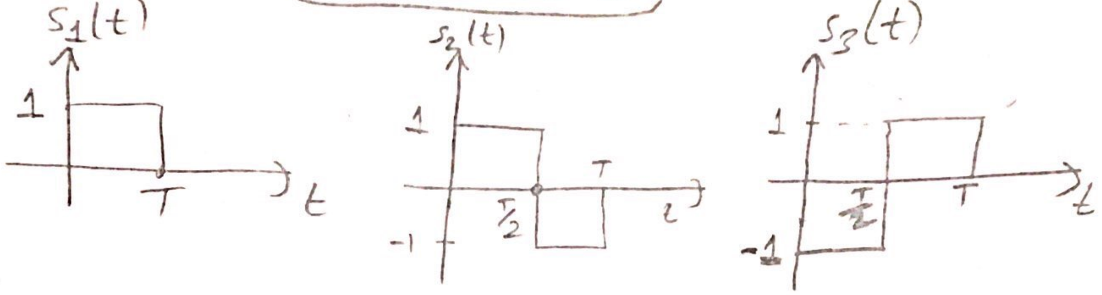
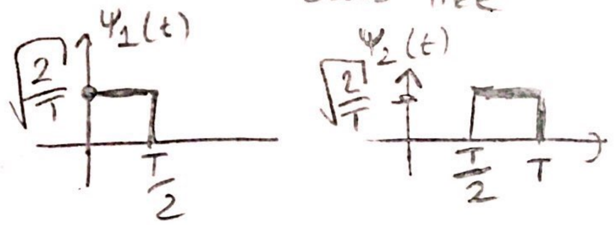
Homework 5

1) a)



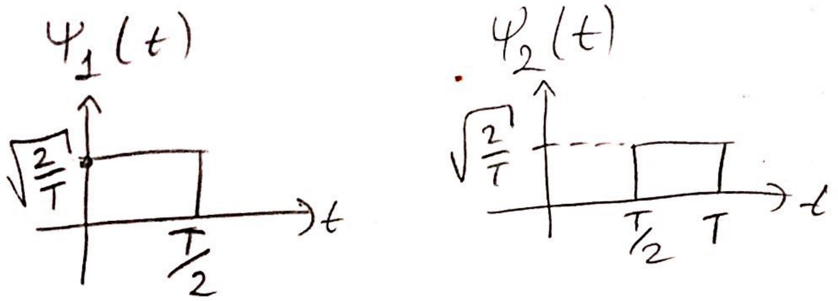
I can use bases like



← orthonormal bases

There dimensionality of the signal space $= \boxed{2}$

b) bases : as shown in part a;

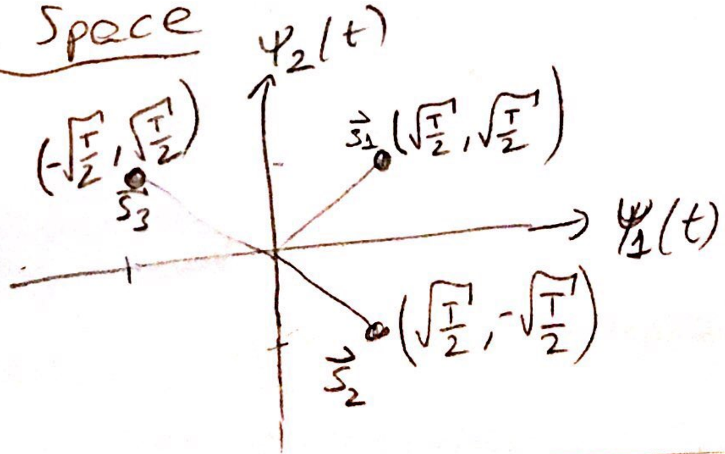


c) $s_1(t) = \sqrt{\frac{T}{2}}\,\psi_1(t) + \sqrt{\frac{T}{2}}\,\psi_2(t) \longleftrightarrow \vec{s_1} = \left[\sqrt{\frac{T}{2}}, \sqrt{\frac{T}{2}}\right]$

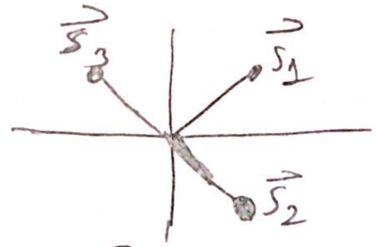
$s_2(t) = \sqrt{\frac{T}{2}}\,\psi_1(t) - \sqrt{\frac{T}{2}}\,\psi_2(t) \longleftrightarrow \vec{s_2} = \left[\sqrt{\frac{T}{2}}, -\sqrt{\frac{T}{2}}\right]$

$s_3(t) = -\sqrt{\frac{T}{2}}\,\psi_1(t) + \sqrt{\frac{T}{2}}\,\psi_2(t) \longleftrightarrow \vec{s_3} = \left[-\sqrt{\frac{T}{2}}, \sqrt{\frac{T}{2}}\right]$

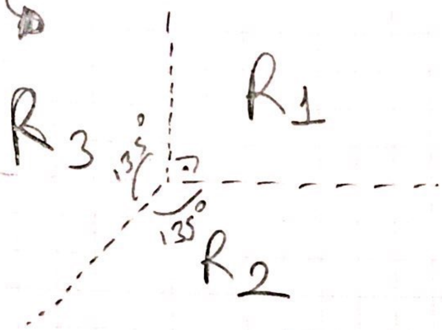
Signal Space



d) My signal space in part c;



Decision boundries



e) $P(Error|m_1) = 1 - p(\vec{s}_1 \text{ decided} | \vec{s}_1 \text{ transmitted})$

