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## HOMEWORK #4

Assigned: Apr. 29, 2021

Due : May,7, 2021, 23:55

- **Remarks: Please keep your answers clear and concise and show all the mathematical derivations that you perform. Each student should write up the solutions entirely on their own. You should list your name and ID on your write-up. If you do not type your solutions in a computer, be sure that your hand-writing is legible, your scan is high-quality and your name and ID are clearly written on your submitted document.**
- **Your solutions should be scanned as a single pdf file (we wont accept other format such as jpeg or multiple files). You should name your pdf file as `first_name_lastname_HW_number` (e.g., Mehmet\_Keskinoz\_HW\_1)**
- **If you have a MATLAB problem, you should also be required to submit your .m file (name .m file as `first_name_lastname_HW_number` and write your name and ID as a commented header in the .m file) .**
- **if you don't have a MATLAB related problem in your homework, just upload your solutions as a single pdf file . Otherwise, you should zip your .m file together with your pdf file (name the zip file as `first_name_lastname_HW_number`) and upload your single zip file to SUCOURSE.**
- **Note that you can only get help from your TAs on MATLAB related questions during their office hours.**
- **If you want to get feedbacks about your homework, you should also submit hand-written (or hard-copy ) of your solutions.**
- **Late submission will not be accepted**

(1) Consider the signal  $s(t)$  shown below.

- a) Determine the impulse response of a filter matched to this signal under AWGN and sketch it as a function of time.
- b) Plot the matched filter output as a function of time
- c) What is the peak value of the output?



(2)

In an additive white Gaussian noise channel with noise power-spectral density of  $\frac{N_0}{2}$ , two equiprobable messages are transmitted by

$$s_1(t) = \begin{cases} \frac{At}{T}, & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$
$$s_2(t) = \begin{cases} A \left(1 - \frac{t}{T}\right), & 0 \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

1. Determine the structure of the optimal receiver.
2. Determine the probability of error.

(3)

A binary PAM communication system employs rectangular pulses of duration  $T_b$  and amplitudes of  $\pm A$  to transmit digital information at a rate of  $R_b = 10^5$  bps. If the PSD of AWGN is  $N_0/2$ , where  $N_0 = 10^{-2}$  W/Hz, determine the value of  $A$  that is required to achieve a probability of bit error  $P_b = 10^{-6}$ .

(4)

A binary communication system employs  $s_0(t) = 0$  for  $0 \leq t \leq T$  to transmit 0, and  $s_1(t) = A$  for  $0 \leq t \leq T$  to transmit 1 under AWGN channel. Assume that symbols are equally likely.

- a) Draw the optimal linear receiver that minimizes the symbol error probability.
- b) Determine the probability of bit error,  $P_B$ , which is obtained by employing the optimal linear receiver drawn in part (a) as a function of  $E_b/N_0$ .

(5)

A binary communication system employs  $s_0(t) = -1$  for  $0 \leq t \leq T$  to transmit 0, and  $s_1(t) = 1$  for  $0 \leq t \leq T$  to transmit 1 under AWGN channel. Assume that  $P(\text{bit } 1) = 2/3$   $P(\text{bit } 0) = 1/3$

1. Determine the optimum threshold at the detector.
2. Determine the average probability of error.
3. Determine the optimum maximum-likelihood decision rule for the detector.
4. Determine the average probability of error for the optimum maximum-likelihood decision rule as a function of  $E_b/N_0$ .