A mean-reverting model to create macroeconomic scenarios for credit risk models

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Abstract

Loan pricing models require forecasts over the life of the loan. The loss reserve calculations proposed by FASB and included in IFRS 9 require lifetime forecasts. In both cases, we cannot create forecasts that assume the current or historic environment persists for many years into the future. Instead, a more reasonable approach is to use macroeconomic scenarios for the near term and then relax onto the long-run average for future years.

In the current work, we create a loan-level forecasting model using an age-vintage-time structure for retail loans, in this case, a small auto loan portfolio. The loan-level age-vintage-time model is similar in structure to an Age-Period-Cohort model, but estimated at the loan-level for greater robustness on small portfolios. The environmental function of time is correlated to macroeconomic factors, which is then extrapolated back in time before the performance data to stabilize the trend of the environment function.

Using prior economic conditions, we create an environmental index with which to calibrate a discrete version of an Ornstein-Uhlenbeck mean-reverting model. Ornstein-Uhlenbeck are best applied to stationary processes, which is true for the environmental function derived from APC-type models. The mean-reverting model is used to transition from the near-term macroeconomic scenario to the long-run average to provide stable lifetime estimates for long-duration loans.

This approach is in line with the explicit goals in the new FASB loan loss accounting guidelines. In addition, this model provides a simple mechanism to transition between point-in-time and through-the-cycle economic capital estimates with an internally consistent model.

Keywords: Forecasting; Mean-reverting Models; Credit Risk; Time series; Age-Period-Cohort Models

1 Introduction

Increasingly, retail lenders require accurate lifetime loss estimates that incorporate current economic conditions, near-term economic scenarios, and reasonable long-term extrapolations. The US Financial Accounting Standards Board (FASB) has proposed just such a view as part of their new procedure for computing lifetime losses. In addition, when lenders price loans and estimate lifetime margin, a similar view is necessary.

In Figure 1 we show the basic concept.

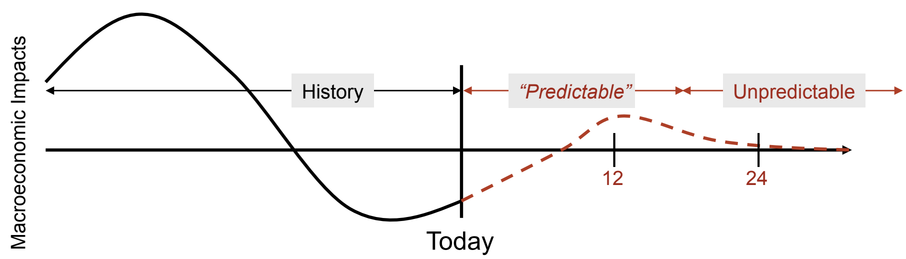

Figure 1: A timeline of macroeconomic impacts representing the assumed transitions from history to a near-term macroeconomic scenario eventually relaxing onto long-run conditions.

For any model that includes macroeconomic factors, $E_i(t)$, we can aggregate those factors into a single index, $H(t)$. Age-Period-Cohort models (APC) [10, 8] directly produce a function of time that captures the net impact of macroeconomic conditions over time via a fixed effect in time rather than explicitly including macroeconomic factors. With either approach, we can take the estimated index $H(t)$ as a starting point for out analysis.

Due to the intrinsic autocorrelation structure of the economy, economists are better than random at predicting six to twelve months forward. Assuming that a favorite scenario is taken by the lender for the near-future, the goal is to create a model that relaxes from the macroeconomic impacts produced by the economist's scenario onto the long-run average impact via an appropriate model.

To create this relaxation onto long-run averages, we could assume a desired functional form. For example, an extrapolation based upon a hyperbolic tangent function could provide the relaxation shown in Figure 1. Although we can design aesthetically desirable extrapolations in this way, we do not obtain any uncertainty bands or possible range of variation.

By adopting a stochastic process for the extrapolation, we can obtain a reasonable extrapolation as well as an uncertainty interval. The Ornstein-Uhlenbeck (O-U) process [11] is the only nontrivial stochastic process that is stationary, Gaussian, and Markovian [3]. The Vasicek model [12] is an Ornstein-Uhlenbeck process as are many other interest rate models.

