CSE 211: Discrete Mathematics	**(Due: 15/12/19)**

Homework #3

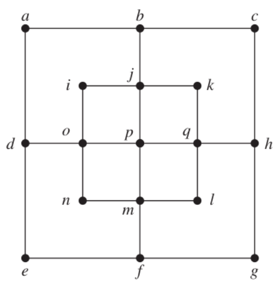
Instructor: Dr. Zafeirakis Zafeirakopoulos *Name:* *Student Id:*
Assistants: Gizem Süngü, Baak Karaka

Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

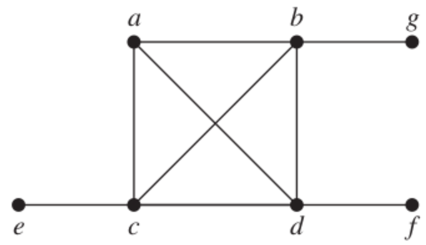
- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework3 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Hamilton Circuits (10+10+10=30 points)

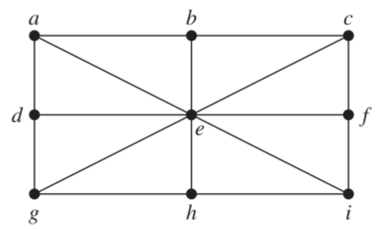
Determine whether there is a Hamilton circuit for each given graph (See Figure 1a, Figure 1b, Figure 1c). If the graph has a Hamilton circuit, show the path with its vertices which gives a Hamilton circuit. If it does not, explain why no Hamilton circuit exists.



(a) The graph G_1



(b) The graph G_2



(c) The graph G_3

Figure 1: The graphs to find Hamilton circuits for Problem 1

(a) ***(Solution)***
Degree of vertex:deg(a) is $= 2$
deg(b) is $= 3$
deg(c) is $= 2$
deg(d) is $= 3$
deg(e) is $= 2$

deg(f) is $= 3$
deg(g) is $= 2$
deg(h) is $= 3$
deg(i) is $= 2$
deg(j) is $= 4$
deg(k) is $= 2$
deg(l) is $= 2$
deg(m) is $= 4$
deg(n) is $= 2$
deg(o) is $= 4$
deg(p) is $= 4$
deg(q) is $= 4$

Hamilton circuit does not exist because a , c, e , g have 2 degree but b,c,h,g,f,d,a,b have already a circuit and a,c,e,g circuit have this b,c,h,g,f,d,a,b circuit which is not possible without passing through b more than once.

(b) ***(Solution)***
deg(a) is $= 3$
deg(b) is $= 4$
deg(c) is $= 4$
deg(d) is $= 4$
deg(e) is $= 1$
deg(f) is $= 1$
deg(g) is $= 1$
Hamilton circuit does not exist because degree of f 1 and any circuit that contains f need to pass through d twice.
(c) ***(Solution)***
deg(a) is $= 3$
deg(b) is $= 3$
deg(c) is $= 3$
deg(d) is $= 3$
deg(e) is $= 8$
deg(f) is $= 3$
deg(g) is $= 3$
deg(h) is $= 3$
deg(i) is $= 3$
Hamilton circuit exist. Possible hamilton circuit is : a,b,c,f,i,h,g,d,e,a

Problem 2: Graph Isomorphism (10+10+10=30 points)

Determine whether each pair of graphs (see Figure 2, Figure 3, Figure 4) is isomorphic or not.
Note: If you answer only "isomorphic" or "not isomorphic", you cannot get points. Show your work.

