INTRODUCTION

The pose of a camera is recovered from given two images which are taken from different viewpoints by estimating the Essential Matrix with different number of points in this lab.

ALGORITHM / PROCEDURE

- We are given with two images and intrinsic camera parameters (K), so we are starting by transforming pixel points in pixel coordinates.
- Select any number of points (for inlab choose 8 points from different planes), for each pair of points obtain ‘Formula 1’ vector and stack them to obtain ‘X’ matrix in Formula 2.

$$a = [x_1x_2 \quad x_1y_2 \quad x_1z_2 \quad y_1x_2 \quad y_1y_2 \quad y_1z_2 \quad z_1x_2 \quad z_1y_2 \quad z_1z_2]^T \tag{1}$$

$$X = [a_1^T;\ a_2^T;\ \dots\ a_8^T] \tag{2}$$

- Then, estimate Essential Matrix (E) by doing svd of transpose of X * X. Last column of V is our Essential Matrix.
- Then, cure this E to satisfy Essential Matrix characterization in Figure 1.

Theorem 1a (Essential Matrix Characterization)
A non-zero matrix E is an essential matrix <u>iff</u> its SVD: $E = U\Sigma V^T$ satisfies: $\Sigma = diag([\sigma_1, \sigma_2, \sigma_3])$ with $\sigma_1 = \sigma_2 \neq 0$ and $\sigma_3 = 0$ and $U, V \in SO(3)$

Figure 1. Essential Matrix Characterization

- Then, find epipoles (e1, e2) and coefficients (L1, L2) of epipolar lines for a specific point (I took first point). Verify by checking below equation Figure 2.

$$l_i^T \mathbf{x}_i = 0$$

$$l_i^T \mathbf{e}_i = 0$$

Figure 2. Verification of Epipolar Points/Lines

- At the end, recover rotation and translation of the camera by using Formula 3.

$$(\hat{T}_1, R_1) = (UR_z(\frac{\pi}{2})\Sigma U^T, UR_z^T(\frac{\pi}{2})V^T) \qquad (\hat{T}_2, R_2) = (UR_z(-\frac{\pi}{2})\Sigma U^T, UR_z^T(-\frac{\pi}{2})V^T) \qquad (3)$$

- For Translation Matrix we need 3x1 from skewed matrix Figure 3.

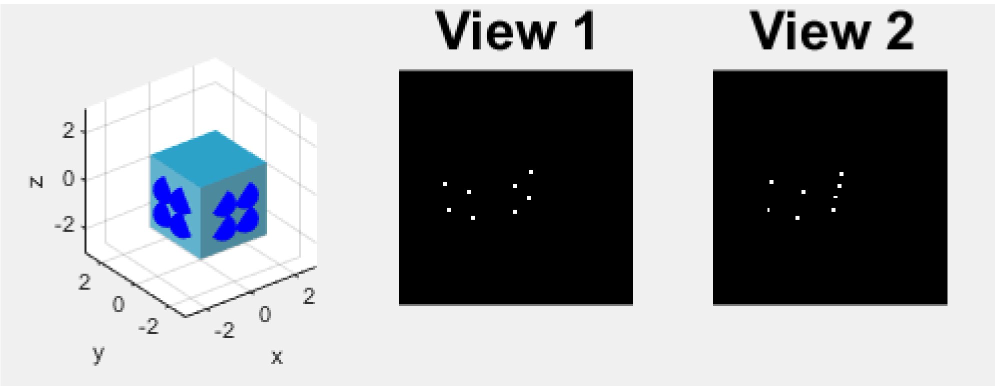
$$\text{Skew}\left(\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}\right) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Figure 3. Skewed Matrix

IN-LAB RESULTS

All the in-lab and post-lab results will be discussed in DISCUSSION part.

