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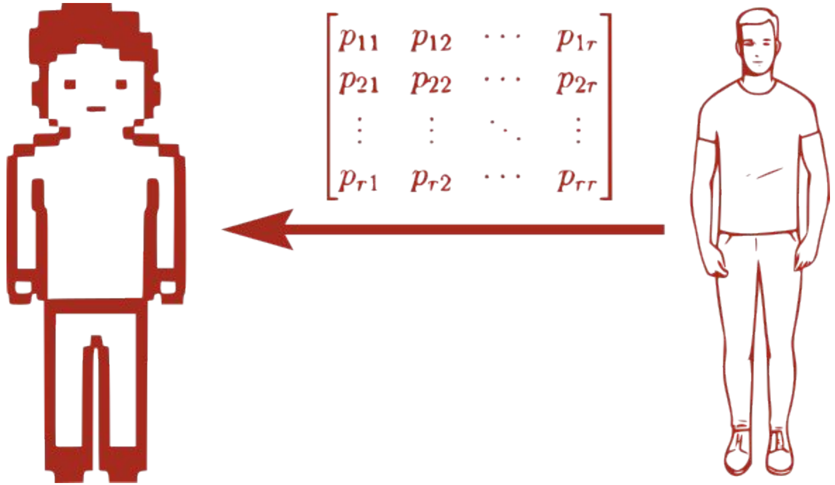
ODTÜ elektrik-elektronik bölümü lisans öğrencisi

21 Aralık 2023

Determining calibration matrix of a linear (pinhole) camera model



Pinhole camera model

"Ford Madox Brown tarafından 19. yüzyıl duvar resminde, tüccar ve amatör gökbilimci William Crabtree 1639 da Venüs'ün Güneş'in önünden geçişini gözlemlerken tasvir edilmiş. Crabtree'ye gerçekleşmesi beklenen bu olayı başka bir amatör gökbilimci Jeremiah Horrocks haber vermiş ve tahmin konusunda yardımcı olmuştu. Bu geçişi gören sadece ikisiydi ve teleskoptan gelen görüntüyü yansıtarak buna şahit oldular. Crabtree bunu Manchester yakınındaki Broughton'daki evinde, Horrock ise otuz beş mil ötedeki Much köyünde gözlemledi."

