Question - 1

a) $(2^n + n^3) \in O(4^n)$

A function $f(n) \in O(g(n))$ iff there exists positive constants c and n_0 s.t $f(n) \leq c.g(n)$ whenever $n \geq n_0$

$f(n) = 2^n + n^3$ $\Rightarrow$ $(2^n + n^3) \leq c.4^n$ for $n \geq n_0$

$g(n) = 4^n$ if we choose $n_0 = 2$, $c = 2$

$2^n + n^3 \leq 2.4^n$ for $n \geq 2$

$2^2 + 2^3 \leq 2.4^2 \Rightarrow 12 \leq 32$ for $n \geq 2$ ✓

Also another approach

if $n \to \infty$ $\quad 2^n + n^3 \leq 2.4^n$

n^3 is negligible because 2^n growth rate bigger than n^3

and $2^n \leq 4^n.c$ ✓ So, $(2^n + n^3) \in O(4^n)$ statement is true

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b) $\sqrt{10n^2 + 7n + 3} \in \Omega(n)$

iff there exists positive constants c and n_0 s.t $c.g(n) \leq f(n)$ for $n \geq n_0$

$f(n) = \sqrt{10n^2 + 7n + 3}$ $\Rightarrow$ $c.n \leq \sqrt{10n^2 + 7n + 3}$

$g(n) = n$ if we choose $c = 2$ $n_0 = 2$

$2.n \leq \sqrt{10n^2 + 7n + 3}$, $n \geq 2$ $\Rightarrow$ $2.2 \leq \sqrt{10.2^2 + 7.2 + 3} \Rightarrow 16 \leq 57$ ✓

Also another approach

if $n \to \infty$ $\quad 2.n \leq \sqrt{10n^2 + 7n + 3}$

negligible because n^2 growth rate bigger than n

$2n \leq \sqrt{10n^2} \rightarrow 2n \leq \sqrt{10}.n$

$\theta(n) = \theta(n)$ ✓

So $\sqrt{10n^2 + 7n + 3} \in \Omega(n)$ statement is true

c) $n^2+n \in o(n^2)$

iff there exists positive constants c, there is a positive integer n_0 such that $f(n) < c.g(n)$ whenever $n \geq n_0$

$f(n) = n^2+n$
$g(n) = n^2$ $\Rightarrow n^2+n < n^2.c$, $n \geq n_0$

$= \frac{n^2+n}{n^2} < \frac{n^2.c}{n^2} \Rightarrow 1+\frac{1}{n} < c$, for $c=1$ its not possible

Also another approach

if $n \to \infty$ $\quad n^2+\underbrace{n}_{\text{negligible}} < n^2.c \Rightarrow n^2 < n^2$ ✗ it is not possible

$n^2+n \in o(n^2)$ statement is false

d) $3\log_2^2 n \in \Theta(\log_2 n^2)$

iff there exist positive constants c_1, c_2 and n_0 s.t

$c_1.g(n) \leq f(n) \leq c_2.g(n)$, $n \geq n_0$

$f(n) = 3\log_2^2 n = 3\log_2(\log_2 n)$

$g(n) = \log_2 n^2 = 2\log_2 n$

$c_1.2\log_2 n \leq 3\log_2(\log_2 n) \leq c_2.2\log_2 n$

It is false because $\log_2(\log_2 n)$ grow rate bigger than $\log_2 n$

$3\log_2^2 n \in \Theta(\log_2 n^2)$ ✗

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c) $(n^3+1)^6 \in O(n^3)$

$f(n) = (n^3+1)^6$
$g(n) = n^3$ $\Rightarrow$ $f(n) \leq c.g(n) \Rightarrow (n^3+1)^6 \leq c.n^3$

$\Rightarrow n^{18} + a_1.n^{17} + a_2.n^{16} \ldots + 1 \leq c.n^3$

if $n \to \infty$ $n^{18} + \underbrace{a_1.n^{17} + a_2.n^{16} \ldots + 1}_{\text{negligible}} \leq c.n^3$

