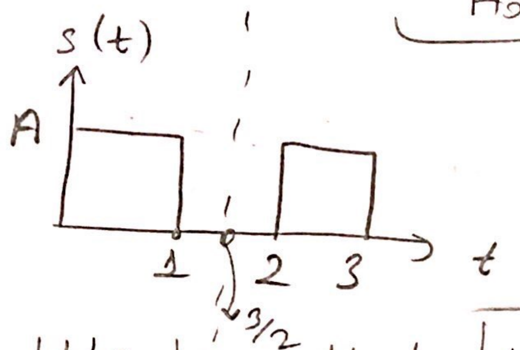
Homework 4

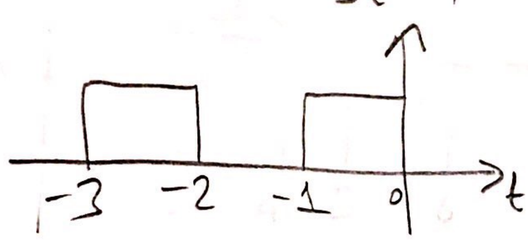
Ege Erdoğan

1)

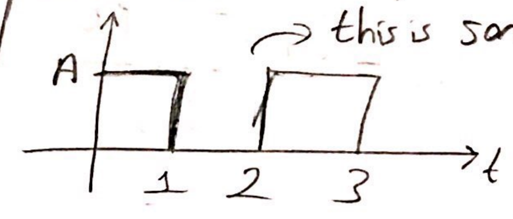


a) We know that $\boxed{h(t) = s(T-t) = s(-(t-T))}$

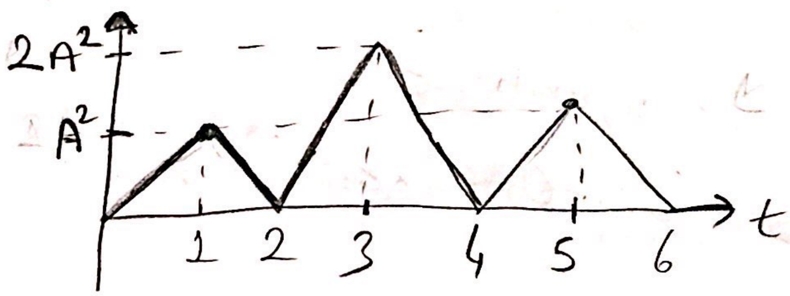
$T = 3$



$s(3-t) = h(t)$

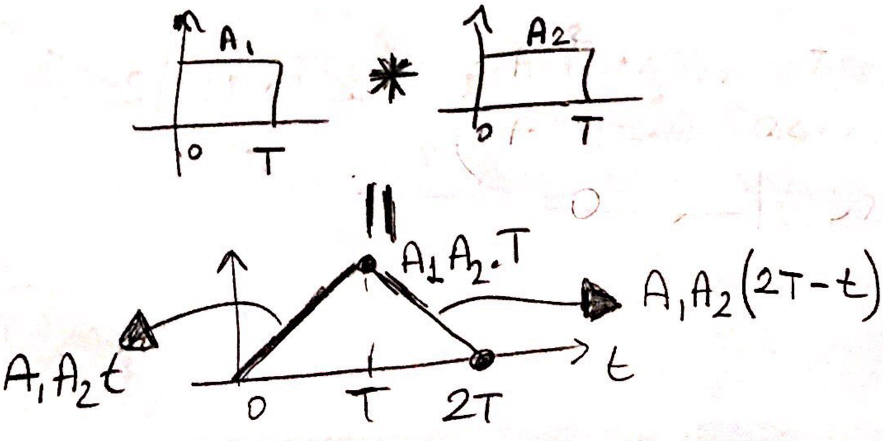


b) $s(t) * h(t) = \int_0^t s(\tau) \cdot h(t-\tau)\, d\tau = \begin{cases} 0, & t<0 \\ A^2 t, & 0 \le t \le 1 \\ A^2(2-t), & 1 \le t \le 2 \\ 2A^2(t-2), & 2 \le t \le 3 \\ 2A^2(4-t), & 3 \le t \le 4 \\ A^2(t-4), & 4 \le t \le 5 \\ A^2(6-t), & 5 \le t \le 6 \\ 0, & \text{else} \end{cases}$



Intuitevely, we know two rect convolution ends up with rectangular function.