"Bilim insanları - Bir keşif destanı" adlı kitaptan

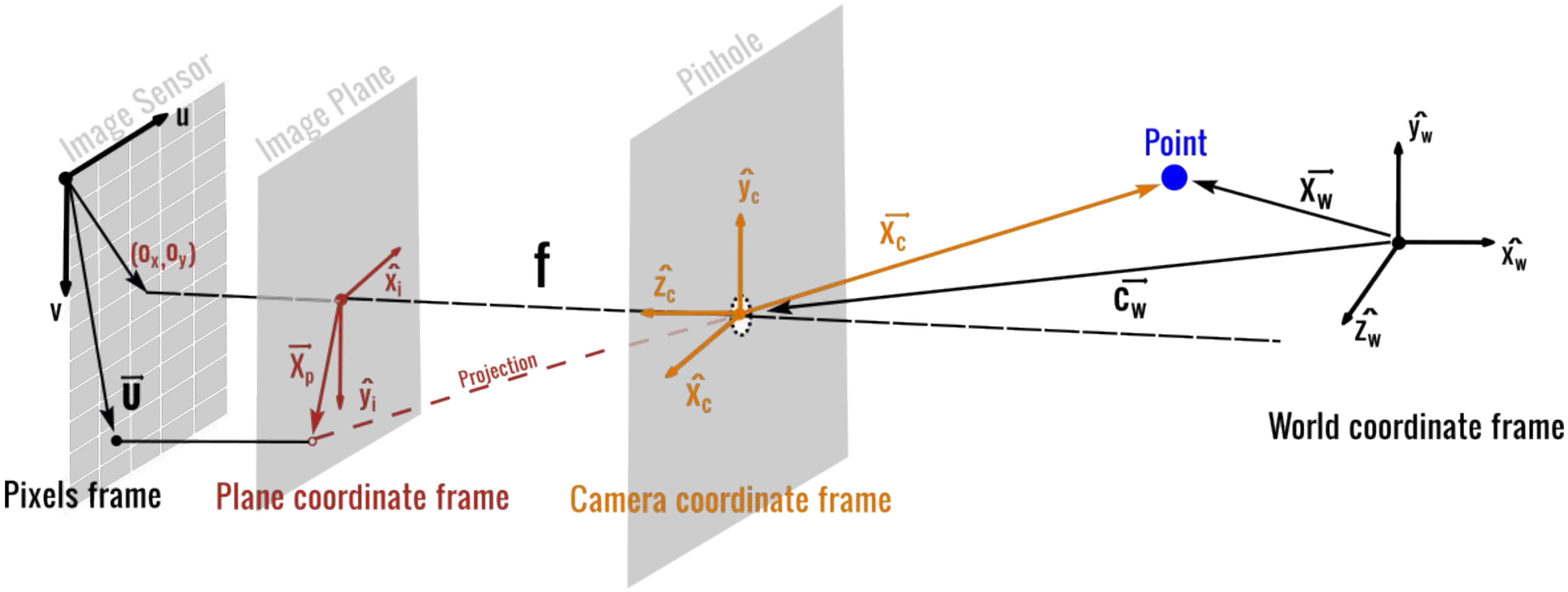
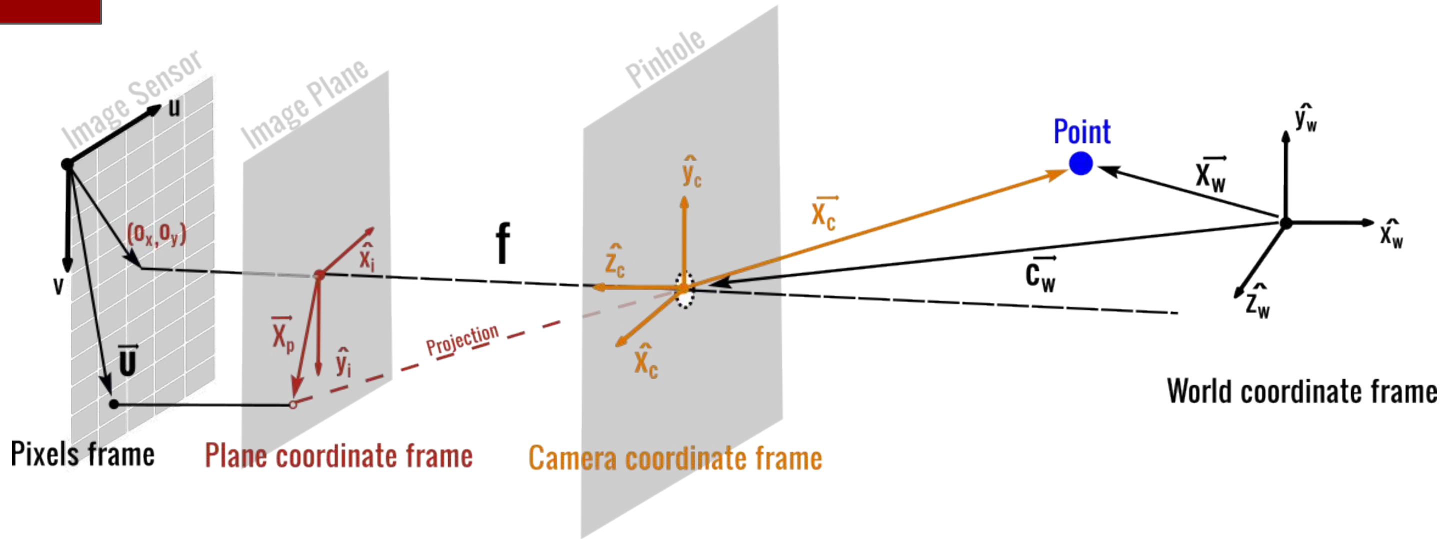


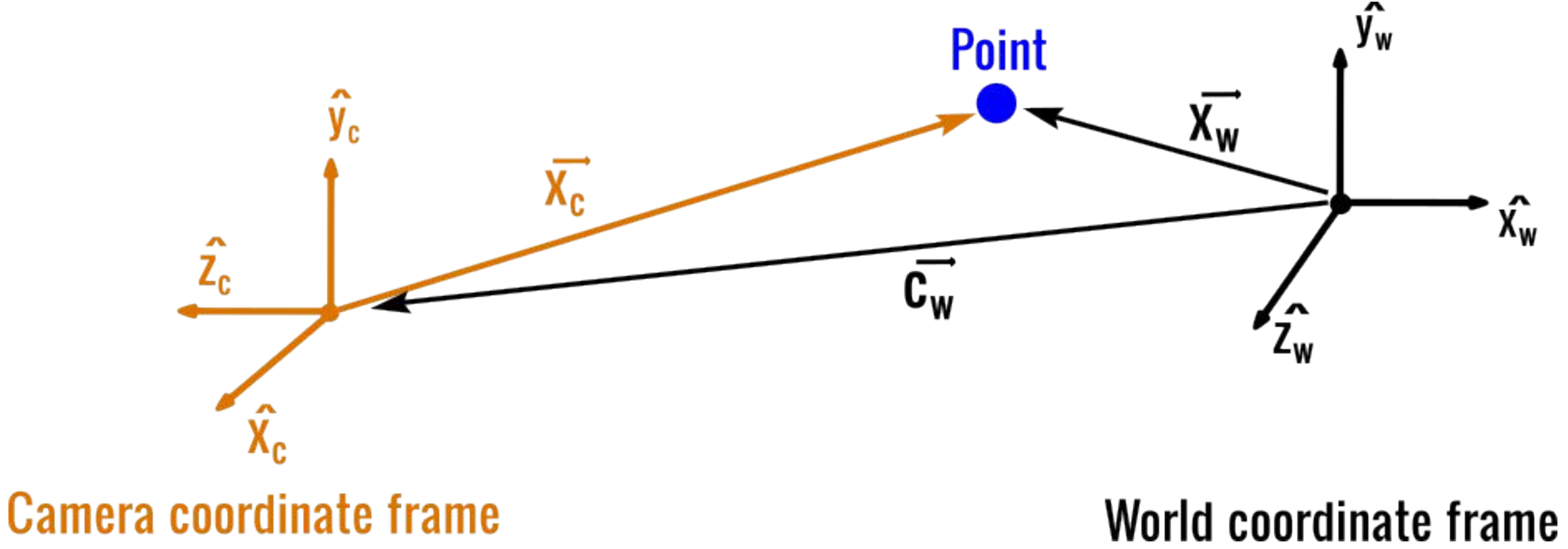
Image Sensor
Image Plane
Pinhole
u
v
(o_x, o_y)
$\hat{x}_i$
$\hat{y}_i$
$\vec{X_p}$
$\vec{U}$
Projection
f
$\hat{y}_c$
$\hat{z}_c$
$\hat{x}_c$
$\vec{X_c}$
Point
$\vec{X_w}$
$\vec{C_w}$
$\hat{y}_w$
$\hat{x}_w$
$\hat{z}_w$
Pixels frame
Plane coordinate frame
Camera coordinate frame
World coordinate frame