I choose 8 points in total from different planes as followed way:



I followed to above algorithm. Firstly, I showed determinants of U and V are equal to 1 to prove our Essential Matrix' U and V in SO(3).

```
Determinant of U:
ans =
    1.0000
Determinant of V:
ans =
    1.0000
```

Then, I verified the Figure 2.

```
First Verification:
   5.8981e-17
Second Verification:
   2.5153e-17
```

They are very approximately equal to 0. Thus, everything is fine right now.

```
True E =
         0   -1.0000         0
   -0.3615         0   -3.1415
         0    3.0000         0
Estimated E =
   -0.0115    0.9509    0.0305
   -0.9036   -0.0347    0.4258
    0.0334   -0.3067   -0.0238
```

I do not know why but some values are off and looks like scaled version of estimated E.

```
True R =
    0.9063         0   -0.4226
         0    1.0000         0
    0.4226         0    0.9063
Estimated R1 & R2 :
R1_est =
   -0.7289   -0.0307    0.6839
    0.0346   -0.9994   -0.0081
    0.6837    0.0177    0.7295
-------------
R2_est =
    0.9917    0.0161   -0.1273
   -0.0079    0.9979    0.0645
    0.1281   -0.0630    0.9898
-------------
```

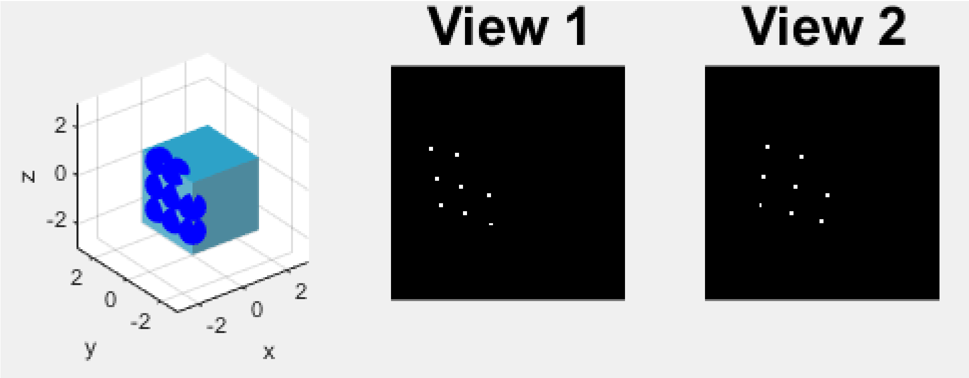
```
True T =
     3
     0
     1
Estimated T1 & T2 :
T1_est =
    0.3079
    0.0312
    0.9509
-------------
T2_est =
   -0.3079
   -0.0312
   -0.9509
```

We can see that R2_est is very close to True R.

We can see that T1_est is very close in terms of y and z directions, but X direction is scaled by 3.

POST-LAB RESULTS

- Firstly, select all 8 points from X plane as followed way:



Verifications:

```
Determinant of U:
ans =
    1.0000
Determinant of V:
ans =
    1.0000
First Verification:
   1.3878e-17
Second Verification:
   1.1102e-16
```

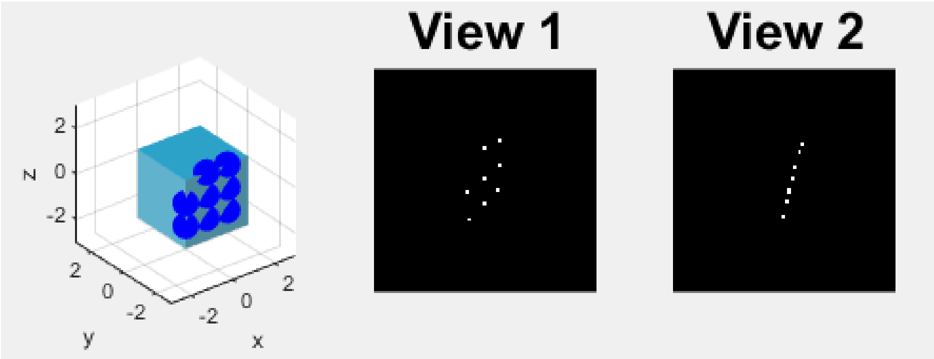
Results:

```
True E =
          0   -1.0000         0
    -0.3615         0   -3.1415
          0    3.0000         0
Estimated E =
     0.0638    0.8813   -0.3410
    -0.8797    0.0975   -0.2696
     0.4082    0.2832   -0.0082
```

```
True R =
    0.9063         0   -0.4226
         0    1.0000         0
    0.4226         0    0.9063
Estimated R1 & R2 :
R1_est =
   -0.8427   -0.1401   -0.5198
   -0.2757   -0.7170    0.6403
   -0.4624    0.6829    0.5656
-------------
R2_est =
    0.9938   -0.0944   -0.0580
    0.0970    0.9944    0.0432
    0.0536   -0.0485    0.9974
```

```
True T =
     3
     0
     1
Estimated T1 & T2 :
T1_est =
   -0.3208
    0.3795
    0.8678
-------------
T2_est =
    0.3208
   -0.3795
   -0.8678
```

