Student Information

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Answer 1

a)

$\mu_0 = 7$, $\overline{X} = 7.8$, $\sigma = 1.4$, $n = 17$ and $\alpha = 0.05$:

$H_0 : \mu = 7$ and $H_A : \mu > 7$

For level $\alpha = 0.05$ test with a right-tail alternative, $Z_\alpha = Z_{0.05} = 1.645$:
Then, the acceptance region is $(-\infty, 1.645]$

$$Test\ statistic\ Z = \frac{7.8 - 7}{\frac{1.4}{\sqrt{17}}} = 2.356$$

As $2.356 > 1.645$, we reject H_0, the null hypothesis. Thus, the service can be regarded as successful.

b)

As $\overline{X} = 7.8$, the new mean will be:

$$\frac{(17 \times 7.8) + 1 - 10}{17} = 7.27$$

For the new mean,

$$Test\ statistic\ Z = \frac{7.27 - 7}{\frac{1.4}{\sqrt{17}}} = 0.795$$

As $0.7952 < 1.645$, we accept the null hypothesis. Thus, the service can not be regarded as successful.

c)

For 45 customers, the new mean will be:

$$\frac{(45 \times 7.8) + 1 - 10}{45} = 7.6$$

For the new mean,

$$Test\ statistic\ Z = \frac{7.6 - 7}{\frac{1.4}{\sqrt{45}}} = 2.875$$

As $2.8749 > 1.645$, we reject the null hypothesis. Thus, the mistake does not affect the success and the service can be regarded as successful.

d)

For $\mu_0 = 8$, test statistic would be negative. Thus, the test statistic would be in the acceptance region and we accept the null hypothesis.

Answer 2

$\mu_0 = 5.8$, $\overline{X} = 6.2$, $\sigma = 1.5$, $n = 55$ and $\alpha = 0.05$

$H_0 : \mu = 5.8$ and $H_A : \mu > 5.8$

For level $\alpha = 0.05$ test with a right-tail alternative, $Z_\alpha = Z_{0.05} = 1.645$

$$Test\ statistic\ Z = \frac{6.2 - 5.8}{\frac{1.5}{\sqrt{55}}} = 0.198$$

As $0.1978 < 1.645$, we accept the null hypothesis. We can not state that the new vaccine protects for a longer duration.

Answer 3

a)

At %95 confidence, $\alpha = 0.05$ and $Z_{\alpha/2} = 1.96$:
The margin of error for $\hat{p}_{red} = 0.48$ at %95 confidence level is:

$$1.96\sqrt{\frac{0.48 \times 0.52}{400}} = 0.049$$

In other words, $\hat{p}_{red}$ is at $48 \pm 4.9\%$
The margin of error for $\hat{p}_{blue} = 0.37$ at %95 confidence level is:

$$1.96\sqrt{\frac{0.37 \times 0.63}{400}} = 0.047$$

In other words, $\hat{p}_{blue}$ is at $37 \pm 4.7\%$

b)

The margin of error for $\hat{p}_{red} - \hat{p}_{blue} = 0.11$ at %95 confidence level is:

$$1.96\sqrt{\frac{0.48 \times 0.52}{400} + \frac{0.37 \times 0.63}{400}} = 0.068$$

In other words, $\hat{p}_{red} - \hat{p}_{blue}$ is at $11 \pm 6.8\%$

c)

Margin of error of Reds is larger.
The reason is, margin of error is at maximum for sample proportion $\hat{p} = 0.5$ and $\hat{p}_{red}$ is closer to 0.5.

d)

With the new size, margins of error will be multiplied with:

$$\sqrt{\frac{400}{1800}} = \frac{\sqrt{2}}{3} = 0.471$$

Thus, new margins will be less than margins with 400 participants.