In this application, we adopt the O-U process just to extrapolate macroeconomic impacts. Any credit risk model with additive macroeconomic impacts [7, 9, 2, 1] can have those impacts collected into an index $H(t)$.

$$H(t) = \sum_{i=1} N c_i E_i(t) + \epsilon_t \tag{1}$$

This is equivalent to the environment function produced via APC.

$$logit(p_i(a, v, t)) = F(a) + G(v) + H(t) \tag{2}$$

Although the O-U process can generate individual stochastic simulations, we will be primarily interested in the closed-form solutions for the expected mean and variance of the process as a very computationally efficient way to meet the modeling needs of lenders. This approach also has the added advantage of merging near-term point-in-time (PIT) estimates with long-run through-the-cycle (TTC) estimates, allowing us to create a single economic capital model that provides both answers.

2 Modeling approach

For any credit risk model that incorporates a set of additive macroeconomic factors, such as shown in Equation 1, the following steps will be appropriate.

1. Create a forecast model that accepts macroeconomic scenarios.
2. Collect the macroeconomic impacts of the credit risk model into a single index.
3. Calibrate a mean-reverting model to the macroeconomic impacts.
4. Obtain a macroeconomic scenario from economist(s).
5. Overlay the mean-reverting model onto the macroeconomic scenario.

The first two steps in this sequence are obvious, so we will focus on the remaining three.

2.1 Ornstein-Uhlenbeck Process

The Ornstein-Uhlenbeck process is a continuous-time stochastic process often described in the context of Brownian motion.

$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t \tag{3}$$

For a studied property x_t, μ is the long-run mean of the process, θ is related to the relaxation time, and σ is related to the variance.

In discrete time, the O-U process simplifies to a structured AR(1) process. In the present context, x_t is replaced with the environmental impacts $H(t)$ from Equation 1.

$$\Delta H(t) = \theta(\mu - H(t))\Delta t + \epsilon_t \tag{4}$$

where

$$\mu = d - \frac{\sigma^2}{2\theta}, \epsilon_t \approx N(0, \sigma). \tag{5}$$

Given this process, the expected mean and variance are

$$E(H(t)) = (1 - e^{-\theta(t-t_0)}\mu + e^{-\theta(t-t_0)}H(t_0) \tag{6}$$

$$Var(H(t)) = \frac{\sigma^2}{2\theta}(1 - e^{-2\theta(t-t_0)} \tag{7}$$

In the limit as $t \to \infty$, this becomes

$$\lim_{t\to\infty} E(H(t)) = \mu \tag{8}$$

$$\lim_{t\to\infty} Var(H(t)) = \frac{\sigma^2}{2\theta} \tag{9}$$

Calibrating the discrete O-U process to loan data is straight-forward, except for the length of history typically available. Under the Basel II minimum requirement of five years of data to create PD models, in most countries at most one economic cycle would be present in the data. This is a dangerously short time interval from which to calibrate the mean and variance of the process.

The solution is simple, though with some caveats. If we assume that the macroeconomic impacts are quantified as in Equation 1 over the period of the loan performance data, typically five years or so, we still have the option to extrapolate Equation 1 backward over previous economic periods. Although loan performance data is rarely available for prior periods, macroeconomic indices are often available for multiple prior decades. The multi-decade extrapolation of $H(t)$ is then used to calibrate the discrete O-U process.

Long-run extrapolations of credit risk models calibrated over comparatively short periods can have trend instabilities. This instability arises naturally from the relationship between the age of the loan a, origination date of the loan v, and performance observation date t wherein $a = t - v$. This issue is discussed in detail by Breeden and Thomas [6] along with suggested solutions. For the rest of the analysis here, we assume that the trend extrapolation issue has been properly addressed. In the present context, we used a simple linear detrending of the backward extrapolation.

Figure 2 provides a visualization of this backward extrapolation, where $H(t)$ is first calibrated against the loan performance data (solid line to the right) and then extrapolated backward through previous decades (long dashes). Over the full extrapolation of $H(t)$, we compute the mean and deviation of $H(t)$ as H_{TTC} and σ_{TTC} respectively. This provides part of the O-U calibration with

