CSE 211: Discrete Mathematics **(Due: 12/11/19)**

Homework #2

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Course Policy: Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework1 directory of the CoCalc project CSE211-2019-2020.

Problem 1: Sets (2+2+2+2+2=10 points)

Which of the following sets are equal? Show your work step by step.

(a) {t : t is a root of x^2 6x + 8 = 0}

(b) {y : y is a real number in the closed interval [2, 3]}

(c) {4, 2, 5, 4}

(d) {4, 5, 7, 2} - {5, 7}

(e) {q: q is either the number of sides of a rectangle or the number of digits in any integer between 11 and 99}
(Solution)
a) t= 4 or t= 2 S1= {4,2}
b) {2.00 , 2.01 , 2.02 ...}
d) {4,2}
e)Sides of rectangle=4 Number of Digits=2
{4,2}
a , d and e are equal

Problem 2: Cartesian Product of Sets (15 points)

Explain why (A × B) × (C × D) and A × (B × C) × D are not the same.
(Solution)
A = {A1}, B = {B1} , C = {C1} , D = {D1}

A × B = {(A1 ,B1)} (C × D) = {(C1 ,D1)}
(A × B) × (C × D) = {(A1 ,B1) , (C1 ,D1)}
(B × C) = {(B1 ,C1)}
A × (B × C)= {(A1 ,B1) ,(A1,C1)}
A × (B × C) × D = {(A1 ,D1) ,(B1,D1) , (C1,D1)}
They are not same

Problem 3: Cartesian Product of Sets in Algorithms (25 points)

Let A, B and C be sets which have different cardinalities. Let (p, q, r) be each triple of A × B × C where p ∈ A, q ∈ B and r ∈ C. Design an algorithm which finds all the triples that are satisfying the criteria: p ≤ q and q ≥ r. Write the pseudo code of the algorithm in your solution.

For example: Let the set A, B and C be as A = { 3, 5, 7 }, B = { 3, 6 } and C = { 4, 6, 9 }. Then the output should be : { (3, 6, 4), (3, 6, 6), (5, 6, 4), (5, 6, 6) }.

(Note: Assume that you have sets of A, B, C as an input argument.)

(Solution)

Algorithm 1: Pseudo Code of Your Algorithm

Input: The sets of A, B, C
if *write a condition* **then**
  Statements
else
  Statements
end
When you want to write a for loop, you can use:
for *write a condition* **do**
end
When you want to write a while loop, you can use:
while *write a condition* **do**
  If you need to return, use **return**
end
For any additional things you have to do while writing your pseudo code, Google "How to use algorithm2e in Latex?".

Problem 4: Relations (3+3+3+3+3+3+3=21 points)

Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where (x, y) $\in$ R if and only if
(a) x $\neq$ y.
(Solution)
Reflexive
We assume that(x,y) $\rightarrow$ (a,a) for reflexive a $\neq$ a is false
It isn't reflexive
Symetric
x $\neq$ y if we assume that (x,y) $\rightarrow$ (y,x) y $\neq$ x is true so it is symetric
Antisymetric
(x, y) $\in$ R and x $\neq$ y ex:(2,3) and (3,2) are elements of this relation.It is symetric and It isn't antisymetric.
Transitive
for (a,b) $\rightarrow$ a $\neq$ b for (b,c) $\rightarrow$ b $\neq$ c for (a,c) $\rightarrow$ a $\neq$ c
There are elements that provide this condition, but not all. For example: (1,3) and (3,1) are in this relation but (1,1) isn't in relation.This relation isn't transitive

(b) xy $\geq$ 1.
(Solution)
Reflexive
if we assume that (x,y) $\rightarrow$ (a,a)== a.a $\geq$ 1
(0,0) doesn't provide this condition.It isn't reflexive
Symetric
for (a,b) $\rightarrow$ a.b $\geq$ 1 for (b,a) $\rightarrow$ b.a $\geq$ 1.Both are same so this relation is symetric.
Antisymetric
Ex:(2,3) and (3,2) are elements of this relation.It is symetric so This relatino isn't antisymetric.
Transitive
for (a,b) $\rightarrow$ a.b $\geq$ 1 (b,c) $\rightarrow$ b.c $\geq$ 1 for (a,c) $\rightarrow$ a.c $\geq$ 1 is true.This relation is transitive

(c) x = y + 1 or x = y - 1.
(Solution)
Reflexive
we assume that (x,y) $\rightarrow$ (a,a)
a=a+1 or a=a-1 F or F = F.This relation isn't reflexive
Symetric
Check (a,b) and (b,a)
(a,b)
a=b+1 or a= b-1 If organized b=a-1 or b=a+1
(b,a)
b=a+1 or b=a-1
(a,b) and (b,a) are same and true.This relation symetric
Antisymetric
Ex:(2,1) in this relation and (1,2) are also in this relation
This is not Antisymetric as we have proved in Symetric.
Transitive
(2,1) and (1,2) are elements of relation but (2,2) isn't element of relation.This relation isn't transitive.