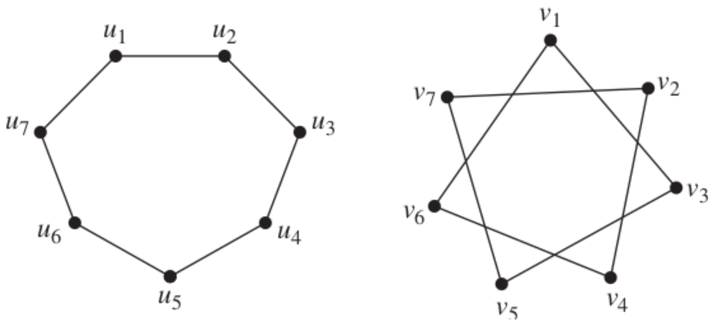


Figure 2: The graphs G_{a1}(left) and G_{a2}(right) to find isomorphism for Problem 2(a)

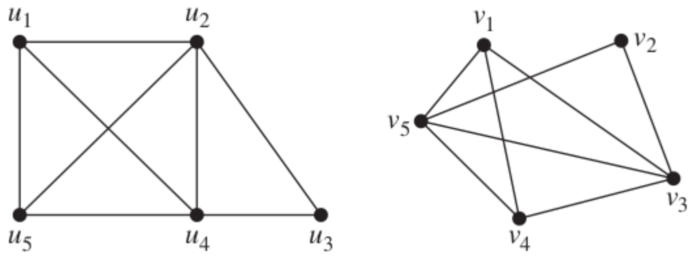


Figure 3: The graphs G_{b1}(left) and G_{b2}(right) to find isomorphism for Problem 2(b)

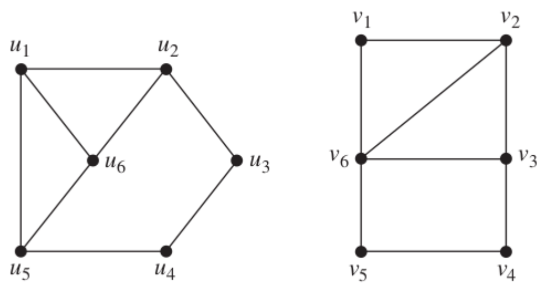


Figure 4: The graphs G_{c1}(left) and G_{c2}(right) to find isomorphism for Problem 2(c)

(a) ***(Solution)***
G_{a1}(left) ;
Set of Vertices($V1$): u1,u2,u3,u4,u5,u6,u7
Set of Edges($E1$) : $(u1, u2)$, $(u2, u3)$, $(u3, u4)$, $(u4, u5)$, $(u5, u6)$, $(u6, u7)$,$(u7, u1)$
G_{a2}(right) ;
Set of Vertices($V2$): v1,v2,v3,v4,v5,v6,v7
Set of Edges($E2$) : $(v1, v3)$, $(v3, v5)$, $(v5, v7)$, $(v7, v2)$, $(v2, v4)$, $(v4, v6)$,$(v6, v1)$
If we compare two set of edges we define one-to-one and onto function
f($u1$) = v1
f($u2$) = v3
f($u3$) = v5
f($u4$) = v7

f($u5$) = v2
f($u6$) = v4
f($u7$) = v6
Then
u1 and u2 are adjacent while f($u1$) = v1 and f($u2$) = v3 are also adjacent
u2 and u3 are adjacent while f($u2$) = v3 and f($u3$) = v5 are also adjacent
u3 and u4 are adjacent while f($u3$) = v5 and f($u4$) = v7 are also adjacent
u4 and u5 are adjacent while f($u4$) = v7 and f($u5$) = v2 are also adjacent
u5 and u6 are adjacent while f($u5$) = v2 and f($u6$) = v4 are also adjacent
u6 and u7 are adjacent while f($u6$) = v4 and f($u7$) = v6 are also adjacent
u7 and u1 are adjacent while f($u7$) = v6 and f($u1$) = v1 are also adjacent
So f is a function make two graphs Isomorphic
Figure 2 is Isomorphic

