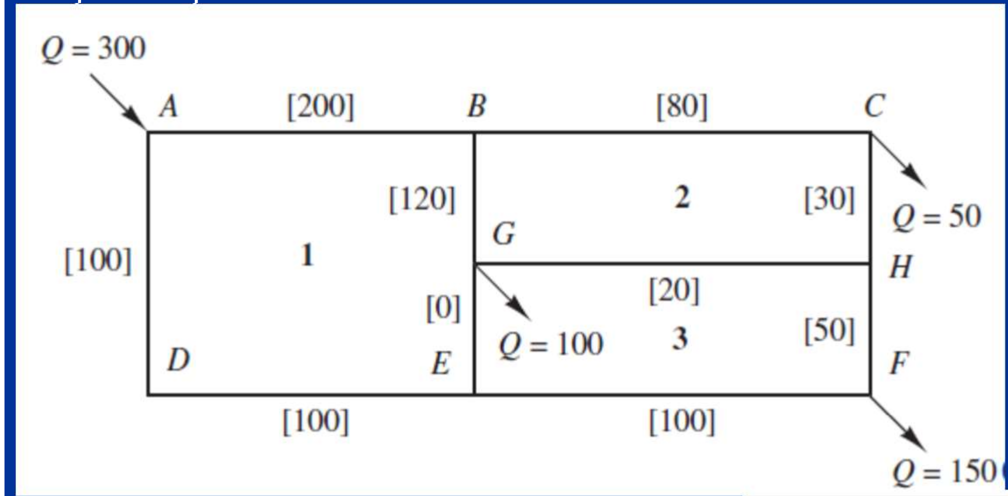
Week 5 & 6: Analysis of pipe networks. Multiple reservoir pipe networks.

Hardy-Cross Method: An example:

Question: A water-supply distribution system for an industrial park is schematically shown in Figure. The demands on the system are currently at junctions *C*, *G*, and *F* with flow rates given in liters per second. Water enters the system at junction *A* from a water storage tank on a hill. The water surface elevation in the tank is 50 m above the elevation of point A in the industrial park. All the junctions have the same elevation as point A. All pipes are aged ductile iron (e=0.26mm) with lengths and diameters provided in the table below. Calculate the flow rate in each pipe. Also determine if the pressure at junction *F* will be high enough to satisfy the customer there. The required pressure is 185 kPa.



Note: Brackets indicate assumed flowrates!.

Pipe	Flow (m^3/sec)	Length (m)	Diameter (m)	*e*/*D*	*f*	*K* (sec^2/m^5)
AB	0.20	300	0.30	0.00087	0.019	194
AD	0.10	250	0.25	0.00104	0.020	423
BC	0.08	350	0.20	0.00130	0.021	1,900
BG	0.12	125	0.20	0.00130	0.021	678
GH	0.02	350	0.20	0.00130	0.021	1,900
CH	0.03	125	0.20	0.00130	0.021	678
DE	0.10	300	0.20	0.00130	0.021	1,630
GE	0.00	125	0.15	0.00173	0.022	2,990
EF	0.10	350	0.20	0.00130	0.021	1,900
HF	0.05	125	0.15	0.00173	0.022	2,990

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Hardy-Cross Method: An example:

Start with iteration 1, loop 1:

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1	*AB*	0.200	194	7.76	38.8	0.205
	BG	0.120	678	9.76	81.3	0.125
	GE	0.000	2,990	0.00	0.0	0.005
	AD	(0.100)	423	(4.23)	(42.3)	(0.095)
	DE	(0.100)	1,630	(16.3)	(163.0)	(0.095)

$$\Delta Q = \frac{(\Sigma h_{fc} - \Sigma h_{fcc})}{2\left(\Sigma \frac{h_{fc}}{Q_c} + \Sigma \frac{h_{fcc}}{Q_{cc}}\right)}$$

Brackets () in here denote the CC direction. Correction term becomes:

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2\left[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})\right]} = \frac{(7.76 + 9.76) - (4.23 + 16.3)}{2\left[(38.8 + 81.3) + (42.3 + 163.0)\right]} = -0.005 \text{ m}^3/\text{sec}$$

Negative sign indicates that CC losses dominate: $\Sigma h_{fcc} > \Sigma h_{fc}$

Thus the ΔQ will be subtracted from CC direction in the next iteration!

Continue with iteration 1, loop 2:

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2	*BC*	0.080	1,900	12.2	152.5	0.078
	CH	0.030	678	0.61	20.3	0.028
	BG	(0.125)	678	(10.6)	(84.8)	(0.127)
	GH	(0.020)	1,900	(0.76)	(38.0)	(0.022)

Since BG is shared by loops 1 and 2, the revised flowrate in loop 1 is used for BG!

