Sabancı University
Faculty of Engineering and Natural Sciences

EE 314 Digital Communications **Spring 2021**
Assoc. Prof. Mehmet Keskinöz

HOMEWORK #5

Assigned: May. 7, 2021
Due : May 24, 2021, 11:55 pm

- ***Remarks: Please keep your answers clear and concise and show all the mathematical derivations that you perform. Each student should write up the solutions entirely on their own. You should list your name and ID on your write-up. If you do not type your solutions in a computer, be sure that your hand-writing is legible, your scan is high-quality and your name and ID are clearly written on your submitted document.***
- ***Your solutions should be scanned as a single pdf file (we wont accept other format such as jpeg or multiple files). You should name your pdf file as first_name_lastname_HW_number (e.g., Mehmet_Keskinoz_HW_1)***
- ***If you have a MATLAB problem, you should also be required to submit your .m file (name .m file as first_name_lastname_HW_number and write your name and ID as a commented header in the .m file) .***
- ***if you don't have a MATLAB related problem in your homework, just upload your solutions as a single pdf file . Otherwise, you should zip your .m file together with your pdf file (name the zip file as as first_name_lastname_HW_number) and upload your single zip file to SUCOURSE.***
- ***Note that you can only get help from your TAs on MATLAB related questions during their office hours.***
- ***If you want to get feedbacks about your homework, you should also submit hand-written (or hard-copy) of your solutions.***
- ***Late submission will not be accepted***

(1)
Three messages m_1, m_2, and m_3 are to be transmitted over an AWGN channel with noise power-spectral density $\frac{N_0}{2}$. The messages are

$$s_1(t) = \begin{cases} 1 & 0 \le 1 \le T \\ 0, & \text{otherwise} \end{cases}$$

$$s_2(t) = -s_3(t) = \begin{cases} 1 & 0 \le t \le \frac{T}{2} \\ -1 & \frac{T}{2} \le 0 \le T \\ 0, & \text{otherwise} \end{cases}$$

1. What is the dimensionality of the signal space?
2. Find an appropriate basis for the signal space (Hint: You can find the basis without using the Gram-Schmidt procedure).
3. Draw the signal constellation for this problem.
4. Derive and sketch the optimal decision regions R_1, R_2, and R_3.
5. Which of the three messages is more vulnerable to errors and why? In other words which of $P(\text{Error} \mid m_i \text{ transmitted})$, $i = 1, 2, 3$ is larger?

(2) Let $p(t) = \frac{2}{\sqrt{T}}[u(t) - u(t - T/4)]$ where $u(t)$ is unit–step function. Over a distortionless AWGN channel with mean of zero and noise PSD of $\frac{N_0}{2} = 10^{-8}$ Watts/Hz, we sent following four signals to achieve a 4-ary signalling scheme : $\varphi_1(t) = p(t)$ for $0 \le t \le T$, $\varphi_2(t) = p(t - T/4)$ for $0 \le t \le T$, $\varphi_3(t) = p(t - T/2)$ for $0 \le t \le T$, $\varphi_4(t) = p(t - 3T/4)$ for $0 \le t \le T$. Assume symbols are equally likely.

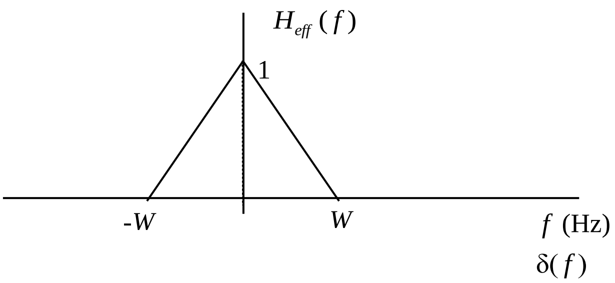
- **a)** Draw the optimal ML receiver which uses minimum number of analog integrators
- **b)** Determine approximate but accurate symbol error probability for the optimal ML receiver.
- **c)** Determine approximate but accurate bit error probability under Gray coding for the optimal ML receiver. Does the use of Gray coding matter? Why?
- **d)** Determine the bandwidth required to employ this modulation. What is bandwidth efficiency?

(3) Consider 16-ary PSK signalling over a distortionless AWGN channel with mean of zero and noise PSD of $\frac{N_0}{2} = 10^{-8}$ Watts/Hz. Also $\frac{E_b}{N_0}$ is determined as 20 dB and symbols are equally likely.

- **a)** Draw the optimal ML receiver which uses minimum number of analog integrators
- **b)** Determine approximate but accurate symbol error probability for the optimal ML receiver.
- **c)** Determine approximate but accurate bit error probability under Gray coding for the optimal ML receiver. Does the use of Gray coding matter? Why?
- **d)** Determine the bandwidth required to employ this modulation. What is bandwidth efficiency?

(4) An analog signal with a bandwidth of 5 KHz is sampled and uniformly quantized to 64 levels. This information is to be transmitted using the 16-ary PAM base-band signaling.

(a) What is the minimum required sampling rate and what is the resulting bit transmission rate?
(b) What is the PAM pulse or symbol transmission rate?
(c) Find the minimum theoretical channel bandwidth that avoids ISI.
(d) If these symbols are transmitted over a channel with the effective transfer function $H_{eff}(f)$ as shown below, determine **<u>minimum value of *W*</u>** so that after the sampling there is no ISI in the digital receiver.



(5) Assume that a digital communication channel has the sampled effective impulse response $\{h_{eff}[k]\}$, with voltage values -0.5, 0.1, 1,-0.2, 0.05 for $k = -2, -1, 0, 1, 2$ respectively.

- **(a)** Design a 3-tap zero forcing equalizer.
- **(b)** Using this equalizer, calculate the ISI values of the equalized channel (i.e., response after equalization) at the sample times $k = \pm 2, \pm 3$.
- **(c)** Verify your findings in parts (a) and (b) using MATLAB.