(b) ***(Solution)***
G_{b1}(left) ;
Set of Vertices($V1$): u1,u2,u3,u4,u5
Set of Edges($E1$) : $(u1, u2)$, $(u1, u4)$, $(u1, u5)$, $(u2, u3)$, $(u2, u4)$, $(u2, u5)$,$(u3, u4)$, $(u4, u5)$
G_{b2}(right) ;
Set of Vertices($V2$): v1,v2,v3,v4,v5
Set of Edges($E2$) : $(v1, v5)$, $(v1, v3)$, $(v1, v4)$, $(v5, v2)$, $(v5, v3)$, $(v5, v4)$,$(v2, v3)$, $(v3, v4)$
If we compare two set of edges we define one-to-one and onto function
f($u1$) = v1
f($u2$) = v5
f($u3$) = v2
f($u4$) = v3
f($u5$) = v4
u1 and u2 are adjacent while f($u1$) = v1 and f($u2$) = v5 are also adjacent
u1 and u4 are adjacent while f($u1$) = v1 and f($u4$) = v3 are also adjacent
u1 and u5 are adjacent while f($u1$) = v1 and f($u5$) = v4 are also adjacent
u2 and u3 are adjacent while f($u2$) = v5 and f($u3$) = v2 are also adjacent
u2 and u4 are adjacent while f($u2$) = v5 and f($u4$) = v3 are also adjacent
u2 and u5 are adjacent while f($u2$) = v5 and f($u5$) = v4 are also adjacent
u3 and u4 are adjacent while f($u3$) = v2 and f($u4$) = v3 are also adjacent
u4 and u5 are adjacent while f($u4$) = v3 and f($u5$) = v4 are also adjacent
So f is a function make two graphs Isomorphic
Figure 2 is Isomorphic

(c) ***(Solution)***
G_{c1}(left) ;
deg($u1$) is = 3
deg($u2$) is = 3
deg($u3$) is = 2
deg($u4$) is = 2
deg($u5$) is = 3
deg($u6$) is = 3
G_{c2}(right) ;
deg($v1$) is = 2
deg($v2$) is = 3
deg($v3$) is = 3
deg($v4$) is = 2
deg($v5$) is = 2
deg($v6$) is = 4
Two graphs don't have same degree sequence
So These two graphs are not isomorphic

Problem 3: Euler Circuits (10+10=20 points)

Determine whether there is a Euler circuit for each given graph (See Figure 5a, Figure 5b). If the graph has a Euler circuit, show the path with its vertices which gives a Euler circuit. If it does not, explain why no Euler circuit exists.

(a) The graph G_{3a} (b) The graph G_{3b}

Figure 5: The graphs to find Euler circuits for Problem 3

(Solution)
A)
In-degree and out-degree of vertex:
deg - (a) is = 1
deg + (a) is = 2
deg - (b) is = 3
deg + (b) is = 3
deg - (c) is = 2
deg + (c) is = 2
deg - (d) is = 4
deg + (d) is = 3
deg - (e) is = 2
deg + (e) is = 2
When G_{3a} is weakly connected , in -out degree is same at each vertex, G_{3a} can be Euler Circuit but some in-out degrees are different so There is no Euler Circuit.
B)
In-degree and out-degree of vertex:
deg - (a) is = 2
deg + (a) is = 2
deg - (b) is = 4
deg + (b) is = 3
deg - (c) is = 2
deg + (c) is = 3
deg - (d) is = 2
deg + (d) is = 2
deg - (e) is = 3
deg + (e) is = 3
deg - (f) is = 3
deg + (f) is = 3
When G_{3a} is weakly connected , in -out degree is same at each vertex, G_{3a} can be Euler Circuit but some in-out degrees are different so There is no Euler Circuit.

Problem 4: Applications on Graphs (20 points)

Schedule the final exams for Math 101, Math 243, CSE 333, CSE 346, CSE 101, CSE 102, CSE 273, and

CSE 211, using the fewest number of different time slots, if there are no students who are taking:

- both Math 101 and CS 211,
- both Math 243 and CS 211,
- both CSE 346 and CSE 101,
- both CSE 346 and CSE 102,
- both Math 101 and Math 243,
- both Math 101 and CSE 333,
- both CSE 333 and CSE 346

but there are students in every other pair of courses together for this semester.
Note: Assume that you have only one classroom.

Hint 1: Solve the problem with respect to your problem session notes.
Hint 2: Check the website

(Solution) Time Slot 1: CS101 , CSE333 , CSE246 because these exams connected other all exams
Time Slot 2:CS211 and Mat243 because CS211 connected all exams except Mat243
Time Slot 3:CS102