$n^{18} < n^3$ ✗ it is false

$(n^3+1)^6 \in O(n^3)$ statement is false

Question -2

a) $2n\log(n+2)^2 + (n+2)^2\log\frac{n}{2} = 4n\log(n+2) + (n+2)^2\log\frac{n}{2}$

$4n\log(n+2) + (n+2)^2(\log n - \log 2)$

* $4n\log(n+2) \in \Theta(n\log n)$
* $(n+2)^2(\log n - \log 2) \in \Theta(n^2\log n)$

$\Theta(n\log n) + \Theta(n^2\log n) \in \Theta(n^2\log n)$

Simplest $\Theta(g(n)) = \Theta(n^2\log n) \Rightarrow g(n) = n^2\log n$

b) $0,001n^4 + 3n^3 + 1$

* $0,001n^4 \in \Theta(n^4)$
* $3n^3 \in \Theta(n^3)$
* $1 \in \Theta(1)$

$\Theta(n^4) + \Theta(n^3) + \Theta(1) \in \Theta(n^4)$

simplest $\Theta(g(n)) = \Theta(n^4) \Rightarrow g(n) = n^4$

Question - 3

a) $\log n$, $n^{\log n}$, $n^{1.5}$

* $\lim\limits_{n \to \infty} \frac{n^{\log n}}{\log n} = \frac{\infty}{\infty} \Rightarrow \frac{\frac{d}{dn}(n^{\log n})}{\frac{d}{dn}(\log n)} \Rightarrow \frac{2n^{\log n} \cdot \log n}{n \cdot \frac{1}{n}}$

$= \lim\limits_{n \to \infty} 2 \cdot n^{\log n} \cdot \log n = \infty$ So, $n^{\log n}$ growth bigger then $\log n$

* $\lim\limits_{n \to \infty} \frac{n^{1.5}}{\log n} = \frac{\infty}{\infty} \Rightarrow \frac{\frac{d}{dn}(n^{1.5})}{\frac{d}{dn}(\log n)} = \lim\limits_{n \to \infty} \frac{\frac{3}{2}\sqrt{n}}{\frac{1}{n}} = \lim\limits_{n \to \infty} \frac{3}{2} \cdot n\sqrt{n} = \infty$

So, $n^{1.5} > \log n$

* $\lim\limits_{n \to \infty} \frac{n^{\log n}}{n^{1.5}} = \frac{\infty}{\infty} = \lim\limits_{n \to \infty} n^{\log n - 1.5} = \infty$, $n^{\log n} > n^{1.5}$

So, orders of growth $n^{\log n} > n^{1.5} > \log n$

b) $n!$, 2^n, n^2

* $\lim\limits_{n \to \infty} \frac{2^n}{n^2} = \frac{\infty}{\infty} = \frac{\frac{d}{dn}(2^n)}{\frac{d}{dn}(n^2)} = \frac{2^n \cdot \log 2}{2n} = \frac{2^{n-1} \cdot \log 2}{n} = \frac{\log 2}{2} \cdot \lim\limits_{n \to \infty} \frac{2^n}{n}$

$= \frac{\log 2}{2} \lim\limits_{n \to \infty} \frac{2^n}{n} = \frac{\infty}{\infty} = \lim\limits_{n \to \infty} \frac{\frac{d}{dn}(2^n)}{\frac{d}{dn}(n)} = \frac{\log 2}{2} \cdot \lim\limits_{n \to \infty} 2^n \cdot \log 2 = \infty$

So, $2^n > n^2$

* $n! \cong \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ for large n (Stirling's formula)

$\lim\limits_{n \to \infty} \frac{n!}{2^n} = \lim\limits_{n \to \infty} \frac{\sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n}{2^n} = \frac{\infty}{\infty} = \lim\limits_{n \to \infty} \sqrt{2\pi n} \cdot \left(\frac{n}{2e}\right)^n = \infty$, $n! > 2^n$

