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EE230 Final Exam June 12, 2023

1. The moment generating function of a random variable X is given as $M_X(s) = ae^{-s} + be^{s} + c$, where a, b, and c are constants. It is given that $\mathbf{E}[X] = 0$, $var(X) = \frac{1}{2}$.
   (a) (4 points) Determine a, b, and c.
   (b) (2 points) What kind (continuous, discrete, mixed) of a rv is X?
   (c) (6 points) Determine and plot the cumulative distribution function $F_X(x)$ of X.
   (d) (5 points) A random variable Z is defined as $Z = X + Y$, where X is the random variable given above and Y is a continuous uniform random variable over the interval $[0, 2]$. It is given that X and Y are independent random variables. Determine $M_Z(s)$.
   (e) (8 points) Determine and plot the cumulative distribution function $F_Z(z)$ of Z.

(a) $M_X(s) = E[e^{sX}] = \int_{-\infty}^{\infty} e^{sX} f_X(x)dx \rightarrow \frac{\partial^n M_X(s)}{\partial s^n} = \int_{-\infty}^{\infty} x^n e^{sX} f_X(x)dx$

Thus, $E[x^n] = \left.\frac{\partial^n M_X(s)}{\partial s^n}\right|_{s=0} \Rightarrow \frac{\partial M_X(s)}{\partial s} = -ae^{-s} + be^{s}$ & $\frac{\partial^2 M_X(s)}{\partial s^2} = ae^{-s} + be^{s}$

$E[x] = (b-a)$, $E[x^2] = a+b \rightarrow \boxed{b-a=0} \rightarrow \boxed{a=b}$ //

$Var(x) = E[x^2] - E[x]^2 = (a+b) - (b-a)^2 = a+b - (b^2 - 2ab + a^2) = a+b+2ab - a^2 - b^2 = \frac{1}{2}$

$b=a \Rightarrow a + a + 2a^2 - a^2 - \cancel{a^2} = \frac{1}{2} \rightarrow 2a = \frac{1}{2} \rightarrow \boxed{a = \frac{1}{2}, \ b = \frac{1}{2}}$ //

$\int_{-\infty}^{\infty} e^{sX} f_c(x)dx \rightarrow$ set $f_c(x) = c\,\delta(x) \rightarrow \int_{-\infty}^{\infty} e^{sX} c\,\delta(x)dx = \mathbf{C}$ //