Outline

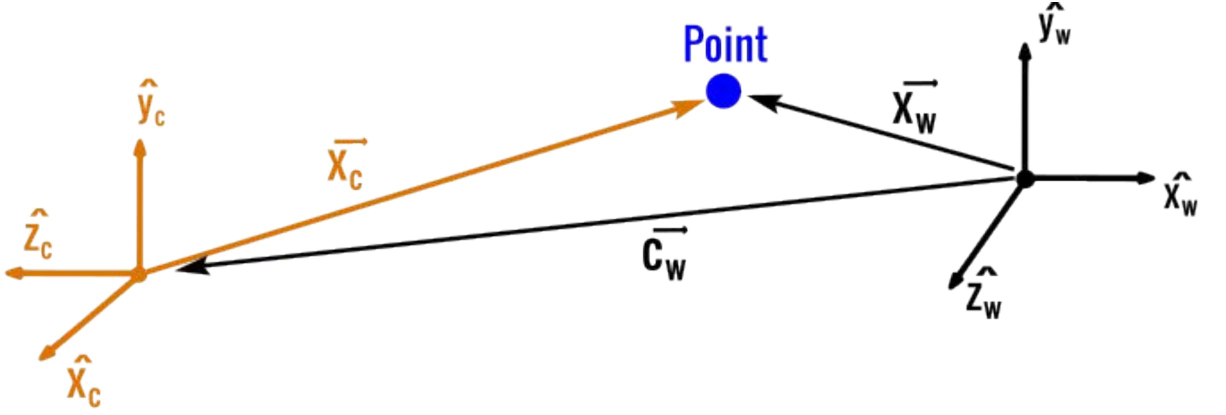


I. Expressing extrinsic matrix
II. Expressing intrinsic matrix
III. Expressing projection matrix
IV. Parameter determination procedure
V. About least squares approach
VI. Python function
VII. Problems to be addresses in the future