- Secondly, select all 8 points from Y plane as followed way:



Verifications:

```
Determinant of U:
ans =
    1.0000
Determinant of V:
ans =
    1.0000
First Verification:
   6.9389e-18
Second Verification:
   3.4694e-18
```

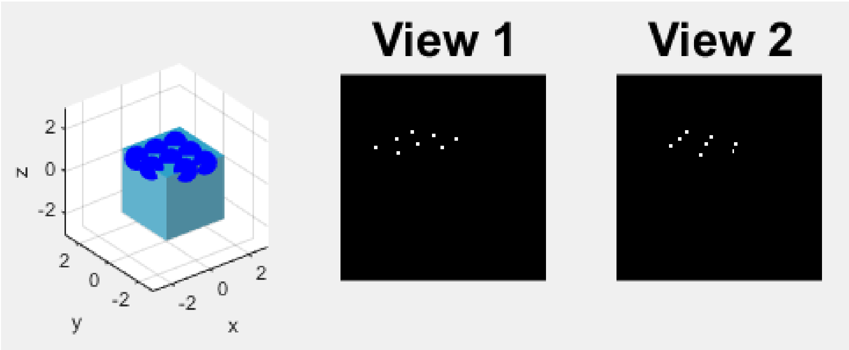
Results:

```
True E =
         0   -1.0000         0
   -0.3615         0   -3.1415
         0    3.0000         0
Estimated E =
   -0.1906    0.9792    0.0314
    0.9624    0.1873    0.1765
    0.0959   -0.0451    0.0134
```

```
True R =
    0.9063         0   -0.4226
         0    1.0000         0
    0.4226         0    0.9063
Estimated R1 & R2 :
R1_est =
    0.9757    0.1863    0.1153
    0.1810   -0.9819    0.0550
    0.1234   -0.0328   -0.9918
-------------
R2_est =
   -0.9548   -0.1783   -0.2380
   -0.2101    0.9708    0.1157
    0.2104    0.1605   -0.9644
```

```
True T =
     3
     0
     1
Estimated T1 & T2 :
T1_est =
    0.0624
   -0.0867
    0.9943
-------------
T2_est =
   -0.0624
    0.0867
   -0.9943
```

- Thirdly, select all 8 points from Z plane as followed way:



Verifications:

```
Determinant of U:
ans =
    1.0000
Determinant of V:
ans =
    -1
First Verification:
  -3.4694e-18
Second Verification:
  -1.7347e-18
```

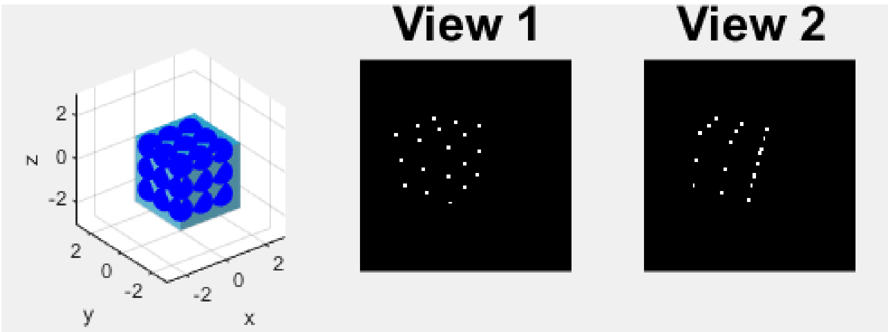
Results:

```
True E =
         0   -1.0000         0
   -0.3615         0   -3.1415
         0    3.0000         0
Estimated E =
   -0.4437    0.8698   -0.0739
   -0.8123   -0.4656   -0.3376
   -0.1729    0.1349   -0.0489
```

```
True R =
    0.9063         0   -0.4226
         0    1.0000         0
    0.4226         0    0.9063
Estimated R1 & R2 :
R1_est =
   -0.8766   -0.4594   -0.1436
    0.4349   -0.8838    0.1725
    0.2061   -0.0888   -0.9745
-------------
R2_est =
    0.7400    0.4220    0.5237
   -0.5000    0.8660    0.0086
    0.4499    0.2682   -0.8518
```

```
True T =
     3
     0
     1
Estimated T1 & T2 :
T1_est =
   -0.2028
   -0.0966
    0.9744
-------------
T2_est =
    0.2028
    0.0966
   -0.9744
```

