

**MIDDLE EAST TECHNICAL UNIVERSITY**

**DEPARTMENT OF ELECTRICAL AND ELECTRONICAL ENGINEERING**

**EE 464 HOMEWORK 3**

**Compensator Design for Buck Converter**

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**GitHub Page:** https://github.com/erdemcanaz/EE464-HW3

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**Introduction**

You are required to design a 200 W full bridge isolating converter with 24 Vin and 200 Vout with 1% output voltage ripple. Switching frequency of the converter is 250kHz

**Question 1:**

*Explain the meaning of input-to-output transfer function and control-to-output transfer function in words.*

In this context, the input-to-output transfer function refers to the relationship between the input voltage (i.e Vin) and the output voltage (i.e. Vout). On the other hand, the control-to-output transfer function describes the relationship between the control signal (such as the duty cycle) and the output voltage Vout.

**Question 2:**

*Obtain the bode plot for control-to-output transfer function of buck converter with and without ESR (rC ) on single graph on MATLAB. How do nonidealities affect the characteristic? Comment on phase margin & gain margin.*

* *L = 8 uH,*
* *C = 8 uF,*
* *rC = 15 mohm*
* *VIN = 5 V*
* *Vo= 3.3 V*
* *Io = 4 A*
* *fsw = 250 kHz*
* *Vref = 1.2 V*
* *****Vosc= 1.8 V*

The bode plot of the given control-to-output transfer function Gp(s) of the buck converter with and without the ESR can be found in Fig. 1. The MATLAB script used can be found in Appendix I.

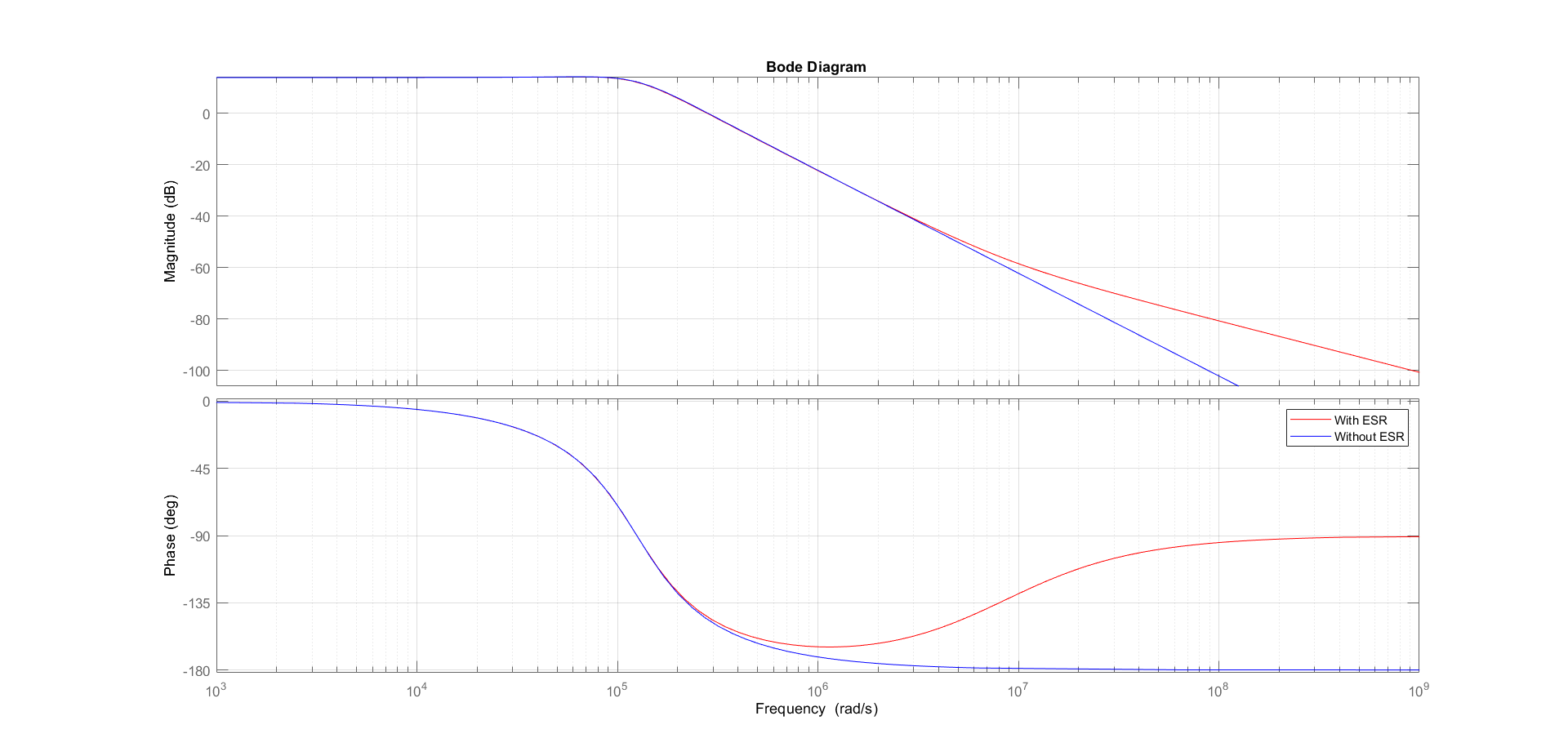


Figure 1 The bode plot of the control-to-output transfer function of the buck converter with and without the ESR

**The main differences between circuits with/without the ESR is, a zero is introduced at around 107 rad/s**. It causes a change in the slope of the magnitude and phase angle behaviors. Lets say magnitude G is;. Gp(s) is of the form

We may replace s with (jw)

Then can be found as;

We can assume the modulus of a complex number can be approximated solely as either real or complex part provided one is much greater magnitude wise (i.e. 10 times). Then the contribution of zero introduced due to ESR can be written as.

It can be further simplified as;

It is obvious that the contribution is zero until . Then it contributes 20dB per decade. Resulting in -40dB/dec slope to become -20dB/dec. For the phase discussion, one can simply consider any complex number as a complex exponential. Knowing real numbers are of the form Ae0 and complex ones are Aei, we can simplify ESR contribution to phase as follows;

Both of gain and phase discussions are well seen in the Fig. 1.

Using MATLAB’s built-in function, gain margins are calculated as infinity for both and phase margins are calculated as and for with and without ESR cases respectively.

The Difference in the phase margins is due to introduced zero has a positive contribution to phase even when .

**Question 3:**

*Identify the pole and zero frequencies for non-ideal buck converter. List them from lower to higher including switching frequency.*

The compact form of the *Gp,ESR(s)* is

The root that make numerator zero is

Whereas roots that make denominator zero are

The frequencies of zeros and poles than found to be

The switching frequency is

Then;

One can use below formulas for the frequency of the poles and zeros directly. [REFERENCE]

*You need to design two different compensators for this application and then you will compare. You can choose different crossover frequencies for two compensators or you can design two compensators with different phase-gain margins having same crossover frequency. It is advised you to apply different options and observe the bode plots and performances. At the end you need to report two of the options and compare performances. Final designs should be stable. An example of a compensator is given in Fig. X.*

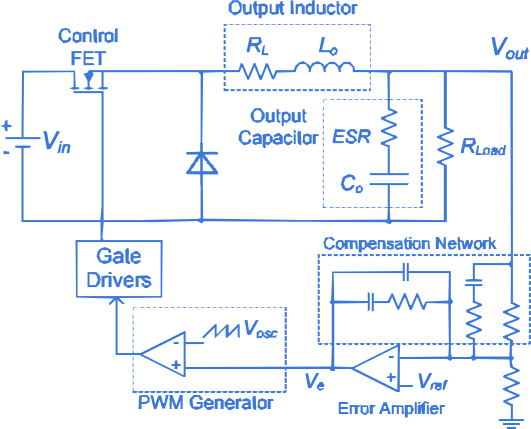
**

Figure x - Simplified circuit diagram of a buck converter with a voltage mode error-amplifier