Expressing extrinsic matrix



$$\boxed{\overrightarrow{X_c} = M_{ext}\overrightarrow{X_w}}$$



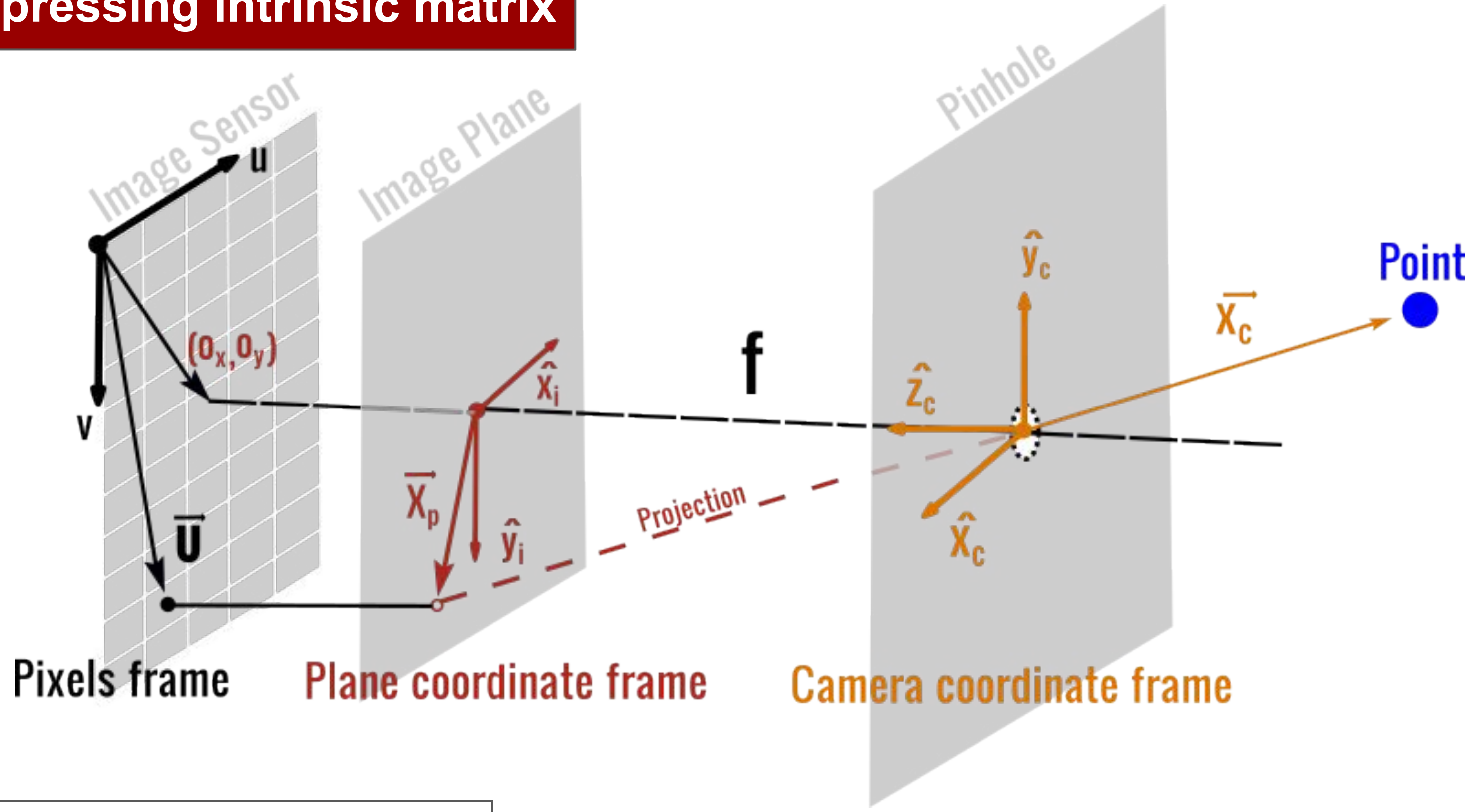
Camera coordinate frame

World coordinate frame

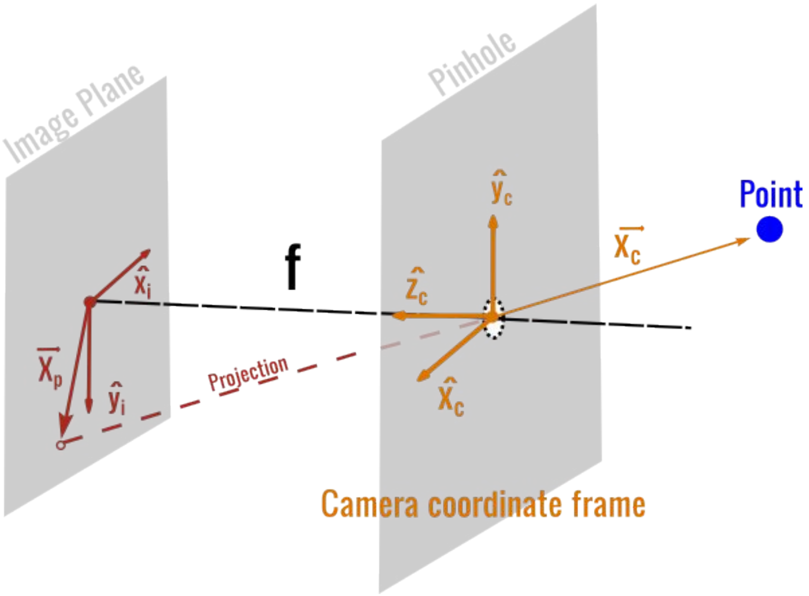
- $$\begin{bmatrix}\hat{x}_c\\ \hat{y}_c\\ \hat{z}_c\end{bmatrix} = \begin{bmatrix}r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{bmatrix}\begin{bmatrix}\hat{x}_w\\ \hat{y}_w\\ \hat{z}_w\end{bmatrix}$$
- $$\vec{X_c} = \begin{bmatrix}x_c\\ y_c\\ z_c\end{bmatrix} = \begin{bmatrix}r_{11} r_{12} r_{13}\\ r_{21} r_{22} r_{23}\\ r_{31} r_{32} r_{33}\end{bmatrix}(\vec{X_w} - \vec{C_w}) = \begin{bmatrix}r_{11} r_{12} r_{13}\\ r_{21} r_{22} r_{23}\\ r_{31} r_{32} r_{33}\end{bmatrix}\begin{bmatrix}x_{w_x}\\ x_{w_y}\\ x_{w_z}\end{bmatrix} + \begin{bmatrix}t_x\\ t_y\\ t_z\end{bmatrix}$$