- Lastly, select all points that are available as followed way:



Verifications:

```
Determinant of U:
ans =
    1.0000
Determinant of V:
ans =
   -1.0000
First Verification:
   5.2042e-18
Second Verification:
   4.8572e-17
```

Results:

```
True E =
         0   -1.0000         0
   -0.3615         0   -3.1415
         0    3.0000         0
Estimated E =
   -0.0043   -0.4130    0.0009
   -0.0339   -0.0138   -0.9993
    0.0091    0.9106   -0.0151
```

```
True R =
    0.9063         0   -0.4226
         0    1.0000         0
    0.4226         0    0.9063
Estimated R1 & R2 :
R1_est =
   -0.9241    0.0088   -0.3820
    0.0154    0.9998   -0.0143
   -0.3818    0.0191    0.9240
-------------
R2_est =
   -0.8962    0.0104    0.4435
   -0.0046   -0.9999    0.0140
   -0.4436   -0.0105   -0.8962
```

```
-------------
True T =
     3
     0
     1
Estimated T1 & T2 :
T1_est =
    0.9107
   -0.0054
    0.4130
-------------
T2_est =
   -0.9107
    0.0054
   -0.4130
```

DISCUSSION

We can see that it is possible to recover pose of a camera by calculating an Essential Matrix with different number of points and from different planes. However, point selection is affecting our results. Therefore, we need to choose our points carefully. Ideally, it is better to choose points from different planes to increase accuracy of calculations.

Our experiments also show the same scenario, when we choose points from different planes our pose accuracy is increasing. However, I could not get exactly the true given results. My results are always look like a down scaled version of the result. I could not understand the main reason of it. Maybe I could not choose the best 8 points from all possible selections.

In addition to these, each time calculations are changing slightly even I did not change the selected points. I could not understand where this randomness comes from.

In short, when all points are selected from same plane, accuracy of the result is decreasing dramatically as it can be seen from above examples like choosing all planes from X planes and so on.

CODES

```
%% Transform pixel coordinates and construct X matrix using Equations 1 and 2
K_inv = inv(K);
u1_new = K_inv * u1;
u2_new = K_inv * u2;

% Picking 8 poins

for i = 1:8
    a(i,:) = [u1_new(1,i)*u2_new(1,i), u1_new(1,i)*u2_new(2,i), u1_new(1,i)*u2_
end

X = [];
for i = 1:8
    X = [X; a(i,:)];
end

% % Picking all poins

% for i = 1:19
%     a(i,:) = [u1_new(1,i)*u2_new(1,i), u1_new(1,i)*u2_new(2,i), u1_new(1,i)*u
% end
%
% X = [];
% for i = 1:19
%     X = [X; a(i,:)];
% end
```

```
%% Estimate E, cure it and check for Essential Matrix Characterization
[U, S, V] = svd(X'*X);
E = V(:, 9);
E_new = [E(1), E(4), E(7);
         E(2), E(5), E(8);
         E(3), E(6), E(9)];
[U2, S2, V2] = svd(E_new);
S2(1,1) = 1;
S2(2,2) = 1;
S2(3,3) = 0;

E_est = U2*S2*V2';

disp("Determinant of U:");
det(U2)
disp("Determinant of V:");
det(V2)
```

```
%% Find epipoles and epipolar lines
epipol1 = null(E_est);
epipol2 = null(E_est');

line1 = E_est' * u2_new(:,1);
line2 = E_est * u1_new(:,1);
```

```
%% Verify epipoles and epipolar lines
ver1 = line1'*epipol1;
ver2 = line2'*epipol2;
disp("First Verification:")
disp(ver1)
disp("Second Verification:")
disp(ver2)
```

```
%% Recover the rotation and the translation
Rz = [0, -1, 0;
      1, 0, 0;
      0, 0, 1];

T1_ = U2*Rz*S2*U2';
R1 = U2*Rz'*V2';

Rz2 = [0,1,0;
       -1, 0,0;
       0,0,1];
T2_ = U2*Rz2*S2*U2';
R2 = U2*Rz2'*V2';

T1 = [T1_(3,2); T1_(1,3); T1_(2,1)];
T2 = [T2_(3,2); T2_(1,3); T2_(2,1)];
```