$$\mu = H_{TTC} \tag{10}$$

and

$$\sigma^2 = \frac{4}{3}\ln(2)\sigma_{TTC}^2 \tag{11}$$

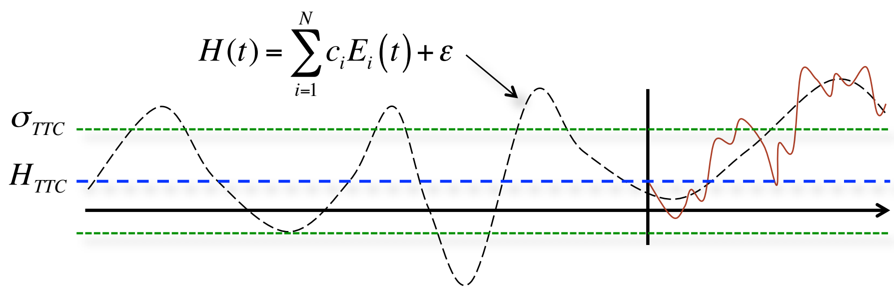

Figure 2: A backward extrapolation of the macroeconomic model fit to a sample of loan portfolio performance. The extrapolation is used to quantify through-the-cycle values of mean and deviation for macroeconomic impacts. The vertical line separates in-sample data to the right from the backward extrapolation to the left.

The last remaining parameter is the relaxation time, for which we have the expression

$$t_{1/2} = \frac{\ln(2)}{\theta} \tag{12}$$

This can also be estimated from $H(t)$, but we can also draw on prior Monte Carlo studies of retail lending models that show $t_{1/2}$ to be on the order of 1.5 to 2 years [5].

Once the calibration is known, the subsequent steps are simply procedural. The practitioner must select an economic scenario from their favorite economist or provider, choose a transition point beyond which the scenario will no longer be taken as a given, and extrapolate from that t_0 using the O-U model.

3 Numerical Example

To provide a numerical example, previous work by the author [4] was extended to include the O-U extrapolation. The same initial steps are followed as before with the addition of the O-U model at the end.

For a small US auto loan portfolio, a loan-level APC model of default rate was created via Equation 2 quantifying a lifecycle function with age of the loan $F(a)$, a credit risk function with vintage $G(v)$, and an environmental function with date $H(t)$ as shown in Figures 3, 4, and 5 respectively.

The environmental function $H(t)$ was fit to macroeconomic data for unemployment rate, changes in house prices, and changes in housing starts. The trend in this model was stabilized via the procedure described in Breeden and Thomas [6] and extrapolated backward through to 1990 when all of the macroeconomic factors first became available for the portfolio region.

The O-U model was calibrated as described above to produce the result in Figure 6. H_{TTC} is shown in the figure as the central dashed line and $+/-\sigma_{TTC}$

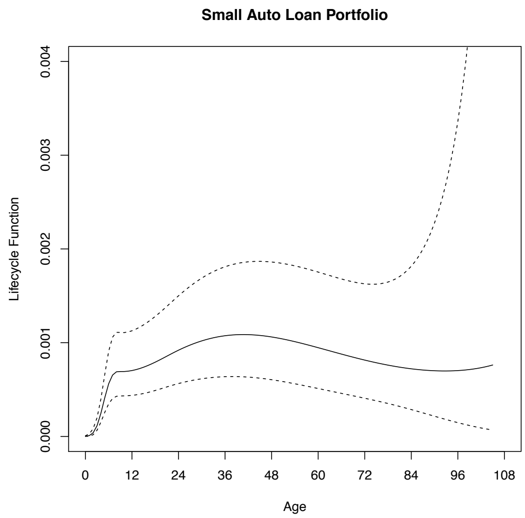

Figure 3: Lifecycle function with account age measured for a small US auto loan portfolio

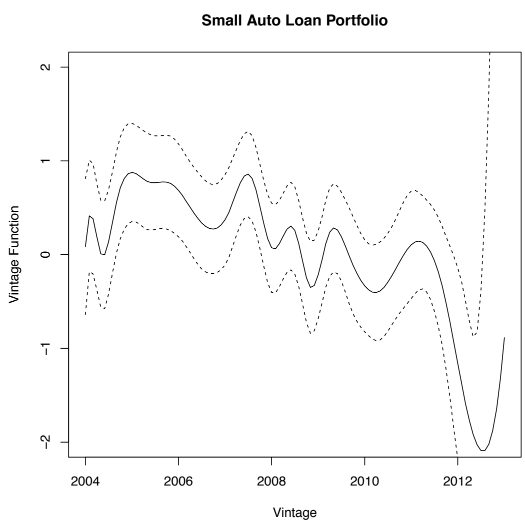

Figure 4: Credit risk function with vintage measured for a small US auto loan portfolio