**Question 4:**

*What does a crossover frequency mean? What is the typical limit for a crossover frequency? Select a crossover frequency for your application, give your reasoning.*

In the application note[REFERENCE], the term “crossover frequency,” also known as the “bandwidth of the loop,” is defined as the point where the gain of the system (i.e. closed loop) equals 0 dB (unity gain). The selection of the crossover frequency is influenced by two factors: the desired response speed to transient changes and the need for effective filtering of switching noise. A higher crossover frequency leads to a faster response, while a lower crossover frequency results in better noise filtering. These two parameters inherently involve a tradeoff; improving one typically compromises the other. It is recommended that the crossover frequency be set between 1/10 to 1/5 of the switching frequency for practical applications. Since the switching frequency is 250kHz, one can choose closed-loop crossover frequency of the buck converter as

**Question 5:**

*There are different types of compensator such as Type-I, Type-II, Type-III A or B. Pick the most suitable one for your design. Give your reasoning.*

Referring to the application notes [REFERENCE], I will use Type-III A&B compensators for my design. With higher component count, it is given that type-III is a more generalized compensator that can be used with less consideration on capacitor types or characteristic frequencies etc.. Also, it’s transient response is told to be relatively fast.

Type I: Simple compensator with a single pole. The topology can be seen in Fig. X. It is discussed that this type is bad at transient response. Which can be also seen in the bode plot since the slope is -20dB/dec starting from w = 0. At higher frequencies, the attenuation is too high.

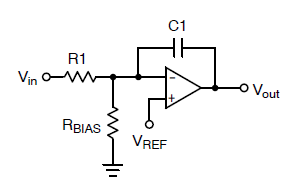


Figure x Type I compensator

The bode plot of the Type-I compensator is given in Fig. X for

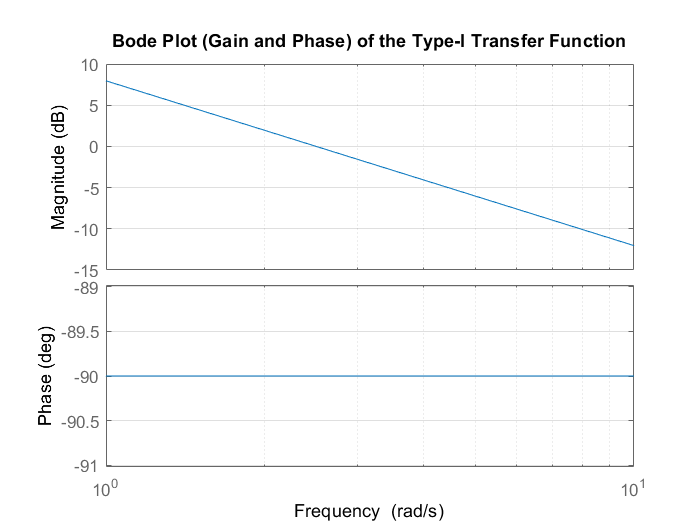


Figure x Type I compensator bode plot for R=50kOhm and C=8uF

Type II: Compensator with a zero-pole pair. The topology can be seen in Fig. X.

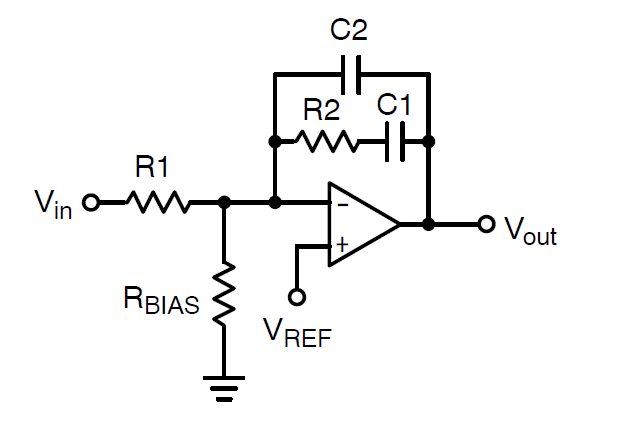


Figure x Type-II compensator

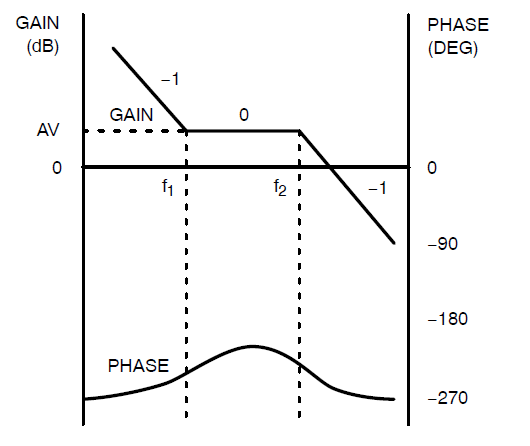


Figure x Type-II compensator typical bode plot

The variables can be found by the following formulas;

Type III: Suggested for a comprehensive solution ensuring unconditional stability with any type of output capacitors and a broad range of ESR values. There are 2 types of type-III compensator. Type A is used when the capacitor ESR is not negligible (i.e. zero is introduced at lower frequencies);

And Type B is used when the EST is negligible (i.e. zero is introduced at higher frequencies) ;

The circuit diagram of type-III compensator is given in Fig. X.

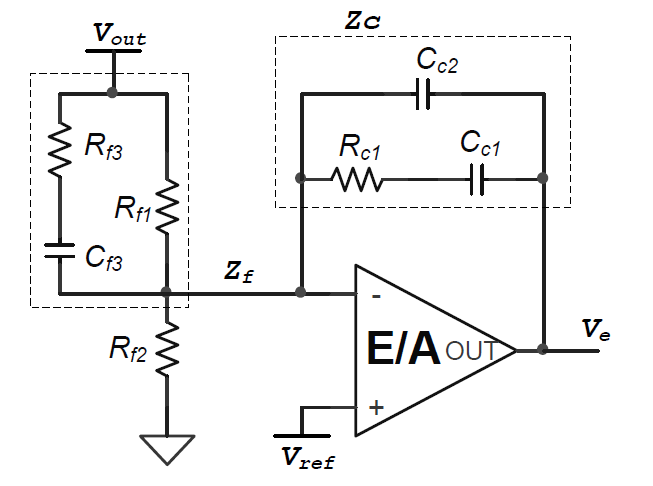
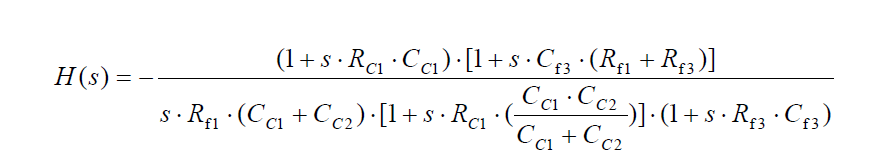
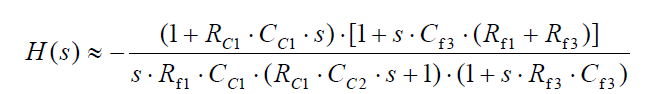


Figure x Type-III compensator

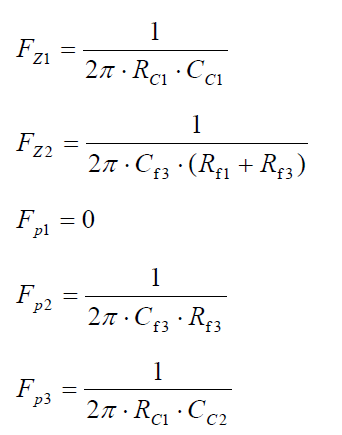
The transfer function H(s) that maps Vout to Ve is given by;



Which can be simplified to (under the assumption of CC2 << CC1



This response has 2 zeros and 3 poles. Their location in Hertz can be found by the following formulas;



**Question 6:**

*Calculate the values of the circuit components for your each compensator. Round up or down the component ratings to the available ratings. For example, 4.68 nF turns into 4.7 nF.*

The parameter names are shown in Fig.X. For those parameters;

Type III-A:

**One should note that with given ESR value of 15mOhm, Type A is not suitable. Thus assume capacitor ESR is 0.2Ohm which is not far away from practical capacitor ESR values. Then can be calculated as;**

With this modification, the type-III-A frequency condition is satisfied. Then using the application note [REFERENCE] we can place poles and zeros of this compensator.

