Sabancı University
Faculty of Engineering and Natural Sciences

EE 314 Digital Communications **Spring 2021**

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HOMEWORK #3

Assigned: Apr. 20, 2021
Due : Apr. 27, 2019, 23:55 pm

- ***Remarks: Please keep your answers clear and concise and show all the mathematical derivations that you perform. Each student should write up the solutions entirely on their own. You should list your name and ID on your write-up. If you do not type your solutions in a computer, be sure that your hand-writing is legible, your scan is high-quality and your name and ID are clearly written on your submitted document.***
- ***Your solutions should be scanned as a single pdf file (we wont accept other format such as jpeg or multiple files). You should name your pdf file as first_name_lastname_HW_number (e.g., Mehmet_Keskinoz_HW_1)***
- ***If you have a MATLAB problem, you should also be required to submit your .m file (name .m file as first_name_lastname_HW_number and write your name and ID as a commented header in the .m file) .***
- ***if you don't have a MATLAB related problem in your homework, just upload your solutions as a single pdf file . Otherwise, you should zip your .m file together with your pdf file (name the zip file as as first_name_lastname_HW_number) and upload your single zip file to SUCOURSE.***
- ***Note that you can only get help from your TAs on MATLAB related questions during their office hours.***
- ***If you want to get feedbacks about your homework, you should also submit hand-written (or hard-copy) of your solutions.***
- ***Late submission will not be accepted***

(1) Consider the set vectors $V = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

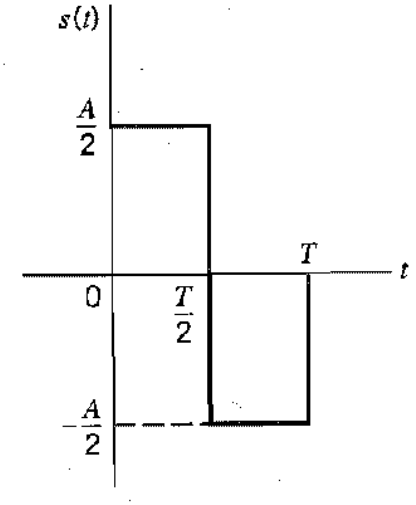
a) Is this vector-set orthogonal?
b) Is this vector-set orthonormal? If not, obtain an orthonormal vector set from V
c) Obtain the representation of following vectors in terms of the orthonormal set of vector found in part (b): $\mathbf{s}_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{s}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ $\mathbf{s}_3 = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$ $\mathbf{s}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
d) Geometrically draw the vector representations found in part (c) with the orthonormal vectors found in (b) as its axes.

(2) Let $p(t)=\frac{2}{\sqrt{T}}[u(t)-u(t-T/4)]$ where $u(t)$ is unit–step function. Over a communication , we sent following four signals to achieve a 4-ary modulation scheme : $\varphi_1(t)=p(t)$ for $0\le t\le T$, $\varphi_2(t)=p(t-T/4)$ for $0\le t\le T$, $\varphi_3(t)=p(t-T/2)$ for $0\le t\le T$, $\varphi_4(t)=p(t-3T/4)$ for $0\le t\le T$

a) Is the signal-set $\{\varphi_1(t),\varphi_2(t),\varphi_3(t),\varphi_4(t)\}$ orthogonal?

b) Is the signal-set $\{\varphi_1(t),\varphi_2(t),\varphi_3(t),\varphi_4(t)\}$ orthonormal?

c) Does the signal-set $\{\varphi_1(t),\varphi_2(t),\varphi_3(t),\varphi_4(t)\}$ form a basis ? If so, obtain the vectorial-representation of the signal below in terms of basis signals.



(3) Suppose that the joint pdf of two-dimensional random variable (*X*,*Y*) is given by

$$f_{XY}(x,y)=\begin{cases} x^2+\frac{xy}{3} & \text{for } 0\le x\le 1 \text{ and } 0\le y\le 2 \\ 0 & \text{Otherwise} \end{cases}$$

Compute the following:

(a) Marginal pdf of *X* and *Y*.

(b) *E*(*X*), *E*(*Y*), σ_X^2 and σ_Y^2

(c) $P(X>0.5)$

(d) $P(Y< X)$

(e) $P(Y<0.5|\ X<0.5)$

(f) Determine the correlation of *X* and *Y*. Are they orthogonal ?

(g) Determine the covariance of *X* and *Y*. Are they uncorrelated ?

(4) Over a binary communication, bit 1 is transmitted by a positive pulse $s(t)$ and bit 0 is transmitted by a zero-pulse pulse (i.e., it is zero for a bit duration) . The probability of transmitting bit 1 or bit 0 is 0.5. To detect the pulses at the receiver, each pulse is sampled at its peak amplitude. Ideally , i.e., in the absence of noise, the sampler output either A_p for bit 1 or 0 for bit 0 where A_p is the peak amplitude of the $s(t)$. However, we have noise in the channel, because of it, the sampler output will be $r = s + n$ where s is noise free signal sample , which is either A_p for bit 1 or 0 for bit 0, and n is zero mean Gaussian RV with variance of σ^2. To recover the transmitted bits from the samples, we decide "1" if the sampler output r is greater or equal to $A_p / 2$, otherwise we decide "0".

(a) Determine the probability of bit error given that we transmit bit 1.

(b) Determine the probability of error given that we transmit bit 0.

(c) Determine the probability of error

(5)

(a) A Gaussian random variable (RV) has $E[X]$=10 and $E[X^2]$=500. Find $P(10 < X \leq 20)$.

(b) Let $Y=A\cos(X)$ where A is constant and X has uniform over $0 \leq x \leq 2\pi$. Find mean, second moment and standard deviation of Y