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2\left[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})\right]} = \frac{(12.2 + 0.61) - (10.6 + 0.76)}{2\left[(152.5 + 20.3) + (84.8 + 38.0)\right]} = +0.002 \text{ m}^3/\text{sec}$$

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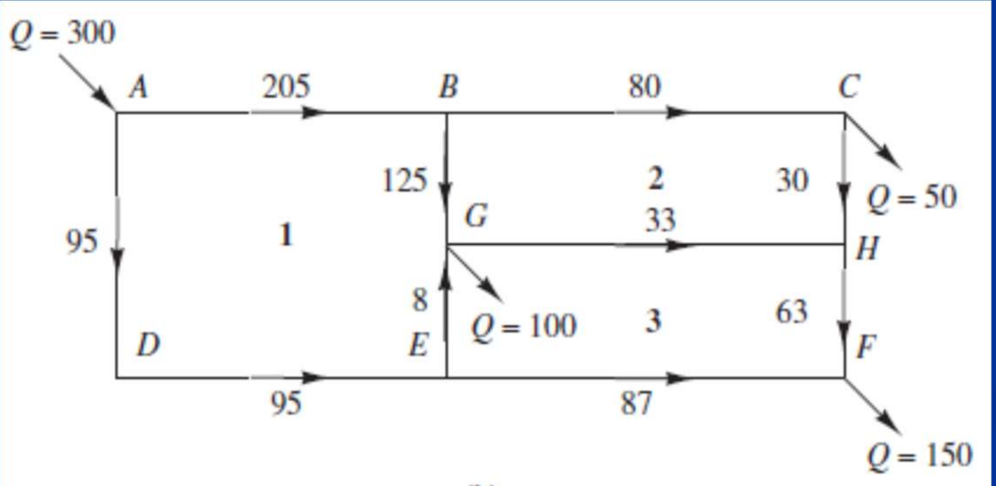
Hardy-Cross Method: An example:

Start with iteration 1, loop 3:

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
3	GH	0.022	1,900	0.92	41.8	0.035
	HF	0.050	2,990	7.48	149.6	0.063
	GE	(0.005)	2,990	(0.07)	(14.0)	0.008
	EF	(0.100)	1,900	(19.0)	(190.0)	(0.087)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2\left[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})\right]} = \frac{(0.92 + 7.48) - (0.07 + 19.0)}{2\left[(41.8 + 149.6) + (14.0 + 190.0)\right]} = -0.013 \text{ m}^3/\text{sec}$$

Counterclockwise head losses dominate so the flow correction is added in the clockwise direction. Note that this is a large enough correction to reverse the flow direction in *GE*; it will be labeled *EG* the next time. So after loop 1:



(b)

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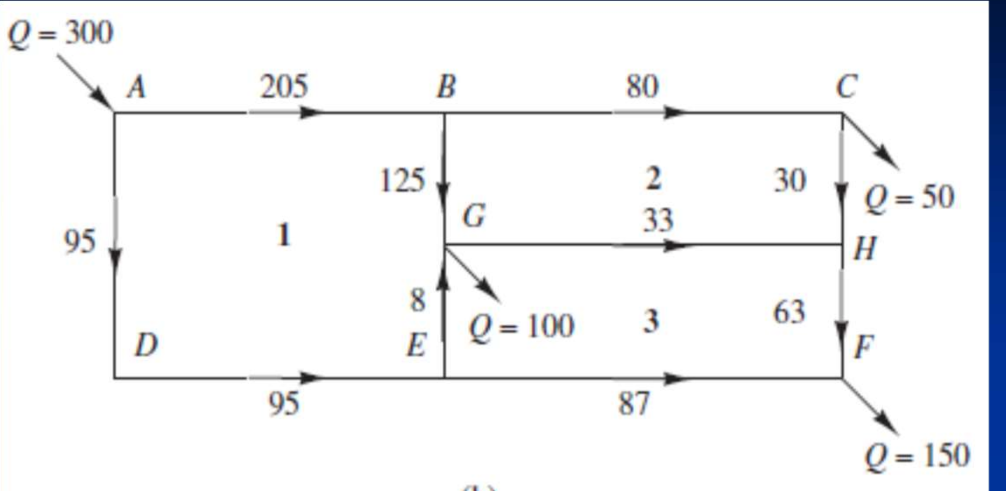
Hardy-Cross Method: An example:



(b)

Continue with iteration 2, loop 1:

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
1	*AB*	0.205	194	8.15	39.8	0.205
	BG	0.127	678	10.9	85.8	0.127
	AD	(0.095)	423	(3.82)	(40.2)	(0.095)
	DE	(0.095)	1,630	(14.7)	(154.7)	(0.095)
	EG	(0.008)	2,990	(0.19)	(23.8)	(0.008)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2\left[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})\right]} = \frac{(8.15 + 10.9) - (3.82 + 14.7 + 0.19)}{2\left[(39.8 + 85.8) + (40.2 + 154.7 + 23.8)\right]} = +0.000\ \mathrm{m^3/sec}$$

