Evolution



MARIN Experiment

Open Issues

- Geometry pre-processing and mesh generation
- *hp*-adaptivity
- Scalable, robust solvers
- Operator splitting (or not)

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Collaboration, Funding, and Employment Opportunities

- Partnering and contracting: erdc.usace.army.mil
  - The Broad Area Announcement (BAA) is a standing call for basic and applied research proposals
  - Cooperative Research and Development Agreements (CRADAs) can be developed to share staff, equipment, and facilities
- Post-doctoral positions:

  www.orau.org/maryland/participants/chl_projects.html
- Army Research Office grants in engineering, physics, and mathematics:

  www.aro.army.mil

Thank You!