CSE 211: Discrete Mathematics **(Due: 24/12/19)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework4 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Nonhomogeneous Linear Recurrence Relations (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.

(Solution)
1)
if $a_n = -2^{n+1}$
$a_{n-1} = -2^n$
2)
-2^{n+1} = -3.2^n + 2^n
$-2^{n+1} = -2.2^n$
3)-2^{n+1} =-2^{n+1} Yes $a_n = -2^{n+1}$ is a solution of the given recurrence relation.

(b) Find the solution with $a_0 = 1$.

(Solution)
1) $a_n = a_n\ (h) + a_n\ (p)$
2)For $a_n\ (h)$
$a_n = 3a_{n-1}$
if we assume that $a_{n-1} = 1$ and a_n =r
r = 3.1 => root
$a_n = c_1.3^n + a_n\ (p)$
3) For $a_n\ (p)$
if we assume that a_n = A.2^n
a_{n-1} = A.2^{n-1}
if we put this equation to general relation we get;
A.2^n = 3.A.2^{n-1} + 2^n
A = 3A/2 +1
A = -2
$a_n\ (p)$ = -2.2^{n-1}
4) $a_n\ (general) = c_1.3^n$ - 2.2^{n-1}
For n= 0 $a_0 = c_1$ - 2 = 1

$c_1 = 3$
5) a_n $(general)$ = 3.3^n - 2.2^{n-1} =3^{n+1} - 2.2^{n-1}

Problem 2: Linear Recurrence Relations (35 points)

Find all solutions of the recurrence relation $a_n = 7a_{n-1}$ - $16a_{n-2}$ + $12a_{n-3}$ + n4^n with a_0 = -2, a_1 = 0, and $a_2 = 5$.

(Solution)
1)This relation is nonhomogenous relation so relation have homogenous and partial part.
a_n (g) = a_n (h) + a_n (p)
2)a_n (h) = $7a_{n-1}$ - $16a_{n-2}$ + $12a_{n-3}$
3)we assume that $a_{n-3} = 1$, a_{n-2} =r , $a_{n-1} = r^2$ and $a_n = r^3$
4) $r^3 = 7r^2$ - 16r + 12
r^3 - $7r^2$ + 16r - 12 =0
$(r-2)$ ^{2}.$(r-3)$ =0
r_1 =3 , r_2 =2 and r_3 =2
5)a_n (h) = c_1 3^n + c_2 2^n + c_3 2^n.n
6)For a_n (p)
we assume that a_n (p) = $(A.n + B)$ $.4^n$
a_{n-1} = $(A.(n-1) + B)$ $.4^(n-1)$
a_{n-2} = $(A.(n-2) + B)$ $.4^(n-2)$
a_{n-3} = $(A.(n-3) + B)$ $.4^(n-3)$
$(A.n + B)$ $.4^n$ = 7$(A.(n-1) + B)$ $.4^(n-1)$ -16$(A.(n-2) + B)$ $.4^(n-2)$ +12 $(A.(n-3) + B)$ $.4^(n-3)$ + n.4^n
if we solve this equation we get that;
A = 16 and B = -80
a_n (p) = $(16.n - 80)$ $.4^n$
7) a_n (g) = c_1 3^n + c_2 2^n + c_3 2^n.n + $(16.n - 80)$ $.4^n$
for n=0
c_1 + c_2 -80 = -2
for n=1
3.c_1 + 2.c_2 + 2.c_3 -256 = 0
for n= 2
9.c_1 + 4.c_2 +8.c_3 -768 =5
if we calculate this three equation we find c_1 c_2 c_3
c_1 = 61 c_2 = 39/2 and c_3 =17
8)a_n (g) = 61. 3^n + 39/2. 2^n + 17. 2^n.n + $(16.n - 80)$ $.4^n$

Problem 3: Linear Homogeneous Recurrence Relations (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.

(Solution)
1)if we assume that $a_{n-2} = 1$, a_{n-1} =r and a_n=r^2
r^2 -2r +2 =0
$\Delta = b^2$ - 4.a.c
Δ = -4 so roots of the relation is complex numbers
r = -(-2) +$\sqrt{\triangle}$ / 2.a
r_1 =-(-2) +$\sqrt{-4}$ / 2 = 1+i
r_2 =-(-2) -$\sqrt{-4}$ / 2 = 1-i
r_1 =1+i and r_2 = 1-i

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.

(Solution)

$a_n\ (h) = c_1\ .\ (1+i)\ ^n + c_2\ .\ (1-i)\ ^n$
for n= 0
$1 = c_1 + c_2$
for n=1
$2 = c_1\ .\ (1+i) + c_2\ .\ (1-i)$
$c_1 + c_2 +$ i.(c_1 - c_2) = 2
c_1 - c_2 = -i
$c_1 + c_2 = 1$

$c_1 = \frac{1-i}{2}$ and c_2=$\frac{1+i}{2}$ so

$a_n\ (h) = \frac{1-i}{2}\ .\ (1+i)\ ^n + \frac{1+i}{2}\ .\ (1-i)\ ^n$