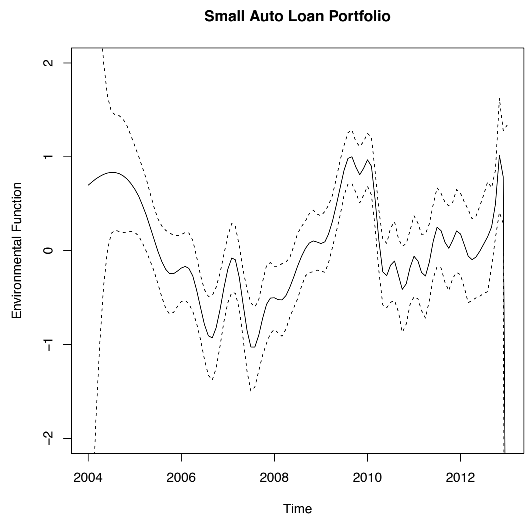

Figure 5: Environmental function with time measured for a small US auto loan portfolio

is shown as the bounding dashed lines. The expected mean of the O-U process is the smooth extrapolation to the future, and the expected deviation of the O-U process is given as the expanding dotted lines.

Figure 7 shows an expanded view with the impact of the provided macroeconomic scenario for the first 12 months, from early 2013 through early 2014, after which the O-U process takes over.

The expected mean and deviations from the O-U process can then be fed as scenarios into the forecast model. For purposes of computing economic capital, higher confidence intervals can be used than the one-standard-deviation bands shown in Figure 7.

4 Conclusions

The application of an Ornstein-Uhlenbeck process allows us to create scenario-based forecasts that relax from current conditions or the end of an accepted macroeconomic scenario onto the long-run average environment over reasonable time scales. This meets the proposed FASB requirement of incorporating current economic conditions into loss forecasts while relaxing onto long-run expectations for lifetime forecasts. From a loan pricing perspective, this provides a mechanism for incorporating current conditions into pricing without the overly pessimistic or optimistic assumption that current conditions persist for the life of the loan.

The choice of an O-U model was natural given the Basel II assumption that long-run PDs exist, and it follows the precedent of using O-U processes in

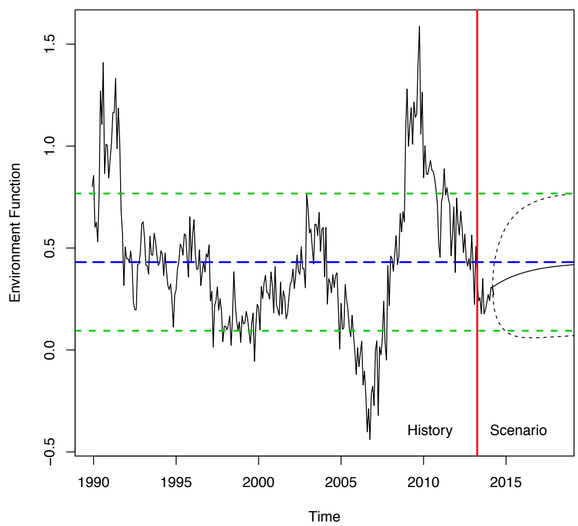

Figure 6: Calibration of the Ornstein-Uhlenbeck process to the backward extrapolation of the macroeconomic fit to $H(t)$ for a small US auto loan portfolio.

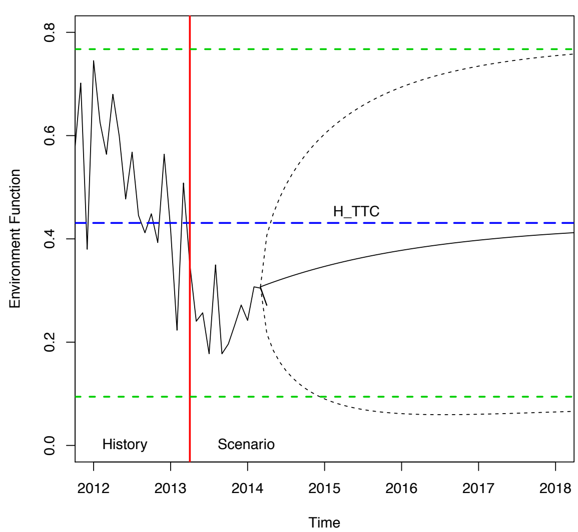

Figure 7: An expansion of Figure 6 highlighting the transition from macroeconomic scenario to the O-U process.

modeling macroeconomic variables. If only expected mean and deviations are required, as was the case here, implementation of a discrete-time O-U model is essentially trivial, once a stable measure of macroeconomic impacts to loan portfolio losses has been obtained.

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