$$\vec{X_c} \equiv \begin{bmatrix}x_c\\ y_c\\ z_c\\ 1\end{bmatrix} = \underbrace{\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_x\\ r_{21} & r_{22} & r_{23} & t_y\\ r_{31} & r_{32} & r_{33} & t_z\\ 0 & 0 & 0 & 1\end{bmatrix}}_{\text{Extrinsic matrix}}\begin{bmatrix}x_{w_x}\\ x_{w_y}\\ x_{w_z}\\ 1\end{bmatrix}$$

Expressing intrinsic matrix



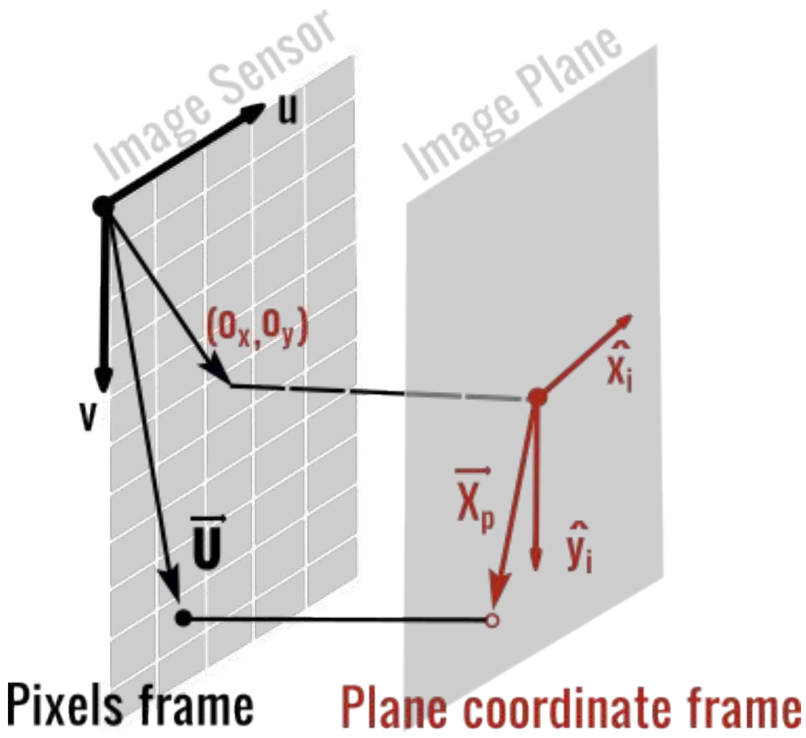
$$\widetilde{U} = M_{int}\overrightarrow{X_c}$$



$$\overrightarrow{x_c} = \langle x_c \, , y_c \, , z_c \, \rangle \qquad \overrightarrow{x_p} = \langle x_p \, , y_p \rangle$$

$$\frac{x_c}{z_c} = \frac{x_p}{f} \;\&\; \frac{y_c}{z_c} = \frac{y_p}{f} \quad \textit{by geometry (note that } z_c < 0)$$

$$x_p = f\,\frac{x_c}{z_c} \qquad \& \qquad y_p = f\,\frac{y_c}{z_c}$$



$$\overrightarrow{x_p} = \langle x_p , y_p \rangle \qquad x_p = f\frac{x_c}{z_c} \quad \& \quad y_p = f\frac{y_c}{z_c}$$

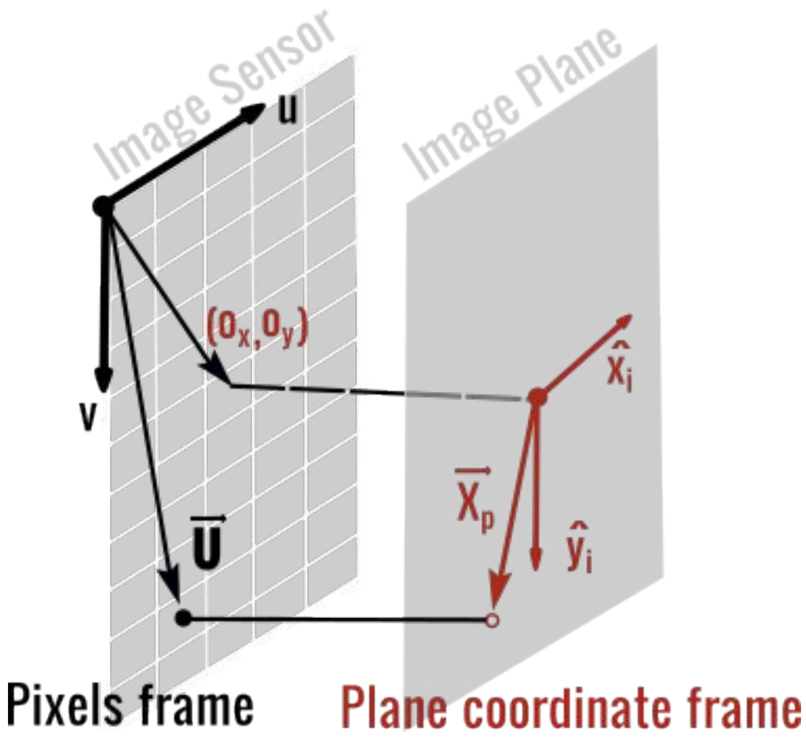
Let's say m_x and m_y denotes the pixel density per distance