Then;

Then the component values to achieve this pole-zero locations are;

|  |  |  |
| --- | --- | --- |
| **Type-III (A) Component selection** | | |
| Component | Desired Value | Component links |
|  |  | [2.2nF](https://www.direnc.net/22nf-63v-seramik-1) |
|  |  | [680 Ohm](https://www.direnc.net/680r-14w-direnc) + [39 Ohm](https://www.direnc.net/39r-14w-direnc) |
|  |  | [2.7 kOhm](https://www.direnc.net/27k-14w-direnc-1) + [220 Ohm](https://www.direnc.net/220r-14w-direnc-11955) |
|  |  | [1.6 kOhm](https://www.direnc.net/15k-14w-direnc-1) |
|  |  | [6.8 kOhm](https://www.direnc.net/68k-14w-direnc-1) |
|  |  | [1nF](https://www.direnc.net/1nf-63v-seramik) + [560pF](https://www.direnc.net/560pf-63v) |
|  |  | [220pF](https://www.direnc.net/220pf-63v) |

Type III-B:

At question 3, , , and are calculated as;

Which satisfies the type-III-B frequency condition. Then using the application note [REFERENCE] we can place poles and zeros of this compensator.

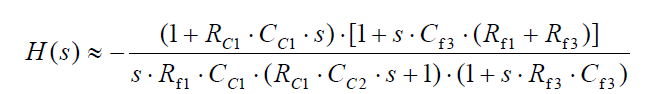
Then the component values to achieve this pole-zero locations are;

The component selection is as follows;

|  |  |  |
| --- | --- | --- |
| **Type-III (B) Component selection** | | |
| Component | Desired Value | Component links |
|  |  | [2.2nF](https://www.direnc.net/22nf-63v-seramik-1) |
|  |  | [390 Ohm](https://www.direnc.net/390r-14w-direnc) |
|  |  | [75 kOhm](https://www.direnc.net/82k-14w-direnc-11877) |
|  |  | [47kOhm](https://www.direnc.net/47k-14w-direnc) |
|  |  | [10 kOhm](https://www.direnc.net/10k-14w-direnc) |
|  |  | [22nF](https://www.direnc.net/22nf-63v-seramik-1) + [10 nF](https://www.direnc.net/10nf-63v-seramik) + [4.7nF](https://www.direnc.net/4-7nf-63v-seramik) |
|  |  | [100pf](https://www.direnc.net/100pf-63v) + [47pf](https://www.direnc.net/47pf-63v) |

**Question 7:**

*Obtain the bode plots of the loop transfer functions for each compensator design. Show and comment on phase margins. You do not need to simulate system for this step. Bode plots of MATLAB transfer functions are acceptable.*



The MATLAB script can be found in Appendix II.

Type III-A:

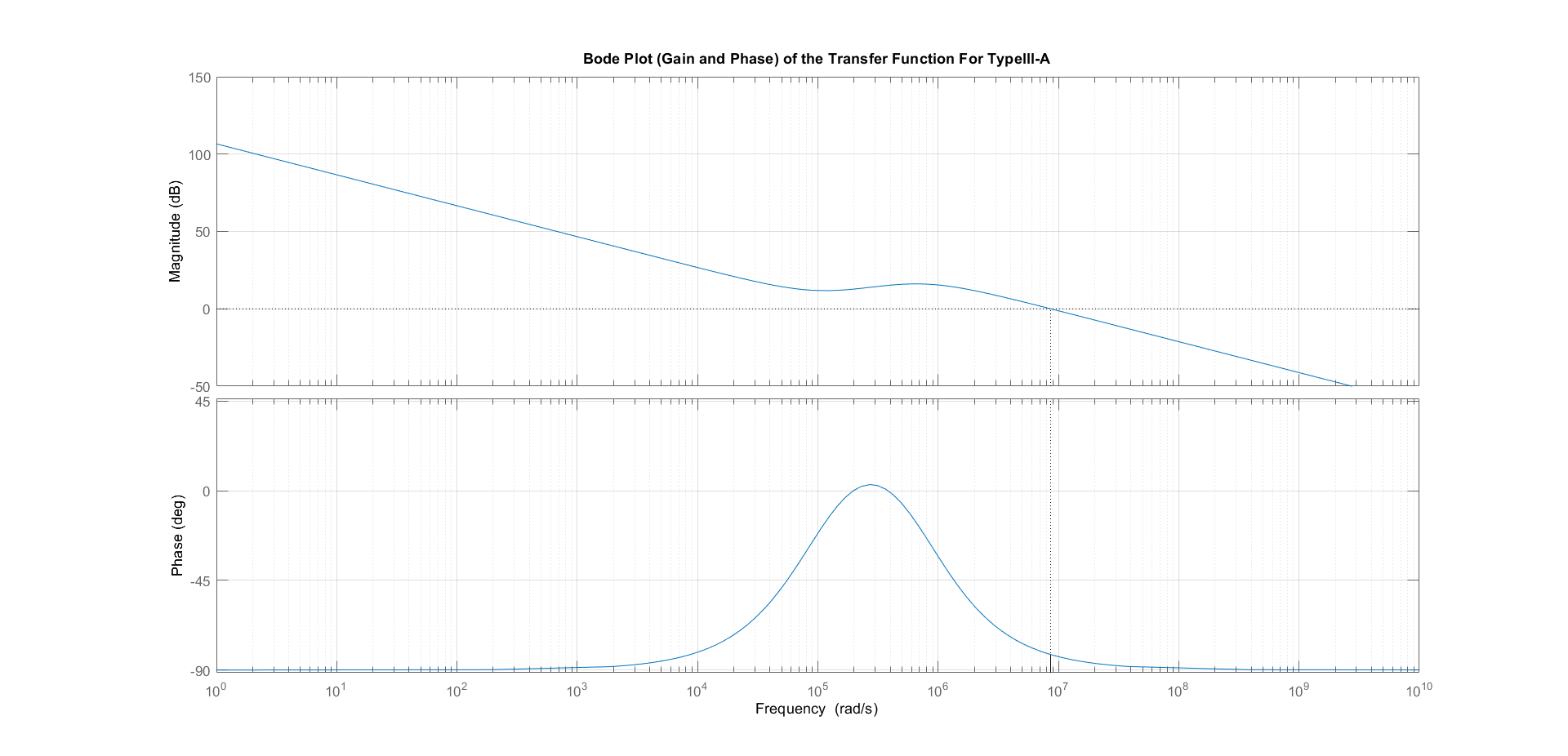


Figure x Bode plot of the designed type-III-A compensator

As shown in Fig X. The phase margin is 97.3 degrees which is pretty safe phase margin.

