CSE 211: Discrete Mathematics **(Due: 24/12/19)**

Homework #4

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework4 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Nonhomogeneous Linear Recurrence Relations (15+15=30 points)

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

(a) Show that whether $a_n = -2^{n+1}$ is a solution of the given recurrence relation or not. Show your work step by step.

(Solution)
1)
if $a_n = -2^{n+1}$
$a_{n-1} = -2^n$
2)
-2^{n+1} = -3.2^n + 2^n
$-2^{n+1} = -2.2^n$
3)-2^{n+1} =-2^{n+1} Yes $a_n = -2^{n+1}$ is a solution of the given recurrence relation.

(b) Find the solution with $a_0 = 1$.

(Solution)
1) $a_n = a_n\ (h) + a_n\ (p)$
2)For $a_n\ (h)$
$a_n = 3a_{n-1}$
if we assume that $a_{n-1} = 1$ and a_n =r
r = 3.1 => root
$a_n = c_1.3^n + a_n\ (p)$
3) For $a_n\ (p)$
if we assume that a_n = A.2^n
a_{n-1} = A.2^{n-1}
if we put this equation to general relation we get;
A.2^n = 3.A.2^{n-1} + 2^n
A = 3A/2 +1
A = -2
$a_n\ (p)$ = -2.2^{n-1}
4) $a_n\ (general) = c_1.3^n$ - 2.2^{n-1}
For n= 0 $a_0 = c_1$ - 2 = 1

$c_1 = 3$
5) a_n $(general) = 3.3^n$ - 2.2^{n-1} $=3^n$ - 2.2^{n-1}

Problem 2: Linear Recurrence Relations (35 points)

Find all solutions of the recurrence relation $a_n = 7a_{n-1}$ - $16a_{n-2} + 12a_{n-3} +$ n4^n with $a_0 =$ -2, $a_1 = 0$, and $a_2 = 5$.

(Solution)
1)This relation is nonhomogenous relation so relation have homogenous and partial part.
a_n $(g) = a_n$ $(h) + a_n$ (p)
1)a_n $(h) = 7a_{n-1}$ - $16a_{n-2} + 12a_{n-3}$

Problem 3: Linear Homogeneous Recurrence Relations (20+15 = 35 points)

Consider the linear homogeneous recurrence relation $a_n = 2a_{n-1}$ - $2a_{n-2}$.
(a) Find the characteristic roots of the recurrence relation.

(Solution)
1)if we assume that $a_{n-2} = 1$, a_{n-1} =r and a_n=r^2
r^2 -2r +2 =0
$\Delta = b^2$ - 4.a.c
$\Delta =$ -4 so roots of the relation is complex numbers
r = -(-2) +$\sqrt{\triangle}$ / 2.a
r = -(-2) +$\sqrt{\triangle}$ / 2.a
r_1 =-(-2) +$\sqrt{-4}$ / 2 = 1+i
r_2 =-(-2) -$\sqrt{-4}$ / 2 = 1-i

(b) Find the solution of the recurrence relation with $a_0 = 1$ and $a_1 = 2$.

(Solution)
a_n $(h) = c_1$. $(1 + i)$ ^{n} $+ c_2$. $(1 - i)$ ^{n}
for n= 0
$1 = c_1 + c_2$
for n=1
$2 = c_1$. $(1 + i) + c_2$. $(1 - i)$
$c_1 + c_2 +$ i.(c_1 - c_2) = 2
c_1 - c_2 = -i
$c_1 + c_2 = 1$

$c_1 = \frac{1-i}{2}$ and c_2=$\frac{1+i}{2}$ so

a_n $(h) = \frac{1-i}{2}$. $(1 + i)$ ^{n} $+$ $\frac{1+i}{2}$. $(1 - i)$ ^{n}