HOMEWORK #1

Assigned: March. 9, 17:00
Due   : March. 16, 23:55

- *Remarks: Please keep your answers clear and concise and show all the mathematical derivations that you perform. Each student should write up the solutions entirely on their own. You should list your name and ID on your write-up. If you do not type your solutions in a computer, be sure that your hand-writing is legible, your scan is high-quality and your name and ID are clearly written on your submitted document.*
- *Your solutions should be scanned as a single pdf file (we wont accept other format such as jpeg or multiple files). You should name your pdf file as first_name_lastname_HW_number (e.g., Mehmet_Keskinoz_HW_1)*
- *If you have a MATLAB problem, you should also be required to submit your .m file (name .m file as first_name_lastname_HW_number and write   your name and ID as a commented header in the .m file) .*
- *if you don't have a MATLAB related problem in your homework, just upload your solutions as a single pdf file . Otherwise, you should zip your .m file together   with   your   pdf   file   (name   the   zip   file   as   as first_name_lastname_HW_number)   and   upload   your   single   zip   file   to SUCOURSE.*
- *Note that you can only get help from your TAs on MATLAB related questions during their office hours.*
- *If you want to get feedbacks about your homework, you should also submit hand-written (or hard-copy) of your solutions.*
- *Late submission will not be accepted*

(1) (20 points)

Let us have two continuous-time systems with $x(t)$ at its input and $y(t)$ at its output.

$$\text{System A: } y(t) = x(t-2) + x(2-t)$$
$$\text{System C: } y(t) = x(t/3)$$

Determine which of the following system properties is hold by each of the these continuous-time systems . Justify your answers.
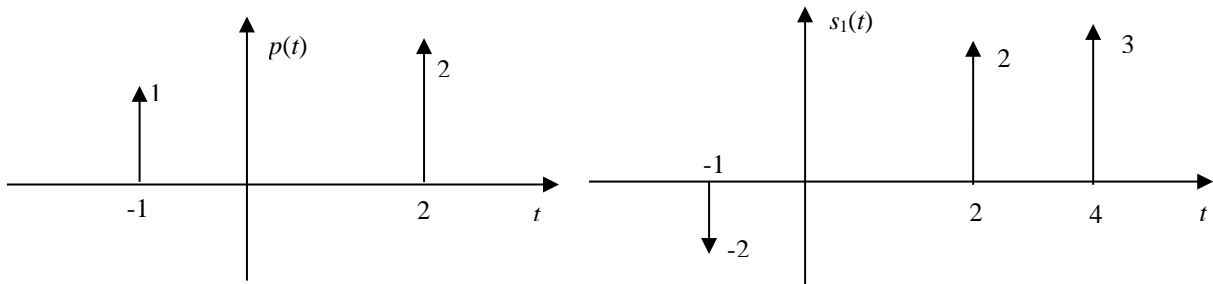(a) Linear
(b) Time Invariant
(c) Memoryless
(d) Casual
(e) Bounded input-bounded output (BIBO) Stable

(2) (20 points)
Simplifythe following expressions as much as possible.

a) $e^{j2\pi t}\left(\delta(t-\frac{1}{4}) + \delta(t-\frac{1}{2}) + \delta(t-1) + \delta(t+\frac{1}{4}) \right)$

b) $\displaystyle\int_{-\infty}^{\infty} \frac{t}{2\tau}\delta(t-2\tau)d\tau$

c) $\displaystyle\int_{-7}^{21} \left(\tan(2t) + e^{-10\pi t}\right)\{\sum_{n=-\infty}^{\infty} \delta(t-8n)\}dt$

d) Determine $\quad y_1(t) = p(t) * s_1(t) \quad$ where * represents the convolution operation, and signals $p(t)$ and $s_1(t)$ are plotted below.


(3)

A filter $H(f)$ has the magnitude (on the left graph) and phase (on the right plot) response shown below. The input $x(t)$ to this filter is $4\cos(30\pi t) - 3\sin(100\pi t) + 7\cos(240\pi t)$. Determine the resulting output.
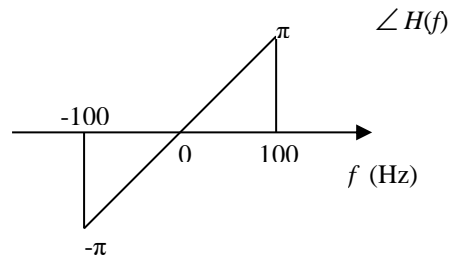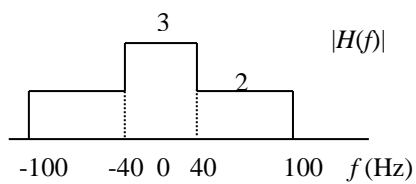
(4)

a) Using convolution, determine and sketch the responses of a linear, time-invariant system with impulse response $h(t) = e^{-t/2}\, u(t)$ to each of the two inputs $x_1(t), x_2(t)$ shown in Figures below. Use $y_1(t)$ to denote the response to $x_1(t)$ and use $y_2(t)$ to denote the response to $x_2(t)$.
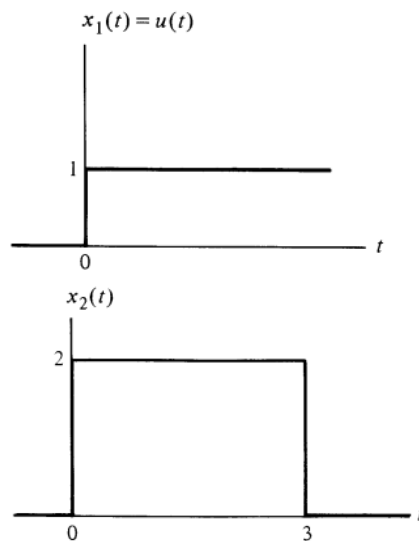
b) Express $x_2(t)$ in terms of $x_1(t)$ and then by taking advantage of the linearity and time-invariance properties, determine how $y_2(t)$ can be expressed in terms of $y_1(t)$. Verify your expression by evaluating it with $y_1(t)$ obtained in part (a) and comparing it with $y_2(t)$ obtained in part (a).

(5)   MATLAB exercise about basic signal operations

In this problem we will simulate some signal operations via simulation i.e. scaling, shifting and their combination , where the sampling interval of 0.001 seconds is used. Please write MATLAB codes to perform the following tasks:

i.   First generate a segment of an exponentially decaying function

$$y(t) = \exp(-\frac{|t|}{4})[u(t) - u(t-4)]$$

In time interval $\begin{bmatrix} -3 & 5 \end{bmatrix}$. Plot $y(t)$

ii.   Apply time scaling to generate new signal $y_1(t) = y(2t)$ and plot it

iii.   Apply time shifting to generate new signal $y_2(t) = y(t+2)$ and plot it

iv.   Apply time scaling, time inversion, and time shifting to generate a new signal $y_3(t) = y(2-2t)$ and plot it

Hints:

-   Use *subplot* command to plot $y(t), y_1(t), y_2(t), y_3(t)$ in a single figure. $\exp()$, *abs()*, *heavyside()* are some useful functions