Type III-B:

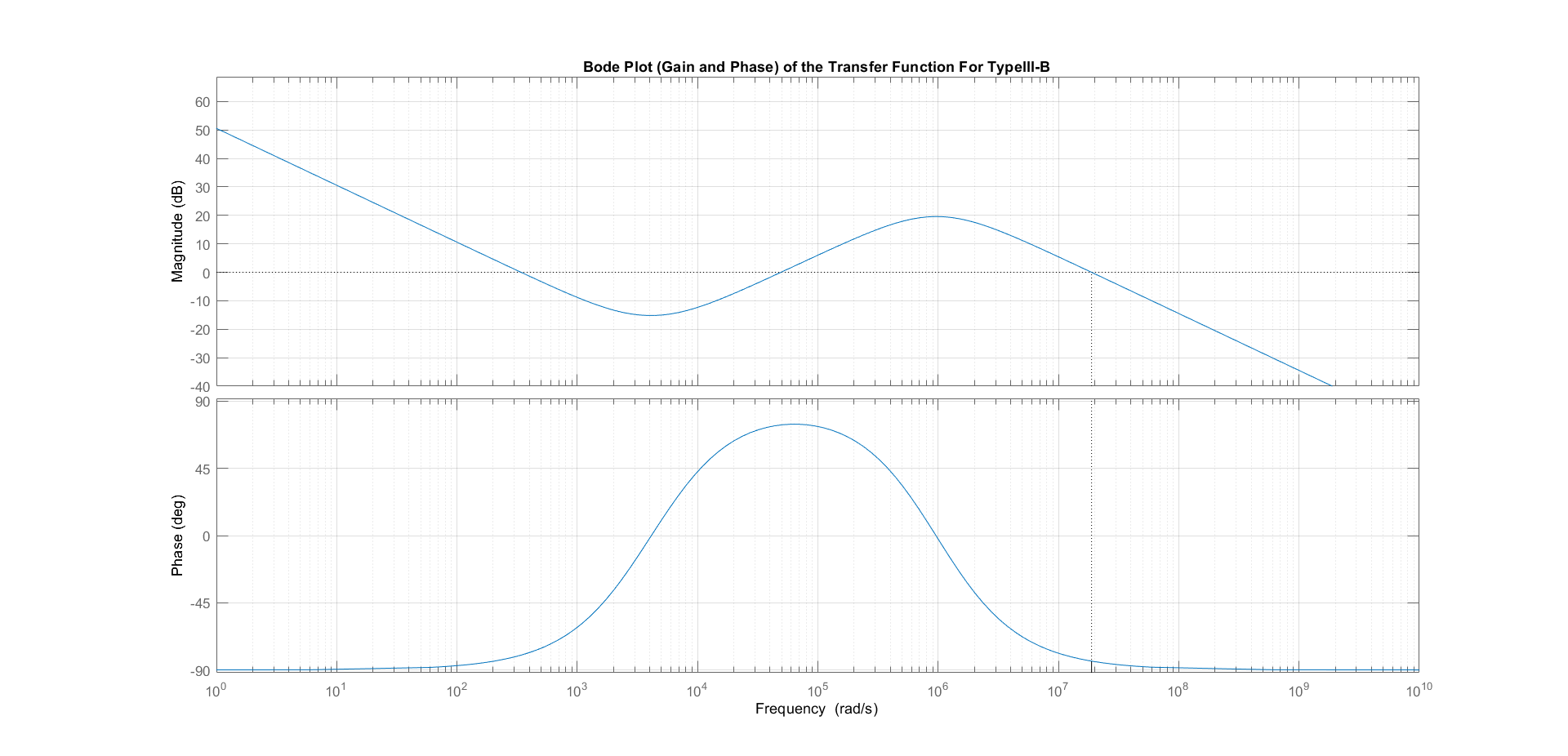


Figure x Bode plot of the designed type-III-B compensator

As shown in Fig X. The phase margin is 95.98 Degrees which is pretty safe phase margin.

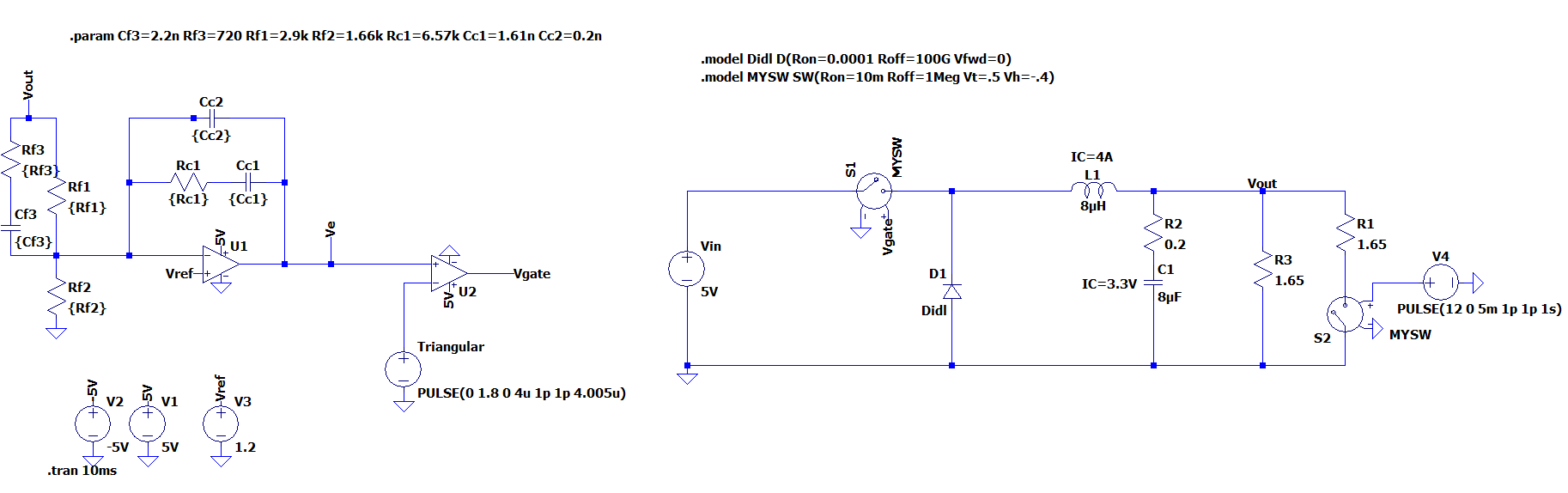
**Question 8:**

*Give the transient waveform for output voltage, inductor current and duty cycle (0<x<1) for each compensator design when load is switched from full to half. Explain what happens at that moment briefly.*

At the switching instance, the average current of the inductor is twice that of the steady-state average current of the load. This surplus energy is transferred to the capacitor, causing a brief voltage spike. During this period, the duty cycle is zero. Subsequently, the average inductor current falls below the final steady-state average current. To compensate for this, the duty cycle is increased for a short duration***. I am genuinely amazed at how effectively this controller operates in terms of transient and steady state response.***

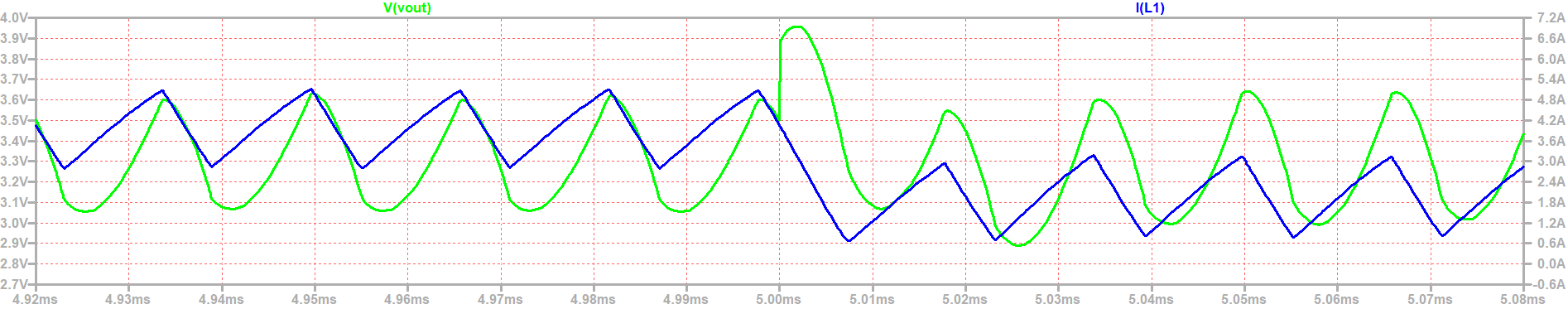
Type III-A:

The simulated circuit is given in Fig. X.