$$\vec{U} = \langle u , v \rangle$$

$$u = m_x x_p + o_x , \qquad v = m_y y_p + o_y$$

$$u = m_x\left(f\frac{x_c}{z_c}\right) + o_x , \qquad v = m_y\left(f\frac{y_c}{z_c}\right) + o_y$$

$$u = f_x\frac{x_c}{z_c} + o_x , \qquad v = f_y\frac{y_c}{z_c} + o_y$$



$$\vec{U} = \langle u\,, v\rangle \qquad u = f_x \frac{x_c}{z_c} + o_x \qquad v = f_y \frac{y_c}{z_c} + o_y$$

$$\vec{U} = \begin{bmatrix} u \\ v \end{bmatrix} \rightarrow \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \tilde{U} = \begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{Intrinsic matrix}} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Expressing projection matrix

$$\overrightarrow{X_c} = M_{ext}\overrightarrow{X_w} \Longrightarrow \tilde{U} = M_{int}\overrightarrow{X_c}$$

"World to camera frame" "Camera frame to pixel"

$$\tilde{U} = \boldsymbol{M}_{\boldsymbol{int}}\left(\boldsymbol{M}_{\boldsymbol{ext}}\overrightarrow{X_w}\right) = \boldsymbol{P}\overrightarrow{X_w}$$

$$\widetilde{U} = \boldsymbol{M}_{int}\left(\boldsymbol{M}_{ext}\overrightarrow{X_w}\right) = \boldsymbol{P}\overrightarrow{X_w}$$

$$\widetilde{U} = \begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{Intrinsic matrix}} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Extrinsic matrix}} \begin{bmatrix} x_{w_x} \\ x_{w_y} \\ x_{w_z} \\ 1 \end{bmatrix}$$

$$\widetilde{U} = \boldsymbol{P}\overrightarrow{X_w}$$

$$\widetilde{U} = \begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}}_{\text{Projection matrix}} \begin{bmatrix} x_{w_x} \\ x_{w_y} \\ x_{w_z} \\ 1 \end{bmatrix}$$

Parameter determination procedure

- Let's say we have n number of points that their pixel correspondences and real world coordinates are known. For i^{th} point we can write;

- $$\begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} x_{w_x} \\ x_{w_y} \\ x_{w_z} \\ 1 \end{bmatrix}$$

- $$z_c = p_{31}x_{w_x} + p_{32}x_{w_y} + p_{33}x_{w_z} + p_{34}$$

$$u^{(i)} = \frac{p_{11}x_w^{(i)} + p_{12}y_w^{(i)} + p_{13}z_w^{(i)} + p_{14}}{p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}}$$

$$v^{(i)} = \frac{p_{21}x_w^{(i)} + p_{22}y_w^{(i)} + p_{23}z_w^{(i)} + p_{24}}{p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}}$$

