EE 314 HW 1

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25331

1) System A: $y(t) = x(t-2) + x(2-t)$

(a) $x_1(t) \rightarrow x_1(t-2) + x_1(2-t) = y_1(t)$

$x_2(t) \rightarrow x_2(t-2) + x_2(2-t) = y_2(t)$

$\alpha x_1(t) + \beta x_2(t) \rightarrow \alpha x_1(t-2) + \alpha x_1(2-t) + \beta x_2(t-2) + \beta x_2(2-t) = y_3(t)$

Since, $y_3(t) = \alpha_1 y_1(t) + \beta y_2(t) \equiv$ this system is $\boxed{\text{Linear}}$

(b) $y(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$

$x(t-t_0) \rightarrow x(t-t_0-2) + x(2-t-t_0)$

These are not equal. So, this system is $\boxed{\text{NOT time invariant}}$.

(c), It is $\boxed{\text{not memoryless}}$ because $y(t)$ is not looking only for present input.

For example $t=0$; $\underset{\text{"0"}}{y(t)} = x(-2) + x(2)$

past input   future input

(d) As we can see in part (c), we need future input. So, this is $\boxed{\text{not causal}}$

(e) Give bounded input to the system, $|x(t)| < \infty$;

$|y(t)| \leq \underbrace{|x(t-2)| + |x(2-t)|}_{\text{These are bounded}} < \infty$

Thus, $|y(t)|$ also bounded. Thus, system is $\boxed{\text{stable}}$.

1: continue:

System C: $y(t) = x\left(\frac{t}{3}\right)$

(a) $x_1(t) \rightarrow x_1\left(\frac{t}{3}\right) = y_1(t)$

$x_2(t) \rightarrow x_2\left(\frac{t}{3}\right) = y_2(t)$

$\alpha x_1(t) + \beta x_2(t) \rightarrow \alpha x_1\left(\frac{t}{3}\right) + \beta x_2\left(\frac{t}{3}\right) = y_3(t)$

Since, $y_3(t) = \alpha y_1(t) + \beta y_2(t)$, This system is $\boxed{\text{Linear}}$

(b) $y(t-t_0) = x\left(\frac{t-t_0}{3}\right)$

$x(t-t_0) \rightarrow x\left(\frac{t}{3} - t_0\right)$

These are not equal.
So, this system is $\boxed{\text{NOT time invariant}}$

(c) It is $\boxed{\text{not memoryless}}$ because $y(t)$ is not looking only for present input.

For exemple, $t=-6$;

$$\underset{-6}{y(t)} = x(-2)$$

future input.

(d) As we can see in part (c), system is $\boxed{\text{not causal}}$ because we need future input.

(e) Give bounded input $|x(t)| < \infty$,

$$|y(t)| \leq \underbrace{\left|x\left(\frac{t}{3}\right)\right|}_{\text{this is still bounded, signal just expanded}} < \infty$$

So, this system is $\boxed{\text{stable}}$.

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2) a) $e^{j2\pi t}\left(\delta(t-\frac{1}{4})+\delta(t-\frac{1}{2})+\delta(t-1)+\delta(t+\frac{1}{4})\right)$

$\underbrace{e^{j2\pi t}\cdot\delta(t-\frac{1}{4})}+e^{j2\pi t}\delta(t-\frac{1}{2})+e^{j2\pi t}\delta(t-1)+e^{j2\pi t}\delta(t+\frac{1}{4})$

$e^{j\frac{\pi}{2}}\delta(t-\frac{1}{4})+e^{j\pi}\delta(t-\frac{1}{2})+e^{j2\pi}\delta(t-1)+e^{-j\frac{\pi}{2}}\delta(t+\frac{1}{4})$

$$=\boxed{j\,\delta(t-\tfrac{1}{4})+(-1)\cdot\delta(t-\tfrac{1}{2})+(1)\cdot\delta(t-1)+(-j)\,\delta(t+\tfrac{1}{4})}$$

b) $\int_{-\infty}^{\infty}\frac{t}{2\tau}\delta(t-2\tau)\,d\tau \rightarrow$ Note: $\boxed{\int_{-\infty}^{\infty}f(x)\cdot\delta(x-a)\,dx=f(a)}$

