CSE 211: Discrete Mathematics	(Due: 27/10/19)

Homework #1

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.

- Do not take any information from Internet.

- No late homework will be accepted.

- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr

- Submit your homework into Assignments/Homework1 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Conditional Statements   (5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
(Solution)

Converse: If I stay at home , it will snow tonight.

Contrapositive: If I dont stay at home , it wont snow tonight.

Inverse:If it doesnt snow tonight, I wont stay at home.

(b) I go to the beach whenever it is a sunny summer day.
(Solution)

Converse:It is a sunny summer day whenever I go to the beach.

Contrapositive:It is not a sunny summer day whenever I dont go to the beach.

Inverse:I dont go to the beach whenever it is not a sunny summer day.

(c) When I stay up late, it is necessary that I sleep until noon.
(Solution)

Converse: When It is necessary that I sleep until noon ,I stay up late.

Contrapositive:When It is not necessary that I sleep until noon,I dont stay up late.

Inverse:When I dont stay up late , it is not necessary that I sleep until noon.

Problem 2: Truth Tables For Logic Operators   (5+5+5=15 points)

Construct a truth table for each of the following compound propositions.

(a) (p ⊕ ¬ q)

(Solution)

p	q	¬ q	$(p \oplus ¬ q)$
T	T	F	T
T	F	T	F
F	T	F	F
F	F	T	T

(b) (p ⟺ q) ⊕ (¬ p ⟺ ¬ r)

(Solution)

p	q	r	(¬ p)	(¬ r)	$(p \Longleftrightarrow q)$	(¬ p ⟺ ¬ r)	$(p \Longleftrightarrow q) \oplus (¬ p \Longleftrightarrow ¬ r)$
T	T	T	F	F	T	T	F
T	F	T	F	F	F	T	T
T	T	F	F	T	T	F	T
T	F	F	F	T	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	T
F	F	F	T	T	T	T	F

(c) $(p \oplus q) \Rightarrow (p \oplus \neg q)$
(Solution)

p	q	$\neg$ q	$(p \oplus$ q$)$	$(p \oplus \neg$ q$)$	$(p \oplus$ q$) \Rightarrow (p \oplus \neg$ q$)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

Problem 3: Logic in Algorithms (10+10+10=30 points)

If x = 1 before the statement is reached, what is the value of x after each of these statements is encountered in a computer program? Why? Show your work step by step.

(a) for i $\Leftarrow$ 1 **to** 10 **do**

       if x + 2 = 3 **then** x := x + 1

    end

(Solution)
1)i=1, X=1 is true because of x+2 = 3; then x=2
2)i=2,X=2 is false because of x+2 = 4 ; then x=2
3)i=3,X=2 is false because of x+2 = 4 ; then x=2
.
.
.
9)i=9, x=2 is false because of x+2= 4 ; then x=2
10)i=10,x=2 is false because of x+2=4 ; then x=2
X=2 Because, after step 2, X does not provide the condition given in any step.

(b) for i $\Leftarrow$ 1 **to** 5 **do**

       if (x + 1 = 2) XOR (x + 2 = 3) **then** x := x + 1

    end

(Solution)
1)i=1 , x=1 , (x+1 = 2) XOR (x+2=3) : 1 XOR 1 =FALSE x=1 still
2)i=2 , x=1 , (x+1 = 2) XOR (x+2=3) : 1 XOR 1 = FALSE x=1 still
3)i=3 , x=1 , (x+1 = 2) XOR (x+2=3) : 1 XOR 1 =FALSE x=1 still
4)i=4 , x=1 , (x+1 = 2) XOR (x+2=3) : 1 XOR 1 =FALSE x=1 still
5)i=5 , x=1 , (x+1 = 2) XOR (x+2=3) : 1 XOR 1 =FALSE x=1 still
X always be 1 because in first step x dont provide the condition in any step

(c) for i $\Leftarrow$ 1 **to** 4 **do**

       if (2x + 3 = 5) AND (3x + 4 = 7) **then** x := x + 1

    end

(Solution)
1)i=1, x=1 (2x+3 = 5) AND (3x + 4 = 7) : 1 AND 1 =TRUE , X:=X+1,X=2
2)i=2, x=2 (2x+3 = 5) AND (3x + 4 = 7) : 0 AND 0 =FALSE , X=2 still
3)i=3, x=2 (2x+3 = 5) AND (3x + 4 = 7) : 0 AND 0 =FALSE , X=2 still
4)i=4, x=2 (2x+3 = 5) AND (3x + 4 = 7) : 0 AND 0 =FALSE , X=2 still

Problem 4: Proof by contradiction   (20 points)

Show that at least three of any 25 days chosen must fall in the same month of the year using a proof by contradiction. Explain your work step by step.
(Solution)
1)p:At least three of any 25 days chosen must fall in same mounth of the year
2)¬p:At least three of any 25 days chosen don't have to fall in same mounth of the year
3)If we detirmine ¬p is false we prove that p is true
4)1 year = 12 mounth
According to ¬p every mounths of the year must has 2 of any 25 days chosen but if we choose 2 day 12*2 =24
, In this situation, we need to choose one more so ¬p is false
5)p is true

Problem 5: Proof by contraposition   (20 points)

Show that if $n^3 + 5$ is odd, then n is even using a proof by contraposition. Explain your work step by step.
Note: Assume that n is an integer.
(Solution)
1)If we prove that (¬ q ⇒ ¬ p) is true we also prove that p ⇒ q true

2) ¬ q ⇒ ¬ p :If n is odd then n3+5 is even
3)Assume that n is odd true
n=2k+1 $k\varepsilon Z$
4)$n^3 + 5 = (2k + 1)^3 + 5 = 8k^3 + 12k^2 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3)$
$p = 4k^3 + 6k^2 + 3k + 3, k\varepsilon Z, p\varepsilon Z$
5)$n^3 + 5 = 2p$
6)(¬ q ⇒ ¬ p) true
also p ⇒ q true