(d) x is a multiple of y.
(Solution)
Reflexive
x=k.y , k$\in$ Z
(a,a) $\rightarrow$ a=k.a k$\in$ Z k=1.It is true.Relation is reflexive.
Symetric
For (a,b) $\rightarrow$ a=k.b , k $\in$ Z $(a \div b)$=k
For (b,a) $\rightarrow$ b=m.a ,m $\in$ Z $(a \div b)$=$(1 \div m)$

$(1 \div m)$ and k must be $\in$ Z but 1/m not $\in$ Z.Relation is not symetric.

Antisymetric
Relation doesn't have any symetric element except (1,1) and (1,1) also provide Antisymetric.Relation is Antisymetric.
Transitive
(a,b) $\rightarrow$ a=k.b (b,c) $\rightarrow$ b=m.c (a,c) $\rightarrow$ a=n.c k,m,n $\in$ Z if a is multiple of b also a is multiple of c
Relation is transitive.

(e) x and y are both negative or both nonnegative.
(Solution)
x $<$ 0 and y $<$ 0 OR x $>$ 0 and y $>$ 0
Reflexive
(a,a) $\rightarrow$ a $<$ 0 and a $<$ 0 OR a $>$ 0 and a $>$ 0.It is true relation is reflexive
Symetric
(a,b) $\rightarrow$ a $<$ 0 and b $<$ 0 OR a $>$ 0 and b $>$ 0
(b,a) $\rightarrow$ b $<$ 0 and a $<$ 0 OR b $>$ 0 and a $>$ 0
They are same so relation is symetric
Antisymetric
Relaiton have symetric elemets like:(1,2) and (2,1) both of them already in relation.So relation isn't antisymetric
Transitive
(a,b) $\rightarrow$ a $<$ 0 and b $<$ 0 OR a $>$ 0 and b $>$ 0
(b,c) $\rightarrow$ b $<$ 0 and c $<$ 0 OR b $>$ 0 and c $>$ 0
(a,c) $\rightarrow$ a $<$ 0 and c $<$ 0 OR a $>$ 0 and c $>$ 0
If a $<$ 0 b must be $<$ 0 then c $<$ 0 So (a,c) is have to in relation.
So relation is transitive.

(f) x $\geq y^2$.
(Solution)
Reflexive
(a,a) $\rightarrow$ a $\geq a^2$ It is not true for negative numbers.Relation isn't reflexive
Symetric
(a,b) $\rightarrow$ a $\geq b^2$
(b,a) $\rightarrow$ b $\geq a^2$ This situation is false.Relation isn't symetric
Antisymetric
Relation only have (1,1) (2,2) (3,3)... symetric elements and this does not disturb the antisymmetric condition.Relation is antisymetric.

Transitive
(a,b) $\rightarrow$ a $\geq b^2$
(b,c) $\rightarrow$ b $\geq c^2$
(a,c) $\rightarrow$ a $\geq c^2$
a $> b^2$ also a c^2.(a,c) in relation elements,Relation is transitive.
(g) x $= y^2$.
(Solution)
Reflexive
(x,y) $\rightarrow$ (a,a) a $= a^2$.
It only provide (0,0) and (1,1) other elements like (2,2) isn't in relation .Relation isn't reflexive
Symetric
(a,b) $\rightarrow$ a $= b^2$
(b,a) $\rightarrow$ b $= a^2$
It only true for (1,1) other elements like (4,2) is in relation but (2,4) isn't in relation.Relation isn't symetric.
Antisymetric
Relation antisymetric.Because relation have only (1,1) symetric elemets and this does not disturb the antisymmetric condition.

Problem 5: Functions (15 points)

If f and f ∘ g are one-to-one, does it follow that g is one-to-one? Justify your answer.
(Solution)

We suppose x1 =x2
f o g(x1) = f o g (x2)
f(g(x1)) = f(g(x2)) then g(x1) =g(x2)
g is one-to-one

Problem 6: Inverse of Functions (7+7=14 points)

Let f be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by f(x) = x^2. Find
(a) f^{-1} ({ x | 0 < x < 1 })
(Solution)

y=x^2 → x=$\sqrt{y}$ → f^{-1}(x) = $\sqrt{x}$

(b)f^{-1} ({ x | x > 4 })
(Solution)
for x: x > 4 also f^{-1}(x) = $\sqrt{x}$