No correction is necessary since convergence criteria is achieved: $\Delta Q = 0.000\mathrm{m^3/s} < 0.005\mathrm{m^3/s}$

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Hardy-Cross Method: An example:

Continue with iteration 2, loop 2:

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
2	*BC*	0.078	1,900	11.6	148.7	0.080
	CH	0.028	678	0.53	18.9	0.030
	BG	(0.127)	678	(10.9)	(85.8)	(0.125)
	GH	(0.035)	1,900	(2.33)	(66.6)	(0.033)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2\left[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})\right]} = \frac{(11.6 + 0.53) - (10.9 + 2.33)}{2\left[(148.7 + 18.9) + (85.8 + 66.6)\right]} = -0.002 \text{ m}^3/\text{sec}$$

Again, since convergence criteria is satisfied, no correction is necessary.

Continue with iteration 2, loop 3:

Loop	Pipe	Q (m^3/sec)	K (sec^2/m^5)	h_f (m)	h_f/Q (sec/m^2)	New Q (m^3/sec)
3	*GH*	0.033	1,900	2.07	62.7	0.033
	HF	0.063	2,990	11.9	188.9	0.063
	EG	0.008	2,990	0.19	23.8	0.008
	EF	(0.087)	1,900	(14.4)	(165.5)	(0.087)

$$\Delta Q = \frac{\Sigma h_{fc} - \Sigma h_{fcc}}{2\left[\Sigma(h_{fc}/Q_c) + \Sigma(h_{fcc}/Q_{cc})\right]} = \frac{(2.07 + 11.9 + 0.19) - (14.4)}{2\left[(62.7 + 188.9 + 23.8) + (165.5)\right]} = -0.000 \text{ m}^3/\text{sec}$$

Again, since convergence criteria is satisfied, no correction is necessary.

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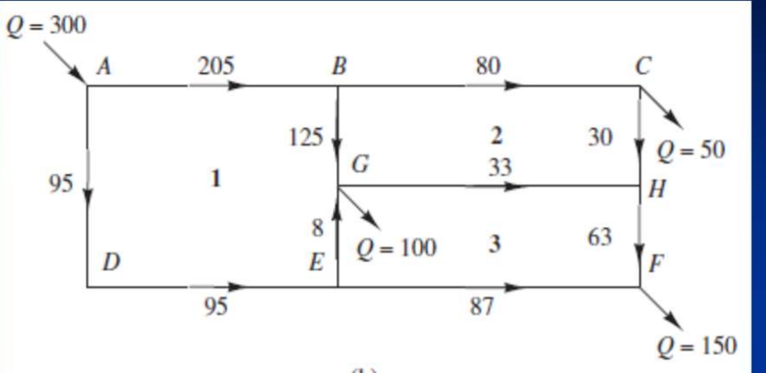
Hardy-Cross Method: An example:

So final dispatch of flow becomes:



(b)

Pipe	Q (L/sec)	Length (m)	Diameter (cm)	h_f (m)	ΔP (kPa)
AB	205	300	30	8.2	80.3
AD	95	250	25	3.8	37.2
BC	80	350	20	12.2	119.4
BG	125	125	20	10.6	103.8
GH	33	350	20	2.1	20.6
CH	30	125	20	0.6	5.9
DE	95	300	20	14.7	143.9
EG	8	125	15	0.2	2.0
EF	87	350	20	14.4	141.0
HF	63	125	15	11.9	116.5

where the last column shows pressure drop due to friction which is calculated using $\Delta P = \gamma h_f$

$$P_A = \gamma h = (9790 \text{ N/m}^3)(50 \text{ m}) = 489.5 \text{ kPa}$$

$$P_F = P_A - \Delta P_{AD} - \Delta P_{DE} - \Delta P_{EF} = 489.5 - 37.2 - 143.9 - 141.0 = 167.4 \text{ kPa}$$

Since this value is less than 185 kPa, the pressure requirement at F won't be satisfied!

Minor losses are not even considered!

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Homework 1:

Write a computer program in your preferred programming language (MATLAB is preferred-easy to learn and use and graphic options are very rich) which solves the example problem solved by Hardy-Cross method. Turn in your code, iteration steps and corresponding results in a neat tabular format. You may add graphs like iteration count vs convergence of Q graphs. You may need functions like xlswrite.

a) Produce the same results first.
b) Then change the convergence criteria to $\Delta Q<10^{-6}$ and repeat calculations. Comment on your results.
c) Add the numbers in your student number to get your FACTOR. Then multiply all numbers given in the question by your FACTOR and repeat same calculations for the convergence criteria to $\Delta Q<10^{-6.}$ Comment on your results.
d) Consider that a pump which supplies 10m of effective head is placed at the midpoint of pipe GH. Repeat the numerical as well as the numerical calculations and compare and comment on the change of results.

DUE 3 weeks!