**

*Figure x: Type-III-A Compensator LT-Spice Simulation for Transient Response to Load Change from Full to Half*

Output voltage and inductor current at the switching instance is given in Fig. X.



*Figure x: Type-III-A Compensator LT-Spice Simulation Results - Inductor Current and Output Voltage During Transient Response to Load Change from Full to Half*

Duty, triangular signal and error signal at the switching instance are given in Fig. X.

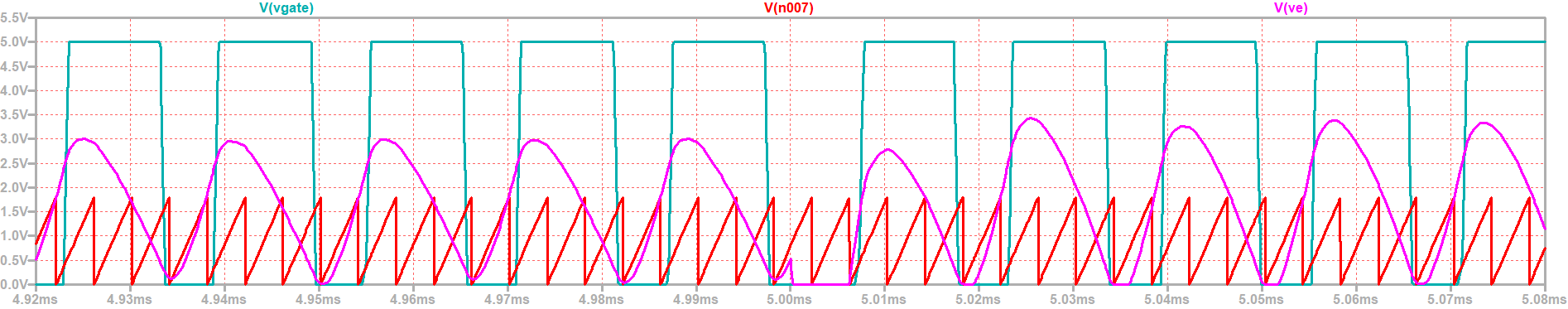


Figure x type-III-A compensator LT-spice simulation results of gate signal, triangular signal and error signal for transient response when load is switched from full to half

***One should note that this output voltage has very low voltage regulation. After checking what is wrong, I realized that the used OP-AMP has very low slew rate for this purpose. After increasing bandwidth and slew while providing a negative rail, I have observed better results as shown in Fig. X and Y***

Output voltage and inductor current at the switching instance is given in Fig. X. when more ideal op-amp model is used

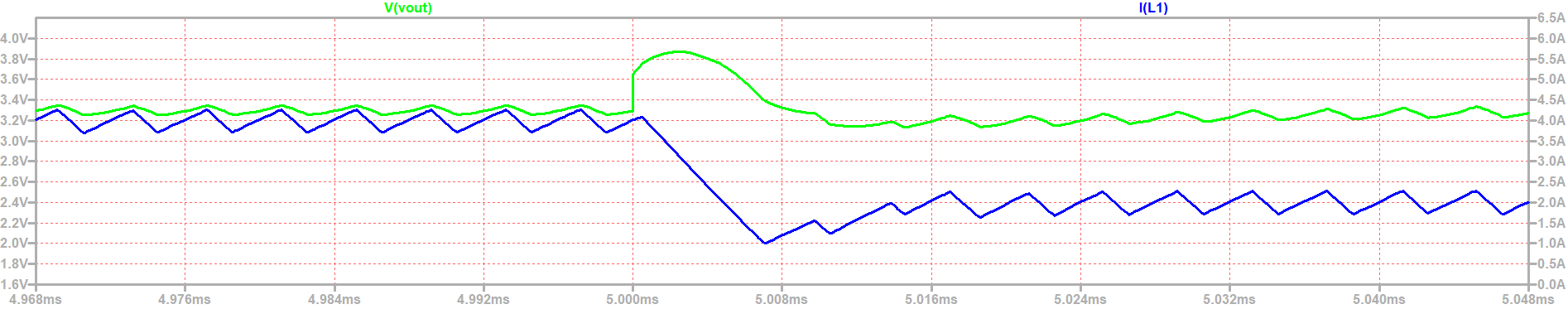
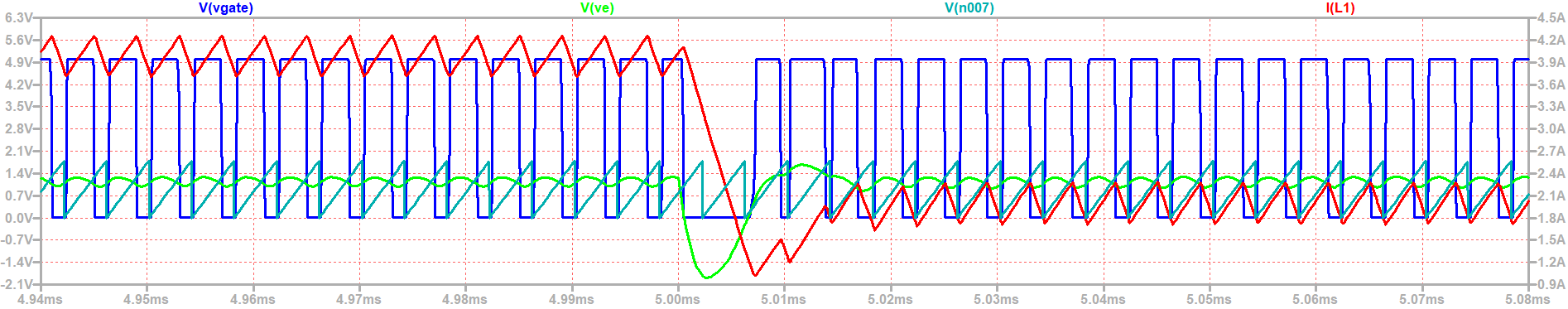


Figure x: Type-III-A Compensator LT-Spice Simulation Results - Inductor Current and Output Voltage During Transient Response to Load Change from Full to Half with More Ideal Op-Amp Using Negative Rail

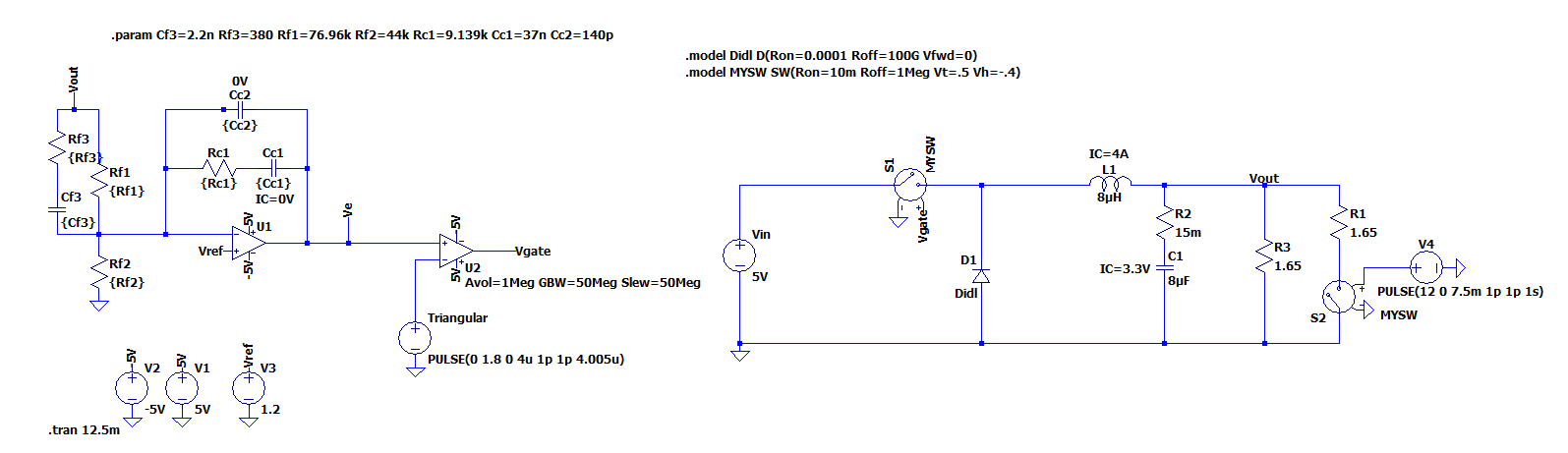
Duty, triangular signal and error signal at the switching instance are given in Fig. X. when more ideal op-amp model is used