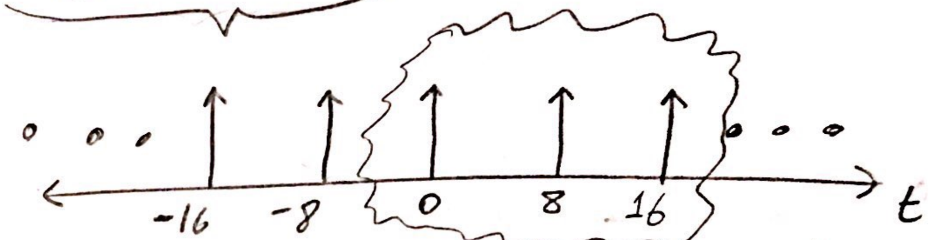
$=\frac{t}{2}\int_{-\infty}^{\infty}\frac{1}{\tau}\delta(t-2\tau)\,d\tau$

$\delta(t-2\tau) = \delta(-2(-\frac{t}{2}+\tau))$

$f(\tau)=\frac{1}{\tau}$

$$=\frac{t}{4}\int_{-\infty}^{\infty}\frac{1}{\tau}\delta(\tau-\frac{t}{2})\,d\tau=\frac{t}{4}\cdot\frac{2}{t}=\boxed{\frac{1}{2}}$$

2) c) $\displaystyle\int_{-7}^{21}\left(\tan 2t + e^{-10\pi t}\right)\cdot\left(\sum_{n=-\infty}^{\infty}\delta(t-8n)\right)dt$



We can only look for t values; 0, 8, 16 because they are the only ones in the range of $-7 \leq t \leq 21$.

Again same property in part (b).

$$\int_{-7}^{21}\Big(\left[\tan 2t\cdot\delta(t-0)\right]+\left[\tan 2t\cdot\delta(t-8)\right]+\left[\tan 2t\cdot\delta(t-16)\right]+\left[e^{-10\pi t}\cdot\delta(t)\right]$$

$$+\left[e^{-10\pi t}\cdot\delta(t-8)\right]+\left[e^{-10\pi t}\cdot\delta(t-16)\right]\Big)\,dt$$

$$=\tan 0+\tan 16+\tan 32+e^{0}+e^{-80\pi}+e^{-160\pi}$$

$$\boxed{=0+\tan 16+\tan 32+1+e^{-80\pi}+e^{-160\pi}}$$

d) $y_1(t) = p(t) * s_1(t)$

$$y_1(t) = [\gamma(t+1) + 2\gamma(t-2)] * [-2\gamma(t+1) + 2\gamma(t-2) + 3\gamma(t-4)]$$

$$= -2\gamma(t+2) + 2\gamma(t-1) + 3\gamma(t-3) - 4\gamma(t-1) + 4\gamma(t-4) + 6\gamma(t-6)$$

$$\boxed{y_1(t) = -2\gamma(t+2) - 2\gamma(t-1) + 3\gamma(t-3) + 4\gamma(t-4) + 6\gamma(t-6)}$$

3) $A\cos(2\pi f_0 t) \longrightarrow |H(f_0)|\cdot A\cdot \cos(2\pi f_0 t + \angle H(f_0))$

$$4\cos(2\pi \times \underbrace{15}_{f_0} \times t) \longrightarrow \boxed{3\cdot 4\cdot \cos\left(2\pi\cdot 15\cdot t + \frac{15\pi}{100}\right)}$$

$$\begin{array}{cc} 100 & \pi \\ 15 & x \end{array} \qquad \frac{15\pi}{100} = x$$

$$\underbrace{-3\sin(2\pi\times 50\times t)}=-3\cos\left(\frac{\pi}{2}-(2\pi\times 50\times t)\right)\longrightarrow 2\cdot(-3)\cdot\cos\left(\frac{\pi}{2}-(2\pi\times 50\times t)+\frac{\pi}{2}\right)$$

$$|||$$

$$\longrightarrow \boxed{2\cdot(-3)\sin\left((2\pi\times 50\times t)+\frac{\pi}{2}\right) = -6\cos(2\pi\times 50\times t)}$$

$$7\cos(2\pi\times 120\times t)\longrightarrow 0$$

"I could write also as cosine"

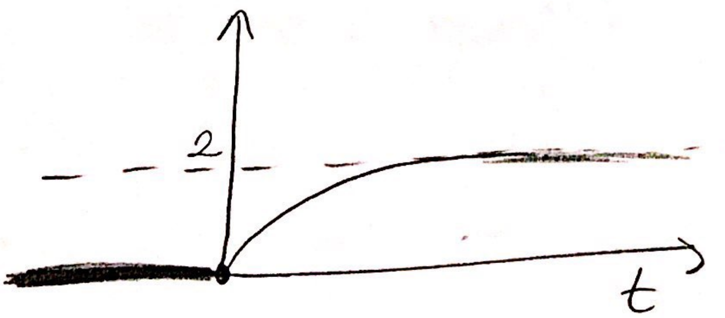
$$\boxed{y(t) = 12\cos\left(30\pi t + \frac{15\pi}{100}\right) - 6\sin\left(100\pi t + \frac{\pi}{2}\right)}$$

