X: bradcasted power

Y: received power

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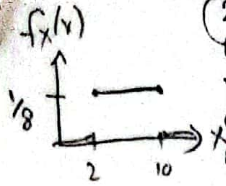
2. TRT Longwave (LW) radio transmitter in Ankara broadcasts globally. Let the transmitted power of this LW radio at any instant be a uniform random variable, X, non-zero between $2 < X < 10$ units. Due to the atmospheric conditions at some particular point in the world, the received power by a LW radio receiver becomes uncertain, represented by the random variable Y. The uncertainity between transmitted and received power is modeled by the conditional pdf $f_{Y|X}(y|x) = \frac{2}{x^2}y$ (Note that the received power must be non-negative and smaller than the transmitted power).

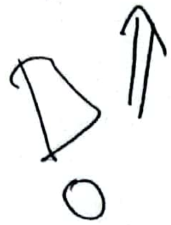


(a) (5 points) Determine the joint pdf between X and Y, $f_{X,Y}(x,y)$. Show the non-zero region for $f_{X,Y}(x,y)$ on the xy-plane.

$f_X = \begin{cases} \frac{1}{8}, & x \in (2,10) \\ 0, & o.w \end{cases}$

(b) (7 points) Determine and plot the marginal pdf of Y, $f_Y(y)$.

(c) (8 points) Determine and plot the conditional pdf(s) of X, given Y, $f_{X|Y}(x|y)$ by indicating the range of values of y for which $f_{X|Y}(x|y)$ is valid.



(d) (5 points) Assume 12 similar LW radio transmitters are built in Ankara with identical power characteristics whose power distributions are uniform between $2 < X_i < 10$ units and they operate independently. If the total transmitter power is defined as $Z = X_1 + ... + X_{12}$, what is the probability of measuring $Z > 74$ units of total power at the output of these transmitters at any instant? [Hint1: Use Central Limit Theorem] [Hint2: For $X \sim uniform[a,b], var(X) = \frac{(b-a)^2}{12}$]

A part of the standard normal table:

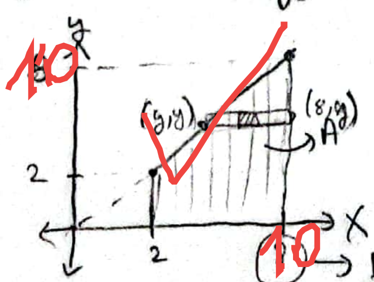
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

ⓐ $f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{2}{x^2}y \rightarrow f_{X,Y}(x,y) = \left(\frac{2y}{x^2}\right) f_X(x) = \frac{y}{4x^2}$ // what about valid region?

(*) We know that $Y > 0$, also Y is X dependent since the received power cannot be greater than X (intiutively). Also the LW transmitter can brodcast 2 to 10 units of power.

$\rightarrow$ by normalization $\int_0^{y_0(x)} f_{Y|X}(y|x)\,dy = 1 \rightarrow \int_0^{y_0(x)} \frac{2y}{x^2}\,dy = \left(\frac{2}{x^2}\right)\left[\frac{y^2}{2}\Big|_0^{y_0(x)}\right] = \frac{2}{x^2}\frac{y_0(x)^2}{2} = \frac{y_0(x)^2}{x^2}$

$y_0(x)^2 = x^2 \rightarrow |y_0(x)| = |x| \rightarrow y_0(x) = x$



$$\boxed{f_{X,Y} = \begin{cases} \frac{y}{4x^2}, & x \in [2,8], y \in [0,x] \\ 0, & o.w \end{cases}}$$ //

This Joint PDF makes sense to me

10 ⊡ should be 10

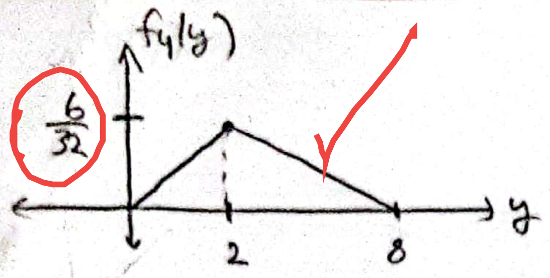
ⓑ $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \begin{cases} \int_2^8 \frac{y}{4x^2}\,dx, & y \in [0,2] \\ \int_y^8 \frac{y}{4x^2}\,dx, & y \in (2,8] \\ 0, & o.w \end{cases}$

(*) $\int \frac{y}{4x^2}\,dx = -\frac{y}{4}x^{-1} = -\frac{y}{4x}$

$$\boxed{f_Y(y) = \begin{cases} \frac{3}{32}y, & y \in [0,2] \\ \frac{8-y}{32}, & y \in [2,8) \\ 0, & o.w \end{cases}}$$ //

$\int_{-\infty}^{\infty} f_Y(y)\,dy = \left(\frac{-y}{32}\right) - \left(\frac{-y}{4y}\right)$

$\frac{1}{4} - \frac{y}{32}$



which is obviously wrong since The area is $\frac{24}{32}$. The normalization is not satisfied.

I could not find what is wrong :) Another approach to try can be This

$$\boxed{f_Y(y) = \int_{-\infty}^{\infty} f_{Y|X}(y,x)\cdot f_X(x)\,dx}$$

total probability law

where $f_{Y|X}(y,x) = \frac{2y}{x^2}$ & $f_X(x) = \frac{1}{8}$ $\Big|$ $x \in [2,10]$, $y \in [0,x]$ zero otherwise

X should be from 2 to 10 but I write 2 to 8 in my solution

$$f_Y(y) = \begin{cases} \frac{y}{10}, & y \in [0,2] \\ \frac{10-y}{40} & \end{cases}$$

c) -8

d) -5