This comes from



c) Maximum value is $\boxed{2A^2}$

at $t=3$

2) $s_1(t) = \frac{At}{T} \quad , 0 \le t \le T$

$$s_2(t) = A\left(1 - \frac{t}{T}\right) = A - \frac{At}{T} \quad , \quad 0 \le t \le T$$

$$z(t) = \int_0^T (s_2(t) - s_1(t)) \cdot r(t)$$

$$E_{s_1} \rightarrow \int_0^T \frac{A^2t^2}{T^2} dt = \frac{A^2t^3}{3T^2} \Big|_0^T = \frac{A^2T}{3}$$

$$E_{s_2} \rightarrow \int_0^T A^2 - \frac{2A^2t}{T} + \frac{A^2t^2}{T^2} dt = \underbrace{A^2t - \frac{\cancel{2}A^2t^2}{T}}_{0} + \frac{A^2t^3}{3T^2} = \frac{A^2T}{3}$$

$E_{s_1} = E_{s_2} \rightarrow$ They have same energy and same probability

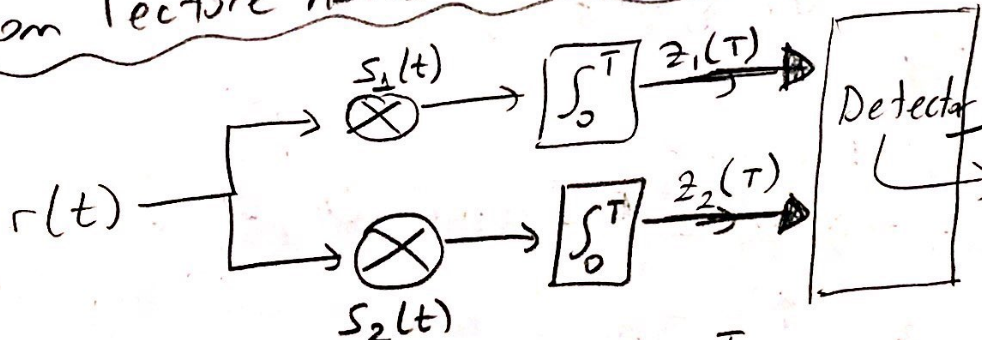
Thus, $z(T) - \left[\frac{\cancel{E_{s_2} - E_{s_1}}}{2}\right] \gtrless 0$

Decision Rule:

received signal

$$\boxed{\int_0^T s_2(t) \cdot r(t) dt \underset{"s_1"}{\overset{"s_2"}{\gtrless}} \int_0^T s_1(t) \cdot r(t) dt}$$

From lecture notes use 2 correlator receiver;



It choose the higher $z_i(T)$.

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \text{ where } E_d = \int_0^T |s_1(t) - s_2(t)|^2 dt$$

$$E_d = \int_0^T \left(\frac{2At}{T} - A\right)^2 dt = \int_0^T \frac{4A^2t^2}{T^2} - \frac{4A^2t}{T} + A^2 dt = \frac{4A^2t^3}{3T^2} - \frac{2A^2t^2}{T} + A^2t \Big|_0^T$$

$$E_d = \frac{4A^2T}{3} - A^2T = \boxed{\frac{A^2T}{3}} \rightarrow \boxed{P_B = Q\left(\sqrt{\frac{A^2T}{6N_0}}\right)}$$

3) For antipodal binary PAM;

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad E_b = A^2 T_b, \quad T_b = \frac{1}{R_b} = 10^{-5}\,\text{sec}$$

inverse Q function where it equals to 10^{-6}.

$$\sqrt{\frac{2E_b}{N_0}} = 4.75$$

$$A^2 T_b = \frac{(4.75)^2 \cdot N_0}{2} \Rightarrow A = \sqrt{\frac{(4.75)^2 \times 10^{-2} \times 10^{5}}{2}}$$