4) a) $y_1(t) = x_1(t) * h(t) = u(t) * \left(e^{-t/2}u(t)\right)$

$$= \int_{-\infty}^{\infty} u(t)\cdot e^{-\frac{(t-\tau)}{2}}\cdot u(t-\tau)\,d\tau$$

$$= \int_0^t e^{-\frac{(t-\tau)}{2}}\,d\tau = 2e^{-\frac{(t-\tau)}{2}}\Big|_0^t = 2-2e^{-\frac{t}{2}}$$

$$y_1(t) = \left(2-2e^{-\frac{t}{2}}\right)u(t)$$



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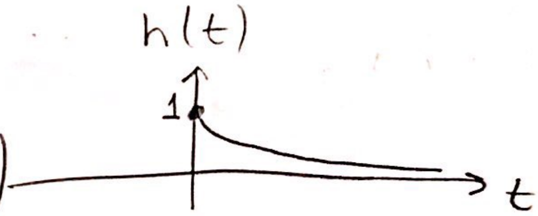
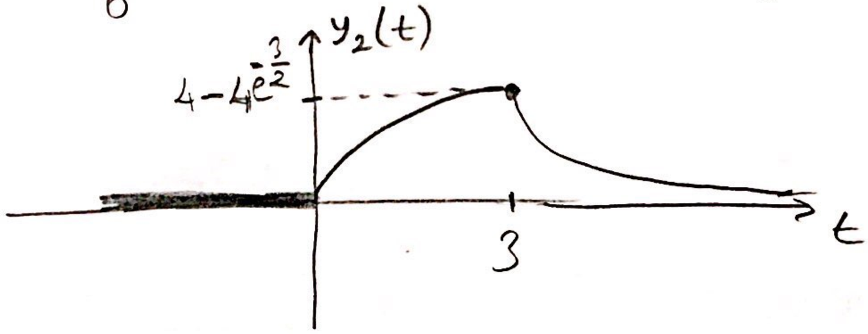
$y_2(t) = x_2(t) * h(t)$



For $t \leq 0$; $y_2(t) = \boxed{0}$ (No overlapping)

for $0 \leq t \leq 3$; $\int_0^t 2e^{-\frac{(t-\tau)}{2}} d\tau = 4e^{-\frac{(t-\tau)}{2}}\Big|_0^t = \boxed{4 - 4e^{-\frac{t}{2}}}$

for $t \geq 3$; $\int_0^3 2e^{-\frac{(t-\tau)}{2}} d\tau = 4e^{-\frac{(t-\tau)}{2}}\Big|_0^3 = \boxed{4e^{-\frac{(t-3)}{2}} - 4e^{-\frac{t}{2}}}$



b) $\boxed{x_2(t) = 2x_1(t) - 2x_1(t-3)}$ LTI

$\boxed{y_2(t) = 2y_1(t) - 2y_1(t-3)}$

For $t < 0$; $y_2(t) = 0$ ✓

"zero"

For $0 \leq t \leq 3$; $y_2(t) = 2y_1(t) - 2y_1(t-3) = 2y_1(t)$

$2y_1(t) = 4 - 4e^{-\frac{t}{2}} = y_2(t)$ ✓

For $t \geq 3$; $y_2(t) = 2y_1(t) - 2y_1(t-3)$

$2y_1(t) - 2y_1(t-3) = \cancel{4} - 4e^{-t/2} - \cancel{4} + 4e^{-\frac{(t-3)}{2}} = \underbrace{4e^{-\left(\frac{t-3}{2}\right)} - 4e^{-\frac{t}{2}}}_{y_2(t)}$ ✓

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```
x = -3:0.001:5;
f = @(x) exp(- abs(x)/4).* (heaviside(x) - heaviside(x-4));

subplot(2,2,1)
plot(x, f(x))
title("y(t)")
xticks([-5:1:5])
xlabel("t")

subplot(2,2,2)
plot(x, f(2.*x))
title("y1(t)");
xticks([-5:1:5])
xlabel("t")

subplot(2,2,3)
plot(x, f(x+2))
title("y2(t)");
xticks([-5:1:5])
xlabel("t")

subplot(2,2,4)
plot(x, f(2 - 2*x))
title("y3(t)");
xticks([-5:1:5])
xlabel("t")

sgtitle("4 Generated Signals")
```