

## Homework #1

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*Name:*

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**Course Policy:** Read all the instructions below carefully before you start working on the assignment, and before you make a submission.

- It is not a group homework. Do not share your answers to anyone in any circumstance. Any cheating means at least -100 for both sides.
- Do not take any information from Internet.
- No late homework will be accepted.
- For any questions about the homework, send an email to gizemsungu@gtu.edu.tr
- Submit your homework into Assignments/Homework1 directory of the CoCalc project CSE211-2019-2020.

**Problem 1: Conditional Statements**

(5+5+5=15 points)

State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.

*(Solution)*

**Converse:** If I stay at home , it will snow tonight.

**Contrapositive:** If I dont stay at home , it wont snow tonight.

**Inverse:**If it doesnt snow tonight, I wont stay at home.

(b) I go to the beach whenever it is a sunny summer day.

*(Solution)*

**Converse:**It is a sunny summer day whenever I go to the beach.

**Contrapositive:**Whenever it is not a sunny summer day. I dont go to the beach.

**Inverse:**I dont go to the beach whenever it is not a sunny summer day.

(c) When I stay up late, it is necessary that I sleep until noon.

*(Solution)*

**Converse:** When It is necessary that I sleep until noon ,I stay up late.

**Contrapositive:**When It is not necessary that I sleep until noon,I dont stay up late.

**Inverse:**When I dont stay up late , it is not necessary that I sleep until noon.

**Problem 2: Truth Tables For Logic Operators**

(5+5+5=15 points)

Construct a truth table for each of the following compound propositions.

(a)  $(p \oplus \neg q)$

(Solution)

$p$	$q$	$\neg q$	$(p \oplus \neg q)$
$T$	$T$	$F$	$T$
$T$	$F$	$T$	$F$
$F$	$T$	$F$	$F$
$F$	$F$	$T$	$T$

(b)  $(p \iff q) \oplus (\neg p \iff \neg r)$

(Solution)

$p$	$q$	$r$	$(\neg p)$	$(\neg r)$	$(p \iff q)$	$(\neg p \iff \neg r)$	$(p \iff q) \oplus (\neg p \iff \neg r)$
$T$	$T$	$T$	$F$	$F$	$T$	$T$	$F$
$T$	$F$	$T$	$F$	$F$	$F$	$T$	$T$
$T$	$T$	$F$	$F$	$T$	$T$	$F$	$T$
$T$	$F$	$F$	$F$	$T$	$F$	$F$	$F$
$F$	$T$	$T$	$T$	$F$	$F$	$F$	$F$
$F$	$T$	$F$	$T$	$T$	$F$	$T$	$T$
$F$	$F$	$T$	$T$	$F$	$T$	$F$	$T$
$F$	$F$	$F$	$T$	$T$	$T$	$T$	$F$

(c)  $(p \oplus q) \Rightarrow (p \oplus \neg q)$

**(Solution)**

$p$	$q$	$\neg q$	$(p \oplus q)$	$(p \oplus \neg q)$	$(p \oplus q) \Rightarrow (p \oplus \neg q)$
$T$	$T$	$F$	$F$	$T$	$T$
$T$	$F$	$T$	$T$	$F$	$F$
$F$	$T$	$F$	$T$	$F$	$F$
$F$	$F$	$T$	$F$	$T$	$T$

**Problem 3: Logic in Algorithms**

(10+10+10=30 points)

If  $x = 1$  before the statement is reached, what is the value of  $x$  after each of these statements is encountered in a computer program? Why? Show your work step by step.

(a) **for**  $i \leftarrow 1$  **to** 10 **do**

**if**  $x + 2 = 3$  **then**  $x := x + 1$

**end**

*(Solution)*

1)  $i=1$ ,  $X=1$  is true because of  $x+2 = 3$ ; then  $x=2$

2)  $i=2$ ,  $X=2$  is false because of  $x+2 = 4$ ; then  $x=2$

3)  $i=3$ ,  $X=2$  is false because of  $x+2 = 4$ ; then  $x=2$

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9)  $i=9$ ,  $x=2$  is false because of  $x+2 = 4$ ; then  $x=2$

10)  $i=10$ ,  $x=2$  is false because of  $x+2 = 4$ ; then  $x=2$

$X=2$  Because, after step 2,  $X$  does not provide the condition given in any step.