*Figure x: Type-III-A Compensator LT-Spice Simulation Results - Gate Signal, Triangular Signal, and Error Signal During Transient Response to Load Change from Full to Half with More Ideal Op-Amp Using Negative Rail*

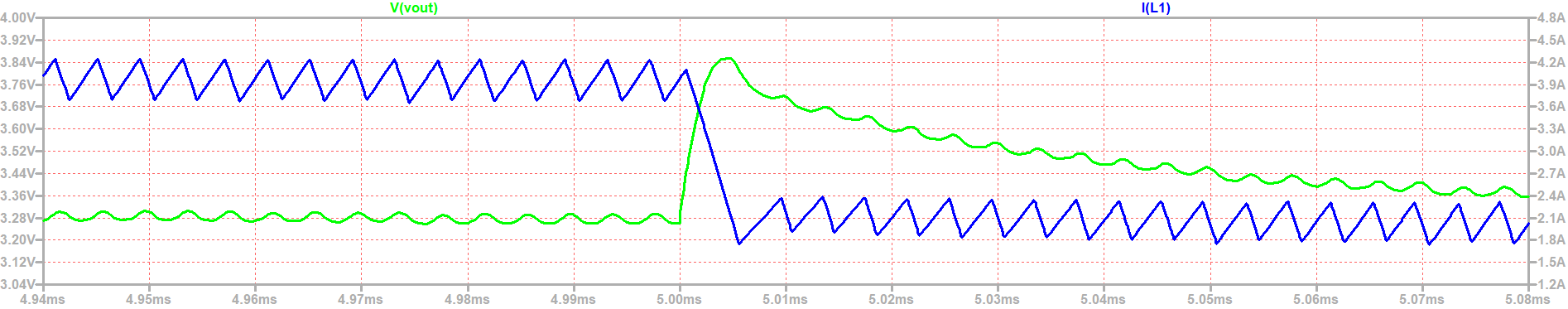
Type III-B:

The simulated circuit is given in Fig. X.

**

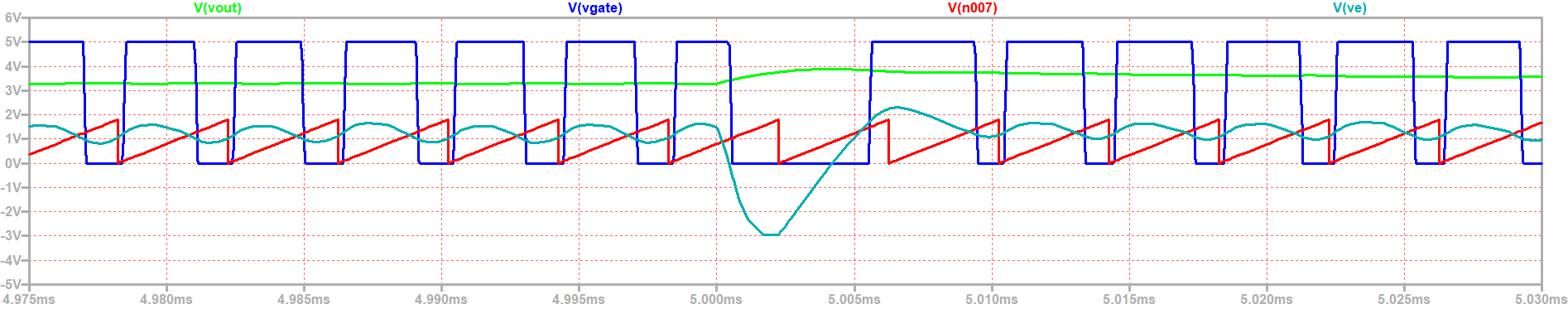
*Figure x: Type-III-B Compensator LT-Spice Simulation for Transient Response to Load Change from Full to Half*

Output voltage and inductor current at the switching instance is given in Fig. X.



*Figure x: Type-III-B Compensator LT-Spice Simulation Results - Inductor Current and Output Voltage During Transient Response to Load Change from Full to Half with More Ideal Op-Amp Using Negative Rail*

Duty, triangular signal and error signal at the switching instance are given in Fig. X.



*Figure x: Type-III-B Compensator LT-Spice Simulation Results - Gate Signal, Triangular Signal, and Error Signal During Transient Response to Load Change from Full to Half with More Ideal Op-Amp Using Negative Rail*

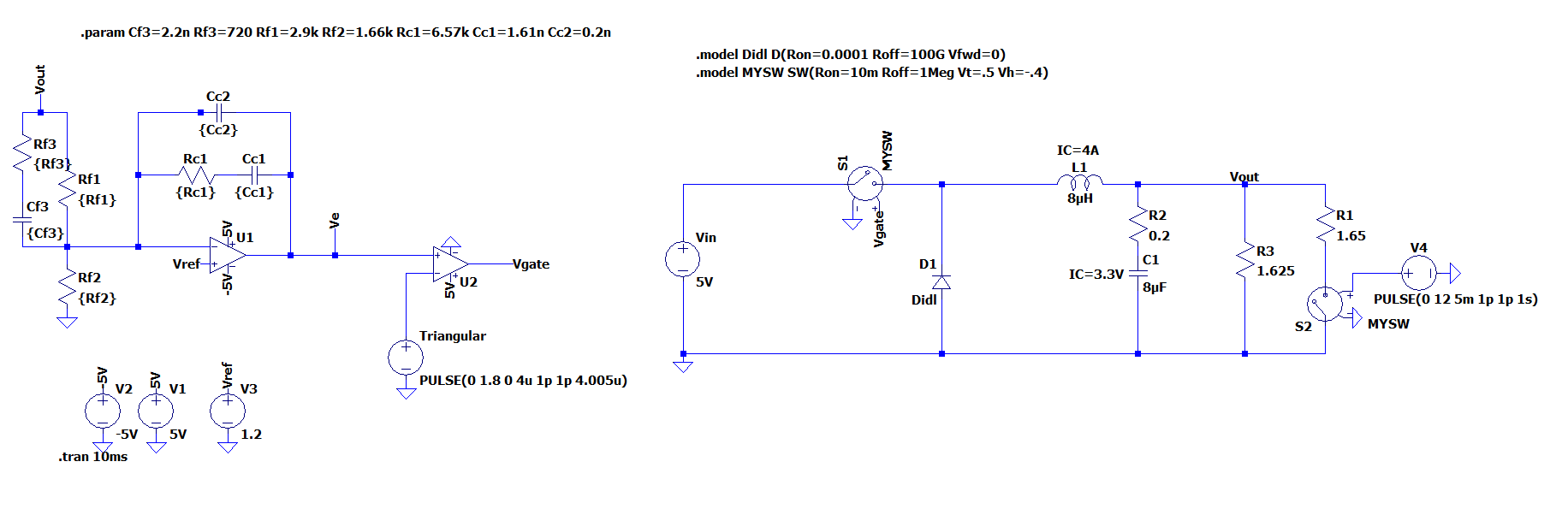
**Question 9:**

*Give the transient waveform for output voltage, inductor current and duty cycle (0<x<1) for each compensator design when load is switched from half to full. Explain what happens at that moment briefly.*

The discussion is exactly the same as in question 8, with the exception that an energy deficit in the inductor is observed rather than a surplus. Additionally, for a short duration, the duty cycle exceeds the steady-state value. Following this, the duty cycle falls below the steady-state value because the inductor current is greater than the steady-state value. During this transition period, output voltage is dropped first then maintained at steady state.

