EE 314 HW 2,

Ege Erdoğan
25331

1) $x(t) = 2 + 5\sin(6\pi t) + 15\cos(12\pi t)$

i) $x(t)$ is periodic, $f_{0_1} = 3$ $f_{0_2} = 6$

$GCD(f_1, f_2) = 3 \rightarrow T = \frac{1}{3}\,s$

fund. period

$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt = 3\int_{-\frac{1}{6}}^{\frac{1}{6}} 4 + \cancel{10\sin(6\pi t)} + \cancel{30\cos(12\pi t)} + \cancel{10\sin(6\pi t)}$

$+ 25\sin^2(6\pi t) + 75\sin(6\pi t)\cos(12\pi t)$

$+ \cancel{30\cos(12\pi t)} + 75\sin(6\pi t)\cos(12\pi t)$

$+ 225\cos^2(12\pi t)$

$\cancel{75\sin(18\pi t)} - \cancel{\sin(6\pi t)}$

$P_x = 3\int_{-\frac{1}{6}}^{\frac{1}{6}} 4 + 25\sin^2(6\pi t) + 225\cos^2(12\pi t)\,dt$

$P_x = 3\left(\frac{4}{3} + \frac{25}{6} + \frac{225}{6}\right) = \boxed{129\ W}$

ii) $x(t) = 2 + 5\sin(2\pi.3.t) + 15\cos(2\pi.6.t)$

$= 2 + \frac{5(e^{j2\pi 3t} - e^{-j2\pi 3t})}{2j} + \frac{15(e^{j2\pi .6t} + e^{-j2\pi 6t})}{2}$

c_0 c_1 1st c_{-1} c_2 2nd c_{-2}

$c_0 = 2$

$C_1 = C_{-1} = \frac{5}{2}$

$C_2 = C_{-2} = \frac{15}{2}$

$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)$

$G_x(f) = 4 + \frac{25}{4}\delta(f+3) + \frac{25}{4}\delta(f-3) + \frac{225}{4}\delta(f-6) + \frac{225}{4}\delta(f+6)$

$P_x = \int_{-\infty}^{\infty} G_x(f)\,df = 4 + \frac{25}{4} + \frac{25}{4} + 2\frac{225}{4} = \boxed{129\ W}$

Both techniques resulted with same result.

2) $y(t)=x^2(t)$, $x(t)=400\,\mathrm{sinc}(400t)$

$y(t)=16\times10^4\,\mathrm{sinc}^2(400t)$

i) Check its energy finite or not;

$E_y=\int_{-\infty}^{\infty} 4^4\times10^8\times\mathrm{sinc}^4(400t)=?$ Use Parseval Th.

$$E_y=\int_{-\infty}^{\infty} Y^2(f)\,df = 16\times10^4\underbrace{\int_{-\infty}^{\infty}\mathrm{tri}^2\left(\frac{f}{400}\right)df}_{\text{area finite}} < \infty$$

$$Y(f)=\frac{16\times10^4}{40\cancel{0}}\,\mathrm{tri}\left(\frac{f}{400}\right)$$

$\mathrm{sinc}\xleftrightarrow{F}\mathrm{rect}(f)$

$\mathrm{rect} * \mathrm{rect} = \mathrm{tri}(f)$

Since Energy is finite, it is $\boxed{\text{Energy Signal}}$.

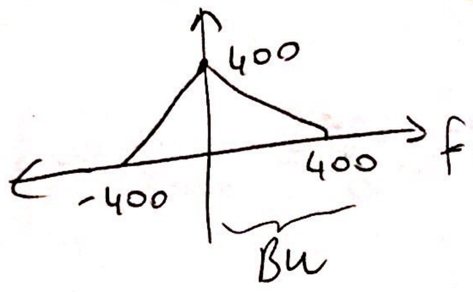
ii) Energy Spectral Density (ESD) $=|Y(f)|^2$

We calculated in the first part;

$$\boxed{|Y(f)|^2 = 16\times10^4\times\mathrm{tri}^2\left(\frac{f}{400}\right)}$$

iii) $Y(f)=400\,\mathrm{tri}\left(\frac{f}{400}\right)$

$\boxed{BW=400\ Hz}$



3) Signal BW = 6k Hz

Min. Sampling frequency → Nyquist Rate : $f_s = 2f_m$

$$\boxed{f_s = 2 \times 6k = 12k\ Hz}$$

With 2k Hz guard band → $\boxed{f_s = 12k + 2k = 14kHz}$

$$H(f) = \begin{cases} K & , \quad |f| < 7k \\ K - K\dfrac{|f| - 7000}{3000} & , \quad 7k < |f| < 10k \\ 0 & , \quad o.w \end{cases}$$