(b) **for**  $i \leftarrow 1$  **to** 5 **do**

**if**  $(x + 1 = 2)$  XOR  $(x + 2 = 3)$  **then**  $x := x + 1$

**end**

*(Solution)*

1)  $i=1$ ,  $x=1$ ,  $(x+1 = 2)$  XOR  $(x+2=3)$  : 1 XOR 1 =FALSE  $x=1$  still

2)  $i=2$ ,  $x=1$ ,  $(x+1 = 2)$  XOR  $(x+2=3)$  : 1 XOR 1 = FALSE  $x=1$  still

3)  $i=3$ ,  $x=1$ ,  $(x+1 = 2)$  XOR  $(x+2=3)$  : 1 XOR 1 =FALSE  $x=1$  still

4)  $i=4$ ,  $x=1$ ,  $(x+1 = 2)$  XOR  $(x+2=3)$  : 1 XOR 1 =FALSE  $x=1$  still

5)  $i=5$ ,  $x=1$ ,  $(x+1 = 2)$  XOR  $(x+2=3)$  : 1 XOR 1 =FALSE  $x=1$  still

$X$  always be 1 because in first step  $x$  dont provide the condition in any step

(c) **for**  $i \leftarrow 1$  **to** 4 **do**

**if**  $(2x + 3 = 5)$  AND  $(3x + 4 = 7)$  **then**  $x := x + 1$

**end**

*(Solution)*

1)  $i=1$ ,  $x=1$   $(2x+3 = 5)$  AND  $(3x + 4 = 7)$  : 1 AND 1 =TRUE,  $X:=X+1$ ,  $X=2$

2)  $i=2$ ,  $x=2$   $(2x+3 = 5)$  AND  $(3x + 4 = 7)$  : 0 AND 0 =FALSE,  $X=2$  still

3)  $i=3$ ,  $x=2$   $(2x+3 = 5)$  AND  $(3x + 4 = 7)$  : 0 AND 0 =FALSE,  $X=2$  still

4)  $i=4$ ,  $x=2$   $(2x+3 = 5)$  AND  $(3x + 4 = 7)$  : 0 AND 0 =FALSE,  $X=2$  still

**Problem 4: Proof by contradiction**

(20 points)

Show that at least three of any 25 days chosen must fall in the same month of the year using a proof by contradiction. Explain your work step by step.

**(Solution)**

- 1)  $p$ : At least three of any 25 days chosen must fall in same month of the year
- 2)  $\neg p$ : At least three of any 25 days chosen don't have to fall in same month of the year
- 3) If we determine  $\neg p$  is false we prove that  $p$  is true
- 4) 1 year = 12 months

According to  $\neg p$  every month of the year must have 2 of any 25 days chosen but if we choose 2 days  $12 \cdot 2 = 24$ , In this situation, we need to choose one more so  $\neg p$  is false

- 5)  $p$  is true

**Problem 5: Proof by contraposition**

(20 points)

Show that if  $n^3 + 5$  is odd, then  $n$  is even using a proof by contraposition. Explain your work step by step.

*Note: Assume that  $n$  is an integer.*

**(Solution)**

- 1) If we prove that  $(\neg q \Rightarrow \neg p)$  is true we also prove that  $p \Rightarrow q$  true

- 2)  $\neg q \Rightarrow \neg p$ : If  $n$  is odd then  $n^3 + 5$  is even

- 3) Assume that  $n$  is odd true

$$n = 2k + 1, k \in \mathbb{Z}$$

$$4) n^3 + 5 = (2k + 1)^3 + 5 = 8k^3 + 12k^2 + 6k + 6 = 2(4k^3 + 6k^2 + 3k + 3)$$

$$p = 4k^3 + 6k^2 + 3k + 3, k \in \mathbb{Z}, p \in \mathbb{Z}$$

$$5) n^3 + 5 = 2p$$

$$6) (\neg q \Rightarrow \neg p) \text{ true}$$

also  $p \Rightarrow q$  true