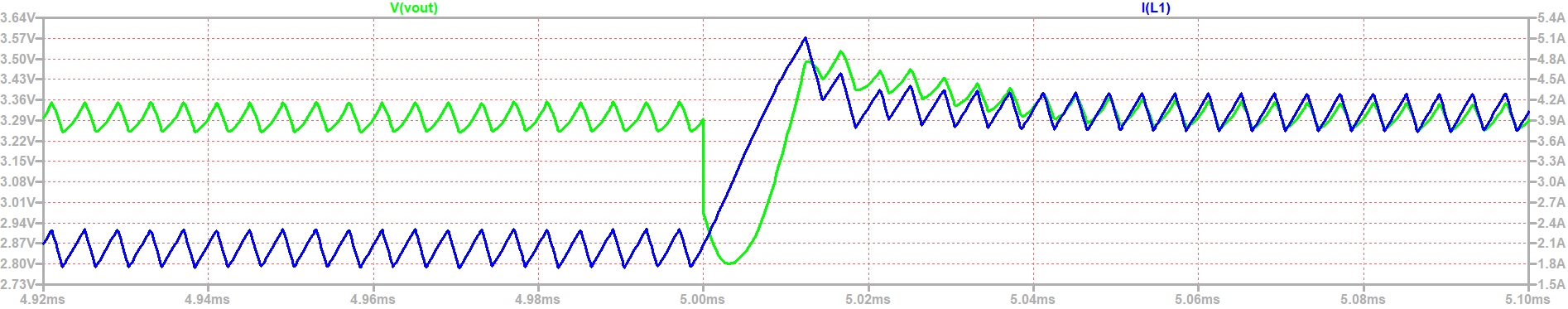
Type III-A:

The simulated circuit is given in Fig. X.



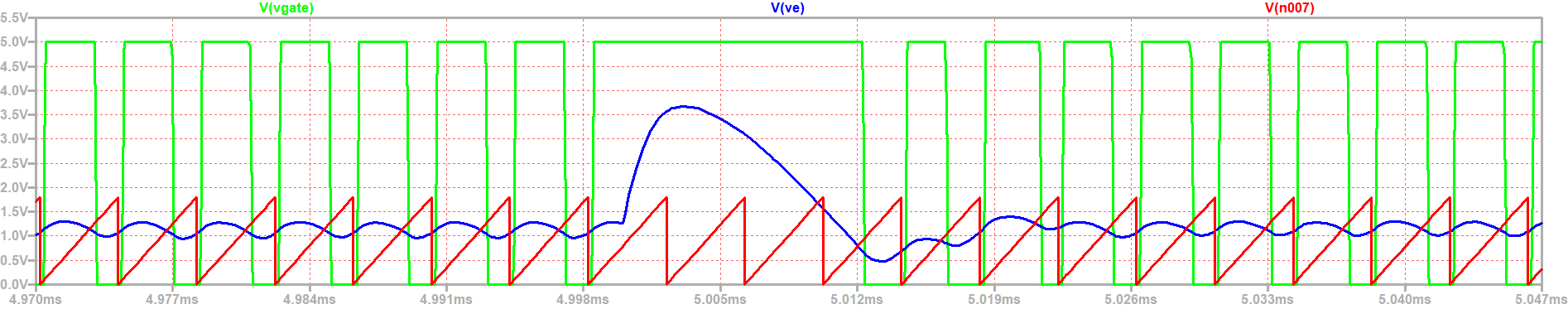
*Figure x: Type-III-A Compensator LT-Spice Simulation for Transient Response to Load Change from Half to Full*

Output voltage and inductor current at the switching instance is given in Fig. X.



*Figure x: Type-III-A Compensator LT-Spice Simulation Results - Inductor Current and Output Voltage During Transient Response to Load Change from Half to Full with More Ideal Op-Amp Using Negative Rail*

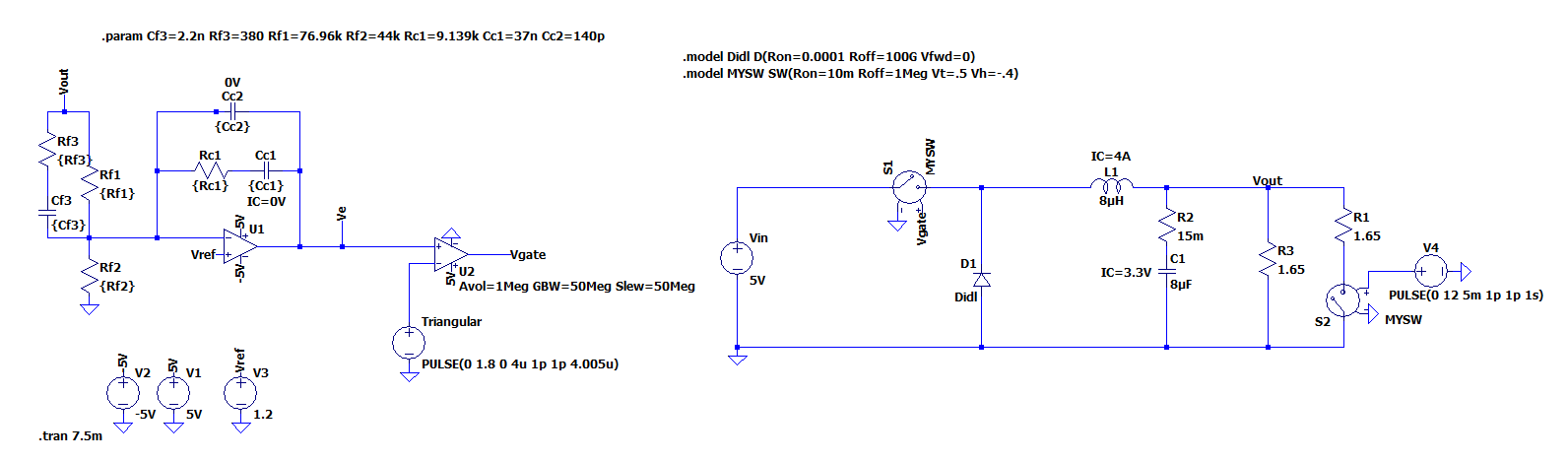
Duty, triangular signal and error signal at the switching instance are given in Fig. X.



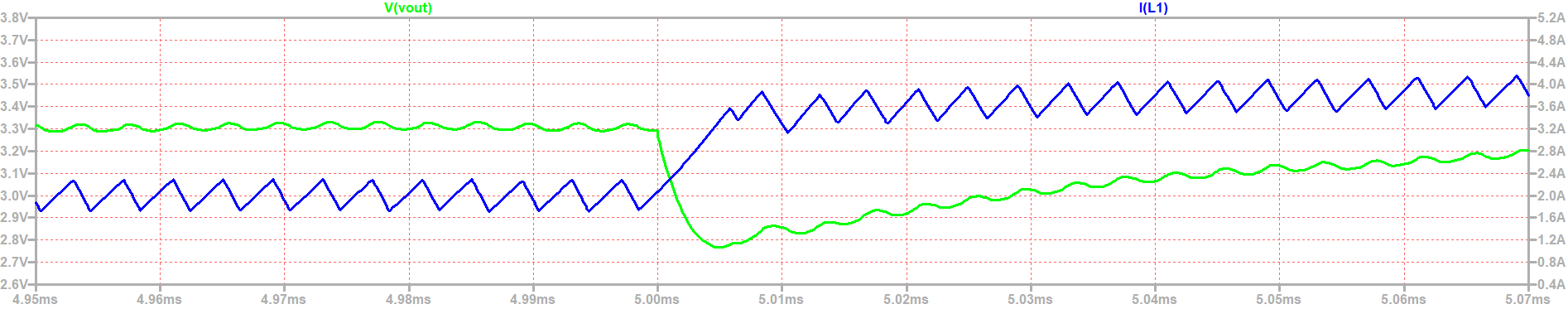
*Figure x: Type-III-A Compensator LT-Spice Simulation Results - Gate Signal, Triangular Signal, and Error Signal During Transient Response to Load Change from Half to Full with More Ideal Op-Amp Using Negative Rail*