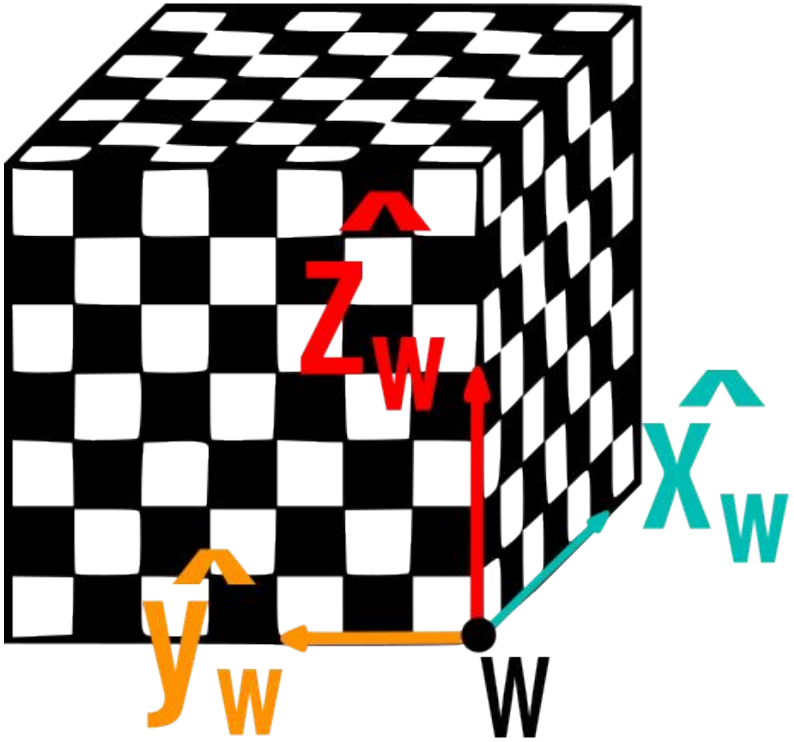
Linear equations are formed

$$\left(p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}\right)u^{(i)} - p_{11}x_w^{(i)} + p_{12}y_w^{(i)} + p_{13}z_w^{(i)} + p_{14} = 0$$

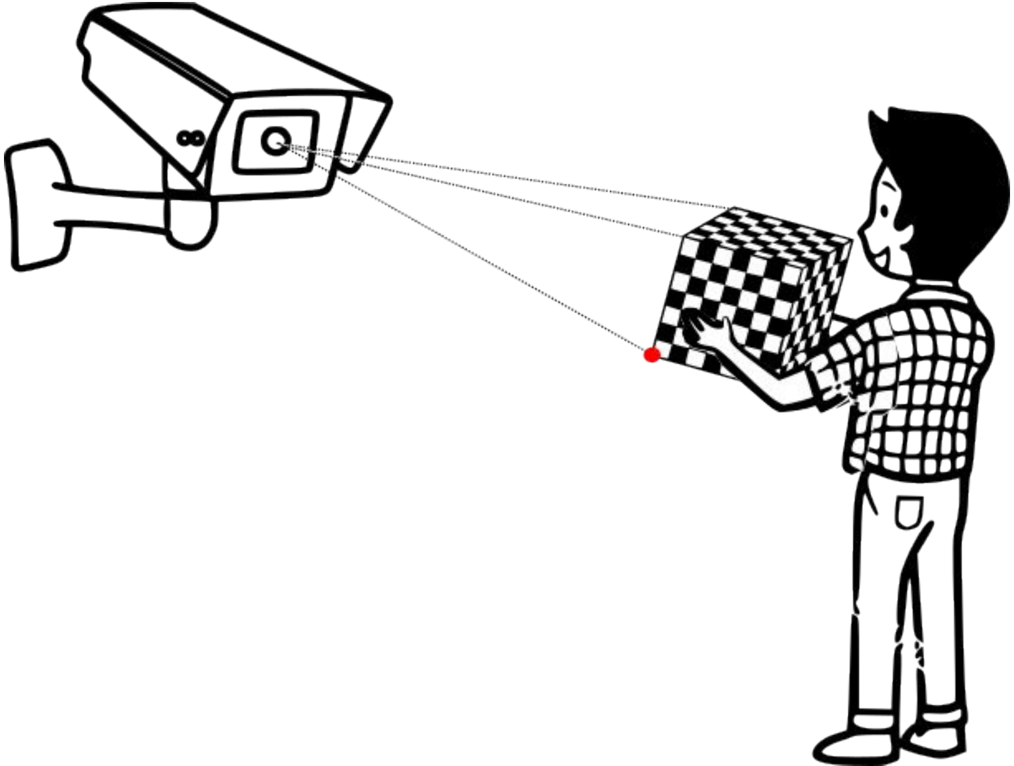
$$\left(p_{31}x_w^{(i)} + p_{32}y_w^{(i)} + p_{33}z_w^{(i)} + p_{34}\right)v^{(i)} - p_{21}x_w^{(i)} + p_{22}y_w^{(i)} + p_{23}z_w^{(i)} + p_{24} = 0$$

$$\underbrace{\begin{bmatrix} x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & 0 & 0 & 0 & 0 & -u_1 x_w^{(1)} & -u_1 y_w^{(1)} & -u_1 z_w^{(1)} & -u_1 \\ 0 & 0 & 0 & 0 & x_w^{(1)} & y_w^{(1)} & z_w^{(1)} & 1 & -v_1 x_w^{(1)} & -v_1 y_w^{(1)} & -v_1 z_w^{(1)} & -v_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & 0 & 0 & 0 & 0 & -u_i x_w^{(i)} & -u_i y_w^{(i)} & -u_i z_w^{(i)} & -u_i \\ 0 & 0 & 0 & 0 & x_w^{(i)} & y_w^{(i)} & z_w^{(i)} & 1 & -v_i x_w^{(i)} & -v_i y_w^{(i)} & -v_i z_w^{(i)} & -v_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & 0 & 0 & 0 & 0 & -u_n x_w^{(n)} & -u_n y_w^{(n)} & -u_n z_w^{(n)} & -u_n \\ 0 & 0 & 0 & 0 & x_w^{(n)} & y_w^{(n)} & z_w^{(n)} & 1 & -v_n x_w^{(n)} & -v_n y_w^{(n)} & -v_n z_w^{(n)} & -v_n \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix}}_{\mathbf{p}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Testing tool - Object of known geometry



