CSE 464
DIGITAL IMAGE PROCESSING
HW2
REPORT

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Task 1

In this task, convolution is implemented. "main.py" file is the one which should be executed. Program does not have any command line arguments.

Once the program is executed, it will ask the file, which involves kernel matrix and image, which will be convoluted.

NOTE: Matrix elements should be separated by commas. There shouldn't be any whitespaces. There is an example file called "kernel.txt".

NOTE 2: Once the kernel is read from file, it is flipped on x and y axis.

Task 2

$$\frac{df}{dx'} = \frac{df}{dx} \frac{dx}{dx'} + \frac{df}{dy} \cdot \frac{dy}{dx'}$$

$$\frac{d^2f}{dx'^2} = \frac{d}{dx'} \cdot \frac{df}{dx'} = \frac{d}{dx'} \cdot \left(\frac{df}{dx} \cdot \frac{dx}{dx'} + \frac{df}{dy} \cdot \frac{dy}{dx'} \right)$$

$$= \frac{d}{dx'} \left(-\frac{df}{dx} \sin\theta - \frac{df}{dy} \cdot \cos\theta \right)$$

$$= -\frac{d}{dx'} \cdot \frac{df}{dx} \sin\theta - \frac{d}{dx'} \cdot \frac{df}{dy} \cdot \cos\theta$$

swap the denominators to obtain chain rule again.

$$= \frac{-d}{dx} \cdot \frac{df}{dx'} \sin\theta - \frac{d}{dy} \cdot \frac{df}{dx'} \cos\theta$$

$$= -\frac{d}{dx} \left(\frac{df}{dx} \cdot \frac{dx}{dx'} + \frac{df}{dy} \cdot \frac{dy}{dx'} \right) \sin\theta - \frac{d}{dy} \left(\frac{df}{dx} \cdot \frac{dx}{dx'} + \frac{df}{dy} \cdot \frac{dy}{dx'} \right) \cos\theta$$

$$= \frac{-d}{dx} \left(\frac{-df}{dx} \sin\theta - \frac{df}{dy} \cos\theta \right) \sin\theta - \frac{d}{dy} \left(\frac{-df}{dx} \sin\theta - \frac{df}{dy} \cos\theta \right) \cos\theta$$

$$= \left(\frac{d^2f}{dx^2} \sin\theta + \frac{d^2f}{dxdy} \cos\theta \right) \sin\theta + \left(\frac{d^2f}{dxdy} \sin\theta + \frac{d^2f}{dy^2} \cos\theta \right) \cos\theta$$

Note: Since,

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

$$\frac{dx}{dx'} = -\sin\theta \quad \text{and} \quad \frac{dy}{dx'} = -\cos\theta$$

$$\frac{df}{dy'} = \frac{df}{dx} \cdot \frac{dx}{dy'} + \frac{df}{dy} \cdot \frac{dy}{dy'}$$

$$\frac{d^2f}{dy'^2} = \frac{d}{dy'} \cdot \frac{df}{dy'} = \frac{d}{dy'} \left(\frac{df}{dx} \frac{dx}{dy'} + \frac{df}{dy} \cdot \frac{dy}{dy'} \right)$$

$$= \frac{d}{dy'} \left(\frac{df}{dx} \cos\theta - \frac{df}{dy} \sin\theta \right)$$

$$= \frac{d}{dy'} \cdot \frac{df}{dx} \cos\theta - \frac{d}{dy'} \cdot \frac{df}{dy} \sin\theta$$

swap the denominators to obtain chain rule again.

$$= \frac{d}{dx} \cdot \frac{df}{dy'} \cos\theta - \frac{d}{dy} \cdot \frac{df}{dy'} \sin\theta$$

$$= \frac{d}{dx} \left(\frac{df}{dx} \cdot \frac{dx}{dy'} + \frac{df}{dy} \cdot \frac{dy}{dy'} \right) \cos\theta - \frac{d}{dy} \cdot \left(\frac{df}{dx} \cdot \frac{dx}{dy'} + \frac{df}{dy} \cdot \frac{dy}{dy'} \right) \cdot \sin\theta$$

$$= \frac{d}{dx} \left(\frac{df}{dx} \cos\theta - \frac{df}{dy} \sin\theta \right) \cos\theta - \frac{d}{dy} \left(\frac{df}{dx} \cdot \cos\theta - \frac{df}{dy} \cdot \sin\theta \right) \sin\theta$$

$$= \left(\frac{d^2f}{dx^2} \cos\theta - \frac{d^2f}{dxdy} \sin\theta \right) \cos\theta - \left(\frac{d^2f}{dxdy} \cos\theta - \frac{d^2f}{dy^2} \sin\theta \right) \cdot \sin\theta$$