Type III-B:

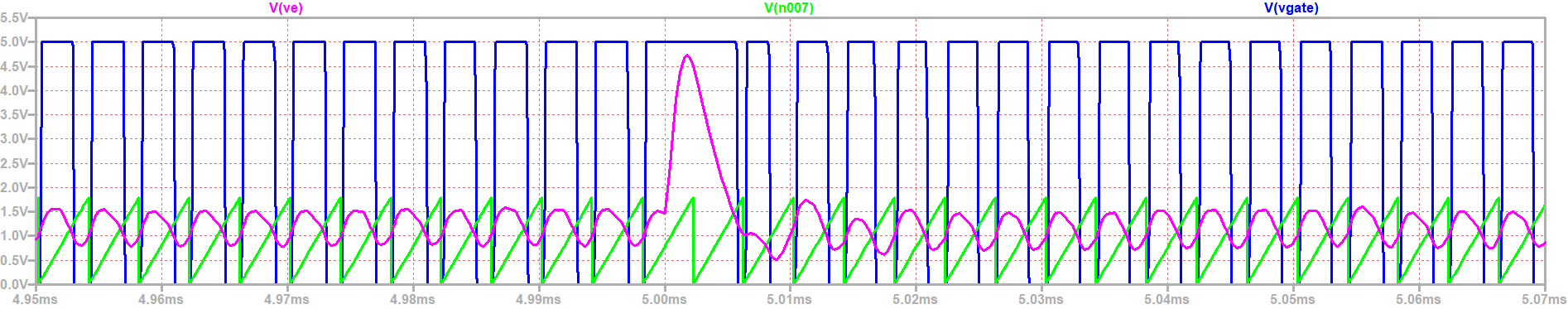
The simulated circuit is given in Fig. X.

****

Output voltage and inductor current at the switching instance is given in Fig. X.



*Figure x: Type-III-B Compensator LT-Spice Simulation Results - Inductor Current and Output Voltage During Transient Response to Load Change from Half to Full with More Ideal Op-Amp Using Negative Rail*



*Figure x: Type-III-B Compensator LT-Spice Simulation Results - Gate Signal, Triangular Signal, and Error Signal During Transient Response to Load Change from Half to Full with More Ideal Op-Amp Using Negative Rail*

**Question 10:**

*Give the transient waveform for output voltage, inductor current, input voltage and duty cycle (0<x<1) for each compensator design when input voltage drops to 4 V as a step change. Explain what happens at that moment briefly. Does your compensator still helps to regulate output voltage?.*

**Conclusion**

**Appendix I:** The MATLAB script to plot bode plot for control-to-output transfer function of buck converter

|  |
| --- |
| clear  clearvars  clc  % Parameters  L\_out = 8e-6; % H  C\_out = 8e-6; % F  R\_c = 15e-3; % Ohm  V\_in = 5; % V  V\_out = 3.3; % V  I\_out = 4; % A  f\_sw = 250e3; % Hz  V\_ref = 1.2; % V  V\_osc = 1.8; % V  %determined variables  R\_load = V\_out/I\_out; % Load resistance  % Transfer function with esr: G\_p\_with\_ESR(s)  numerator\_with\_ESR = V\_in\*R\_load \* [C\_out \* R\_c, 1];  denominator\_with\_ESR = [L\_out \* C\_out \* (R\_load + R\_c), (L\_out + R\_load \* C\_out \* R\_c), R\_load];  G\_p\_with\_ESR = tf(numerator\_with\_ESR, denominator\_with\_ESR);  % Transfer function without esr: G\_p\_without\_ESR(s)  numerator\_without\_ESR = V\_in\*R\_load \* [0, 1];  denominator\_without\_ESR = [L\_out \* C\_out \* R\_load, L\_out, R\_load];  G\_p\_without\_ESR = tf(numerator\_without\_ESR, denominator\_without\_ESR);  % Plot Bode plots  figure;  bode(G\_p\_with\_ESR, 'r', G\_p\_without\_ESR, 'b');  legend('With ESR', 'Without ESR');  grid on;  % Display transfer functions as fractions  disp('Transfer function with ESR:');  G\_p\_with\_ESR  disp('Transfer function without ESR:');  G\_p\_without\_ESR  % Gain margin and phase margin  [Gm\_with\_ESR, Pm\_with\_ESR, Wcg\_with\_ESR, Wcp\_with\_ESR] = margin(G\_p\_with\_ESR);  [Gm\_without\_ESR, Pm\_without\_ESR, Wcg\_without\_ESR, Wcp\_without\_ESR] = margin(G\_p\_without\_ESR);  % Display margins  disp('With ESR:');  fprintf('Gain Margin: %.2f dB\n', 20\*log10(Gm\_with\_ESR));  fprintf('Phase Margin: %.2f degrees\n', Pm\_with\_ESR);  disp('Without ESR:');  fprintf('Gain Margin: %.2f dB\n', 20\*log10(Gm\_without\_ESR));  fprintf('Phase Margin: %.2f degrees\n', Pm\_without\_ESR); |

**Appendix II:** The MATLAB script to plot bode plot requested at question seven.

|  |
| --- |
| %Q7-Type III A & B bode plots  clear  clearvars  clc  close all  is\_type\_A = true;  if is\_type\_A  %Values for TypeIII-A: R\_f1, R\_C1, R\_f3, C\_C1, C\_C2, C\_f3  R\_f1 = 2.9e3; % Replace with your specific value  R\_C1 = 6.57e3; % Replace with your specific value  R\_f3 = 720; % Replace with your specific value  C\_C1 = 1.61e-9; % Replace with your specific value  C\_C2 = 0.2e-9; % Replace with your specific value  C\_f3 = 2.2e-9; % Replace with your specific value  else  % Values for TypeIII-B: R\_f1, R\_C1, R\_f3, C\_C1, C\_C2, C\_f3  R\_f1 = 79.96e3; % Replace with your specific value  R\_C1 = 9.139e3; % Replace with your specific value  R\_f3 = 380; % Replace with your specific value  C\_C1 = 37e-9; % Replace with your specific value  C\_C2 = 140e-12; % Replace with your specific value  C\_f3 = 2.2e-9; % Replace with your specific value  end  % Numerator coefficients  numerator = [R\_C1 \* C\_C1 \* C\_f3 \* (R\_f1 + R\_f3) , (R\_C1 \* C\_C1 + C\_f3 \* (R\_f1 + R\_f3)), 1 ];  % Denominator coefficients  denominator = [R\_f1 \* R\_C1 \* C\_C1 \* C\_C2 \* R\_f3 \* C\_f3, R\_f1 \* C\_C1 \* (R\_C1 \* C\_C2 + R\_f3 \* C\_f3), R\_f1 \* C\_C1, 0];  % Transfer function  transfer\_function = tf(numerator, denominator)  % Create the Bode plot and calculate margins  figure;  [~,~,~,wcg] = bode(transfer\_function);  [GM, PM, Wcg, Wcp] = margin(transfer\_function);  % Plot Bode plot with gain and phase margins  margin(transfer\_function);  % Add title and grid for better readability  title('Bode Plot (Gain and Phase) of the Transfer Function For TypeIII-A');  grid on;  % Display gain and phase margins  disp(['Gain Margin (GM): ', num2str(20\*log10(GM)), ' dB']);  disp(['Phase Margin (PM): ', num2str(PM), ' degrees']);  disp(['Gain Crossover Frequency (Wcg): ', num2str(Wcg), ' rad/s']);  disp(['Phase Crossover Frequency (Wcp): ', num2str(Wcp), ' rad/s']); |