Testing procedure



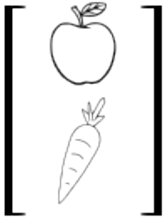
About least squares approach

- Ayse says that she has bought **two apples** and **three carrot** for 3.49₺.
  ☞It is known that at that time apple and carrot cost 0.47₺ and 0.85₺ per item respectively.

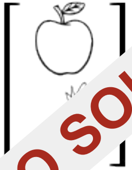
- Ahmet says that she has bought **seven apples** and **five carrot** for 7.60₺.
  ☞It is known that at that time apple and carrot cost 0.55₺ and 0.75₺ per item respectively.

- Ezgi says that she has bought **two apples** and **four carrot** for 4.20₺.
  ☞It is known that at that time apple and carrot cost 0.50₺ and 0.80₺ per item respectively.

$$\begin{bmatrix}2 & 3\\ 7 & 5\\ 2 & 4\end{bmatrix}$$



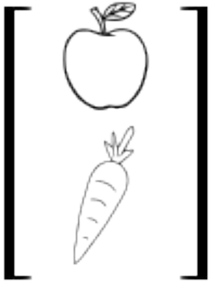
$$= \begin{bmatrix}3.49\\ 7.60\\ 4.20\end{bmatrix} \Longrightarrow \begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}$$



$$= \begin{bmatrix}107/220\\ 923/1100\\ -71/550\end{bmatrix}$$

NO SOLUTION

$$\begin{bmatrix}2 & 3\\7 & 5\\2 & 4\end{bmatrix}$$



$$= \begin{bmatrix}3.49\\7.60\\4.20\end{bmatrix}$$

- Solving by least square approach

 = **0.51**

 = **0.80**

Python function

- Considering each data results in two equations and there are 12 unknowns, n ≥ 6

$$\left[\left[u^{(1)}, v^{(1)}, x^{(1)}, y^{(1)}, z^{(1)}\right], \ldots, \left[u^{(n)}, v^{(n)}, x^{(n)}, y^{(n)}, z^{(n)}\right]\right]$$

$$\Downarrow$$

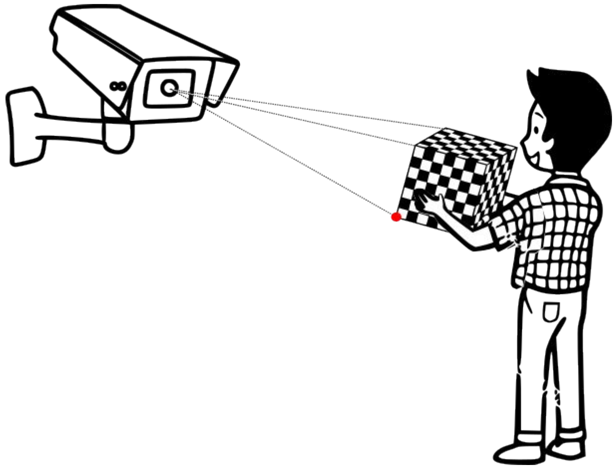
def calculate_projection_matrix_using_test_data(.)

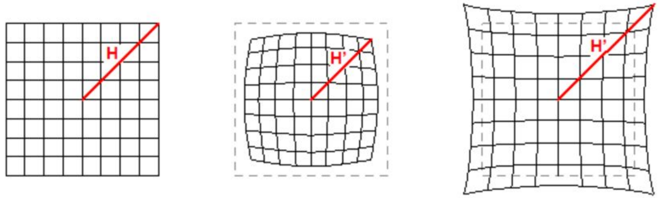
$$\Downarrow$$

$$[p_{11}\ , p_{12}\ , p_{13}\ , p_{14}\ , p_{21}\ , \ldots p_{34}]$$

Expressing intrinsic matrix

- The position of the real world coordinate system is ambiguous
- There are procedures to extract intrinsic and extrinsic matrices from projection matrix. Which is not utilized for now
- Distortions due to lenses are neglected. Ideally, this distortions should be eliminated first by an additional function. Than the corrected version should be passed.





Reference and educational source

First Principles of Computer Vision

@firstprinciplesofcomputerv3258 · 50,2 B abone · 151 video

First Principles of Computer Vision is a lecture series presented by ...

fpcv.cs.columbia.edu

Camera Calibration | Uncalibrated Stereo