BW = 10k Hz

$BW + f_m \leq f_s$

$\downarrow$ 10k $\quad \downarrow$ 6k

$16k \leq f_s$

min 16k Hz for perfect reconstruction

$K \cdot 16k = 1$

$$\boxed{K = \frac{1}{16K}}$$

4) a) $P = 0.005 \rightarrow l \geq \log\frac{1}{2p} \rightarrow l \geq \underbrace{\log_2 \frac{1}{0.01}}_{\approx 6.64}$

$l \geq 6.64 \Rightarrow$ $\boxed{l \text{ is minimum } 7 \text{ bits.}}$

b) Nyquist rate → $\boxed{F_s = 2 \times 5k = 10k\ Hz}$

Bitrate $R = l \cdot f_s = 7 \times 10k\ Hz = \boxed{70.000 \text{ bits/sec}}$

c) $2^k = 16 \Rightarrow k = 4$ bits/symbol

$R_s = R/k = \dfrac{70.000}{4} = 17500$ symbols/sec

4) continue:

d) $(BW)_{min} = \frac{R_s}{2} = \frac{17500}{2} = 8750 \text{ Hz}$

---

5) $\mu = 255$

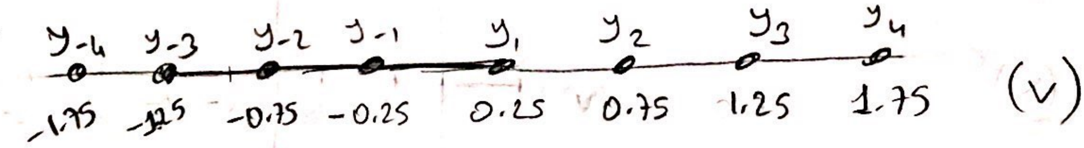
$V_{pp} = 4V \rightarrow$ from $-2V$ to $2V$

$$y = y_{max} \cdot \frac{\ln\left(1 + \mu \frac{|x|}{|x|_{max}}\right)}{\ln(1+\mu)} \cdot \mathrm{sgn}(x)$$

$$y = \frac{\ln\left(1 + 255 \frac{|x|}{2}\right)}{\ln(256)} \cdot y_{max} \rightarrow 2V$$

$$q = \frac{2V_p}{2} = \frac{2 \times 2}{8} = \boxed{0.5V}$$

levels

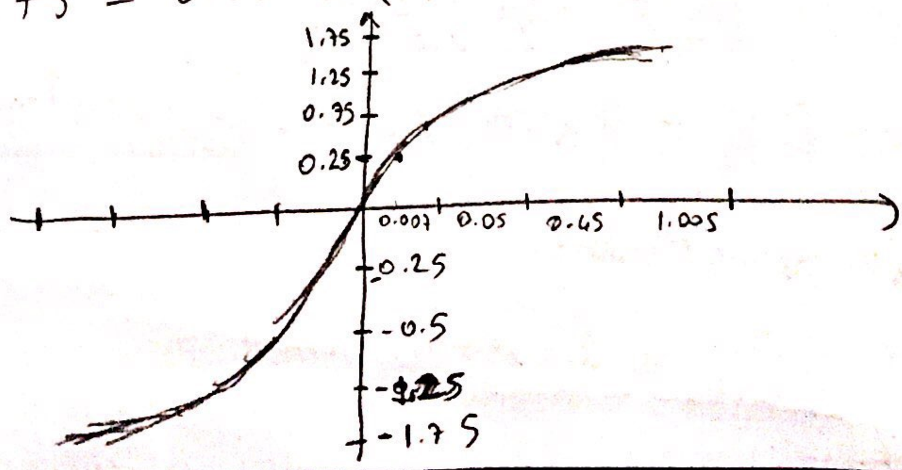


$y_1 = 0.25 = 0.36 \ln(1 + 127.5|x|) \Rightarrow x_1 \approx 0.007$

$y_2 = 0.75 = 0.36 \ln(1 + 127.5|x|) \Rightarrow x_2 \approx 0.05$

$y_3 = 1.25 = 0.36 \ln(1 + 127.5|x|) \Rightarrow x_3 \approx 0.49$

$y_4 = 1.75 = 0.36 \ln(1 + 127.5|x|) \Rightarrow x_4 \approx 1.005$



CamScanner ile tarandı