So, orders of growth $n! > 2^n > n^2$

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c) $n\log n, \sqrt{n}$

$$\lim_{n\to\infty} \frac{n\log n}{\sqrt{n}} = \frac{\infty}{\infty} = \frac{\cancel{\sqrt{n}}.\sqrt{n}.\log n}{\cancel{\sqrt{n}}} = \lim_{n\to\infty} \sqrt{n}.\log n = \infty$$

So, orders of growth $n\log n > \sqrt{n}$

d) $n.2^n, 3^n$

$$\lim_{n\to\infty} \frac{3^n}{n.2^n} = \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{\left(\frac{3}{2}\right)^n}{n} = \frac{\frac{d}{dn}\left(\left(\frac{3}{2}\right)^n\right)}{\frac{d}{dn} n} = \lim_{n\to\infty}\left(\frac{3}{2}\right)^n.\ln\frac{3}{2}$$

$$= (\ln 3 - \ln 2).\lim_{n\to\infty}\left(\frac{3}{2}\right)^n = \infty$$

So, orders of growth $3^n > n.2^n$

e) $\sqrt{n+10}, n^3$

$$\lim_{n\to\infty} \frac{n^3}{\sqrt{n+10}} = \frac{\infty}{\infty} = \lim_{n\to\infty} \frac{n^3}{\sqrt{n}.\sqrt{1+\frac{10}{n}}} = \lim_{n\to\infty} \frac{n^{5/2}}{\sqrt{1+\frac{10}{n}}} = \frac{\lim_{n\to\infty} n^{5/2}}{\lim_{n\to\infty}\sqrt{1+\frac{10}{n}}} \quad \infty$$

$$= \infty.\frac{1}{\lim_{n\to\infty}\sqrt{1+\frac{10}{n}}} = \infty.\frac{1}{\sqrt{\lim_{n\to\infty} 1 + \lim_{n\to\infty}\frac{10}{n}}} = \infty.\frac{1}{\sqrt{1+\lim_{n\to\infty}\frac{10}{n}}} = \infty.\frac{1}{\sqrt{1+\left(10.\lim_{n\to\infty}\frac{1}{n}\right)}}$$

$$= \infty.\frac{1}{\sqrt{1+10.\frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} n}}} = \infty.\frac{1}{\sqrt{1+\frac{10}{\lim_{n\to\infty} n}}} \quad \leftarrow 0 \quad \infty = \infty.\frac{1}{\sqrt{1+0}}$$

$$= \infty.\frac{1}{1} = \infty$$

So, orders of growth $n^3 > \sqrt{n+10}$

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Question - 4

a) Basic operation is B[i, j] != B[j, i] comparison

b) $\sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} 1 = \sum_{i=0}^{n-2} [(n-1)-(i+1)+1]$

$= \sum_{i=0}^{n-2} (n-1-i) = (n-1)+(n-2)+\dots+1 = \frac{(n-1).n}{2}$ times

c) $\sum_{i=0}^{n-2} (n-1-i) = \frac{(n-1).n}{2} = \frac{n^2-n}{2} \Rightarrow$ Time complexity is $O(n^2)$

Question - 5

a) Basic operation is C[i, j] = C[i, j] + A[i, k] * B[k, j]

b) $\sum_{i=0}^{n-1} \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} 1 = (n-1)n^2$ times

c) $(n-1)n^2 = n^3 - n^2 \longrightarrow$ Time complexity is $O(n^3)$

Question - 6

```
find-pairs (A[0,1...,n-1], desiredNumber)
    for i=0 to n-1 do
        for j=i+1 to n-1 do
            if A[i] * A[j] = desiredNumber
                print{A[i], A[j]}
```

Time complexity $= \sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 1 = \frac{n(n-1)}{2} = \frac{n^2-n}{2}$

Time Complexity is $O(n^2)$