NOTE: $\frac{dx}{dy'} = \cos\theta \qquad \frac{dy}{dy'} = -\sin\theta$

As result:

We try to prove that

$$\frac{d^2f}{dx^2} + \frac{d^2f}{dy^2} = \frac{d^2f}{dx'^2} + \frac{d^2f}{dy'^2}$$

So we will sum two of equations that we have found

$$\frac{d^2f}{dx'^2} = \left(\frac{d^2f}{dx^2}\sin\theta + \frac{d^2f}{dxdy}\cos\theta\right)\sin\theta + \left(\frac{d^2f}{dxdy}\sin\theta + \frac{d^2f}{dy^2}\cos\theta\right)\cos\theta$$

$$\frac{d^2f}{dy'^2} = \left(\frac{d^2f}{dx^2}\cos\theta - \frac{d^2f}{dxdy}\sin\theta\right)\cos\theta - \left(\frac{d^2f}{dxdy}\cos\theta - \frac{d^2f}{dy^2}\sin\theta\right)\sin\theta$$

$+$

$$\frac{d^2f}{dx^2}\cos^2\theta + \frac{d^2f}{dx^2}\sin^2\theta + \frac{d^2f}{dy^2}\cos^2\theta + \frac{d^2f}{dy^2}\sin^2\theta$$

$$= \underbrace{(\sin^2\theta + \cos^2\theta)}_{1}\left(\frac{d^2f}{dx^2}\right) + \underbrace{(\sin^2\theta + \cos^2\theta)}_{1}\left(\frac{d^2f}{dy^2}\right)$$

$$= \frac{d^2f}{dx^2} + \frac{d^2f}{dy^2}$$

So,

$$\frac{d^2f}{dx^2} + \frac{d^2f}{dy^2} = \frac{d^2f}{dx'^2} + \frac{d^2f}{dy'^2}$$

Task 3

a)

If

filter (image 1 + image 2) = filter (image 1) + filter (image 2)

that means our filter is linear.

$$h(x,y) = 3f(x,y) + 2f(x-1,y) + 2f(x+1,y) - 17f(x,y-1) + 99f(x,y+1)$$

Let's choose two images called f_1 and f_2

$f_1(x,y) = a$ $\quad$ $f_2(x,y) = g$

$f_1(x-1,y) = b$ $\quad$ $f_2(x-1,y) = h$

$f_1(x+1,y) = c$ $\quad$ $f_2(x+1,y) = i$

$f_1(x,y-1) = d$ $\quad$ $f_2(x,y-1) = j$

$f_1(x,y+1) = e$ $\quad$ $f_2(x,y+1) = k$

First, we are going to sum these images and apply the filter on final image.

$f_1 + f_2 = f_3$

$f_3(x,y) = a+g$

$f_3(x-1,y) = b+h$

$f_3(x+1,y) = c+i$

$f_3(x,y-1) = d+j$

$f_3(x,y+1) = e+k$

$\Longrightarrow$ $f_3(x,y) \cdot h(x,y) + f_3(x-1,y) \cdot h(x-1,y) + f_3(x+1,y) \cdot h(x+1,y) + f_3(x,y-1) \cdot h(x,y-1) + f_3(x,y+1) \cdot h(x,y+1)$

$= 3a + 3g + 2b + 2h + 2c + 2i - 17d - 17j + 99e + 99k$

And now, we are going to sum these images separately, after we apply filter on them

$$f_1(x,y) \cdot h(x,y) + f_1(x-1,y) \cdot h(x-1,y) + f_1(x+1,y) \cdot h(x+1,y) +$$
$$f_1(x,y-1) \cdot h(x,y-1) + f_1(x,y+1) \cdot h(x,y+1) = 3a + 2b + 2c - 17d + 99e$$

$$f_2(x,y) \cdot h(x,y) + f_2(x-1,y) \cdot h(x-1,y) + f_2(x+1,y) \cdot h(x+1,y) +$$
$$f_2(x,y-1) \cdot h(x,y-1) + f_2(x,y+1) \cdot h(x,y+1) = 3g + 2h + 2i - 17j + 99k$$

If we sum these two results,

$$3a + 3g + 2b + 2h + 2c + 2i - 17d - 17j + 99e + 99k$$

We can easily see that, the result is exactly the same. That means the filter is linear.

b) $$\begin{bmatrix} 0 & 2 & 0 \\ -17 & 3 & 99 \\ 0 & 2 & 0 \end{bmatrix} \xrightarrow[\text{x and y}]{\text{Flip around}} \begin{bmatrix} 0 & 2 & 0 \\ 99 & 3 & -17 \\ 0 & 2 & 0 \end{bmatrix}$$ //