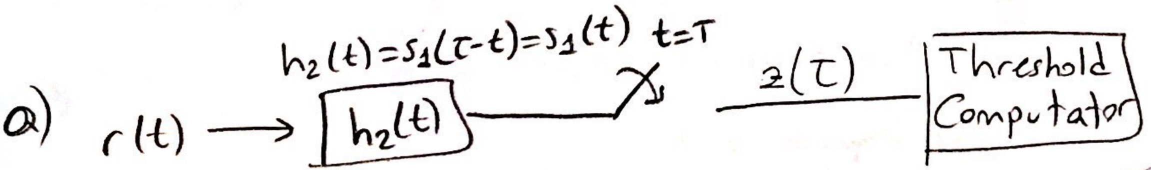
$$\boxed{A = 106.21}$$

4) $s_1(t) = A$ for $0 \le t < T$ for transmitting 1

$s_0(t) = 0$ for $0 \le t < T$ for transmitting 0

We know that this is an "ON-OFF Keying" from lecture notes.

a)



$$z(T) \underset{0}{\overset{1}{\gtrless}} \gamma_0$$

where, $E_{s_0} = 0$, $E_{s_1} = A^2 T = E_d$

$$E_b = \frac{1}{2} E_{s_0} + \frac{1}{2} E_{s_1} = \frac{A^2 T}{2}$$

$$\gamma_0 = \frac{E_{s_1} - E_{s_0}}{2} = \frac{A^2 T}{2}$$

b) $$P_b = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = Q\left(\sqrt{\frac{A^2 T}{2N_0}}\right) = \boxed{Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

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5) $s_0(t) = -1, \quad 0 \le t \le T$ for "0"

$s_1(t) = 1, \quad 0 \le t \le T$ for "1"

$P(0) = \frac{1}{3}, \quad P(1) = \frac{2}{3}$

$$\gamma_0 = \underbrace{\cancel{\frac{a_1+a_2}{2}}}_{\text{this part zero}} + \frac{\sigma_0^2}{\underbrace{a_1 - a_2}_{2a_1}} \ln\left(\frac{P(1)}{P(0)}\right), \text{ where } \sigma_0^2 = \frac{N_0}{2} \cdot E_d$$

$$E_d = \int_0^T |s_1(t) - s_0(t)|^2$$

$E_d = 2E_b$ (antipodal)

$s_1^2 = E_s$ where $E_s = E_b$

$s_2^2 = -E_s$

$$\boxed{\gamma_0 = \frac{N_0}{4\sqrt{E_b}} \cdot \ln(2)}$$

b) $$P_b = P(\overset{\text{bit}}{0}) \cdot \underbrace{\int_{\gamma_0}^{\infty} f(\overset{\text{received}}{r} | s = \overset{-\sqrt{E_b}}{-1})}_{P(N(\sqrt{E_b}, N_0/2) \text{ where } < \gamma_0)} + P(\overset{\text{bit}}{1}) \underbrace{\int_{-\infty}^{\gamma_0} f(\overset{\text{received}}{r} | s = 1^{\sqrt{E_b}})}_{P(N(\sqrt{E_b}, \frac{N_0}{2}) \text{ where } < -\gamma_0)}$$

$$\boxed{P_b = \frac{1}{3} Q\left(\frac{\sqrt{E_b} - \gamma_0}{\sqrt{\frac{N_0}{2}}}\right) + \frac{2}{3} Q\left(\frac{\sqrt{E_b} + \gamma_0}{\sqrt{\frac{N_0}{2}}}\right)}$$

where $\gamma_0 = \frac{N_0}{4\sqrt{E_b}} \cdot \ln(2)$

c) When we have maximum-likelihood we should have equal probability of $s_0(t)$ and $s_1(t)$. Therefore,

$$\text{received} \quad \frac{p(r|s_1)}{p(r|s_2)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)} = 1$$

Also, we know that from lecture notes, for antipodal ML detection;

$$\boxed{z \underset{0}{\overset{1}{\gtrless}} 0}$$

d) We know from part b,

$$P_b = p\,Q\left(\frac{\sqrt{E_b} - \gamma_0}{\sqrt{\frac{N_0}{2}}}\right) + (1-p)\,Q\left(\frac{\sqrt{E_b} + \gamma_0}{\sqrt{\frac{N_0}{2}}}\right)$$

We know for maximum likelihood we have equally likely antipodal signals. Thus,

$$\gamma_0 = \cancelto{0}{\frac{a_1 + a_2}{2}} + \frac{\sigma_0^2}{a_1 - a_2} \cancelto{0}{\ln\left(\frac{p(s_1)}{p(s_0)}\right)} = 0$$

Thus,

$$P_b = \frac{1}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\boxed{P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$