Student Information

Name : Mehmet Erdeniz Aydoğdu

ID : 2380103

Answer 1

a)

$E(Blue) = 2 \times \frac{2}{3} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} = \frac{5}{3} = 1.67$
$E(Yellow) = 1 \times \frac{2}{6} + 2 \times \frac{2}{6} + 3 \times \frac{2}{6} = 2$
$E(Red) = 1 \times \frac{2}{8} + 2 \times \frac{2}{8} + 3 \times \frac{3}{8} + 5 \times \frac{1}{8} = \frac{5}{2} = 2.5$

b)

For both cases, each dice has values, say x, y, z. Then their expected values will be equal to $E(x + y + z)$ which is equal to $E(x) + E(y) + E(z)$ by the linearity of expected value.

- For two red and one yellow dice case, $E(2\ Red + Yellow) = 2 \times E(Red) + E(Yellow) = 7$
- For two yellow and one blue dice case, $E(2\ Yellow + Blue) = 2 \times E(Yellow) + E(Blue) = 5.67$

In order to maximize the total value, first option should be chosen, as its expected value is higher.

c)

When 4 is guaranteed, $E(Blue)$ will be equal to 4. Because of that change, $E(2\ Yellow + Blue) = 8$

This time, second option's expected value is higher, so second option should be chosen.

d)

Given that the value is 3, the probability that the rolled die is red is $P(Red|3)$, also:

$$P(Red|3) = \frac{P(3|Red) \times P(Red)}{P(3)}$$

$$\frac{P(3|Red) \times P(Red)}{P(3)} = \frac{P(3|Red) \times P(Red)}{P(3|Red) \times P(Red) + P(3|Yellow) \times P(Yellow) + P(3|Blue) \times P(Blue)}$$

by Baye's Law and the law of total probability.

As $P(3|Red) = \frac{3}{8}$, $P(3|Yellow) = \frac{1}{3}$, $P(3|Blue) = \frac{1}{6}$, $P(Red) = P(Yellow) = P(Blue) = \frac{1}{3}$

$$P(Red|3) = \frac{\frac{3}{8} \times \frac{1}{3}}{\frac{3}{8} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{3}} = \frac{3}{7}$$

e)

For the total value 6, there are two combinations.

Red	Yellow
3	3
5	1

- $P(3,3) = \frac{3}{8} \times \frac{1}{3} = \frac{1}{8}$
- $P(5,1) = \frac{1}{3} \times \frac{1}{8} = \frac{1}{24}$

$\frac{1}{8} + \frac{1}{24} = \frac{1}{6}$ is the probability that the total value will be 6 when a single red die and a single yellow die is rolled together.

Answer 2

a)

By the third line of Table 1, $P(A = 0, I = 2) = 0.17$

b)

As there are probabilities for only A = 0 and A = 1, $P(A = 2, I = 0) = 0$

c)

$P(A = 0, I = 2) + P(A = 1, I = 1) = 0.17 + 0.11 = 0.28$

d)

$P(A = 1, I = 0)+P(A = 1, I = 1)+P(A = 1, I = 2)+P(A = 1, I = 3) = 0.12+0.11+0.22+0.15 = 0.6$

e)

- $P_A(0) = P(0,0) + P(0,1) + P(0,2) + P(0,3) = 0.08 + 0.13 + 0.17 + 0.02 = 0.4$
- $P_A(1) = P(1,0) + P(1,1) + P(1,2) + P(1,3) = 0.12 + 0.11 + 0.22 + 0.15 = 0.6$
- $P_I(0) = P(0,0) + P(1,0) = 0.08 + 0.12 = 0.2$
- $P_I(1) = P(0,1) + P(1,1) = 0.13 + 0.11 = 0.24$
- $P_I(2) = P(0,2) + P(1,2) = 0.17 + 0.22 = 0.39$
- $P_I(3) = P(0,3) + P(1,3) = 0.02 + 0.15 = 0.17$

I\A	0	1	2	3	$P_A(a)$
0	0.08	0.13	0.17	0.02	0.4
1	0.12	0.11	0.22	0.15	0.6
$P_I(i)$	0.2	0.24	0.39	0.17	1

f)

In order to electric outages be independent, $P_A(a) \times P_I(i) = P(a,i)$ for all cases.

As cases other than $P_A(0) \times P_I(0) = P(0,0)$ and $P_A(1) \times P_I(0) = P(1,0)$ does not satisfy the condition, electric outages in Ankara and Istanbul are not independent.