From Lecture Notes

$= 1 - P(r_0 \geq 0, r_1 \geq 0 \mid \vec{s}_1 \text{ transmitted})$

$$= 1 - \left(P\left(\frac{\sqrt{T}}{2} + n_0 \geq 0\right)\right)^2 = \boxed{2Q\left(\sqrt{\frac{T}{2N_0}}\right) - Q^2\left(\sqrt{\frac{T}{2N_0}}\right)}$$

$P(Error|m_2) = 1 - P(r_0 \geq 0, r_1 \overset{?}{\leq} 0 \mid \vec{s}_2 \text{ transmitted})$

$$= 1 - \left[P\left(\frac{\sqrt{T}}{2} + n_0 \geq 0\right) \cdot P\left(\frac{\sqrt{T}}{2} + n_1 \leq 0\right)\right]$$

$$\left(1 - P\left(\frac{\sqrt{T}}{2} + n_1 \geq 0\right)\right)$$

$$= 1 - \left[Q\left(-\sqrt{\frac{T}{2N_0}}\right) - Q^2\left(-\sqrt{\frac{T}{2N_0}}\right)\right]$$

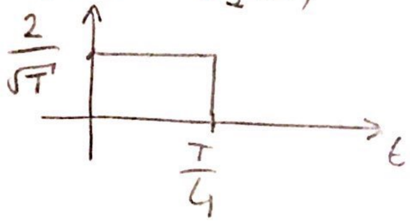
$$1 - Q\left(\sqrt{\frac{T}{2N_0}}\right) - 1 + 2Q\left(\sqrt{\frac{T}{2N_0}}\right) - Q^2\left(\sqrt{\frac{T}{2N_0}}\right)$$

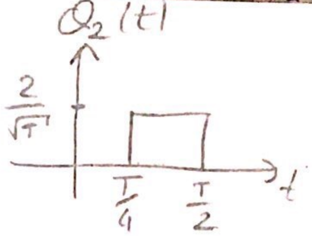
symmetric decision region

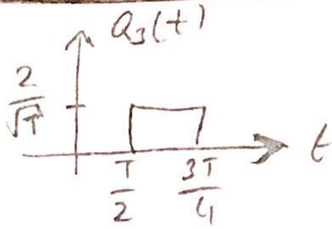
$$= \boxed{Q\left(\sqrt{\frac{T}{2N_0}}\right) - Q^2\left(\sqrt{\frac{T}{2N_0}}\right)} \rightarrow P(Error|m_2) = P(Error|m_3)$$

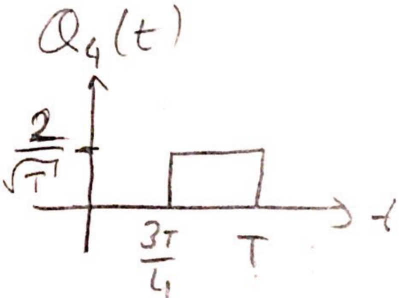
* M_1 is more vulnorable to errors because it has smaller decision region.

2) $P(t) = Q_1(t)$





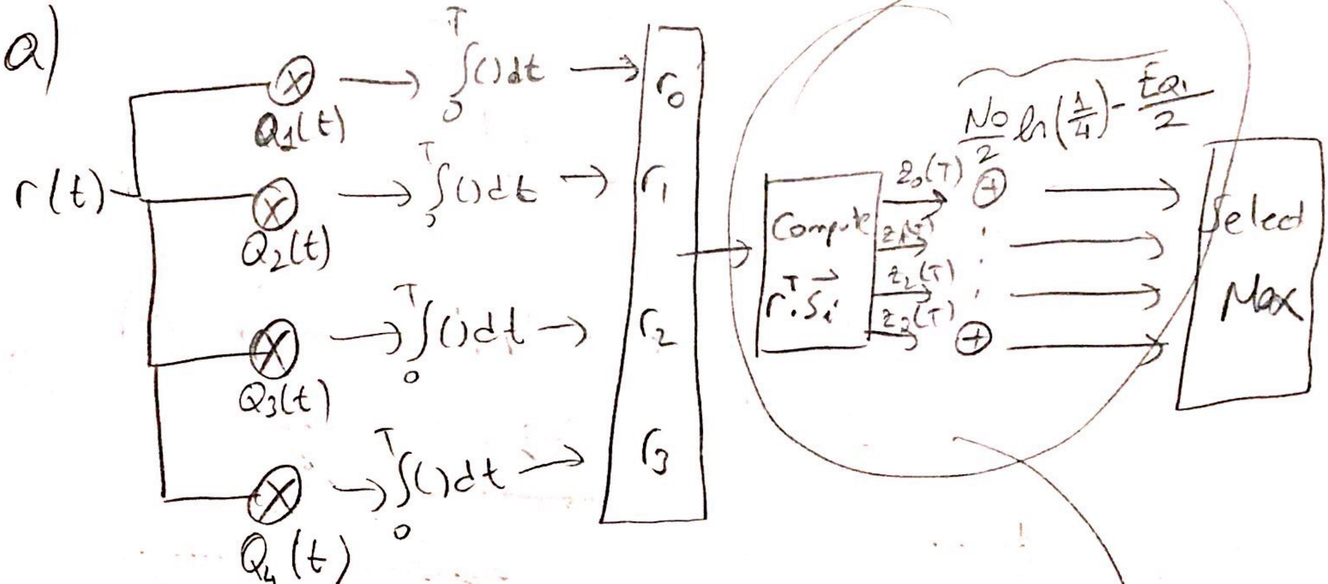




$$P(Q_1) = P(Q_2) = P(Q_3) = P(Q_4) = \frac{1}{4}$$

$\{Q_1(t), Q_2(t), Q_3(t), Q_4(t)$ forms an orthonormal basis.

a)

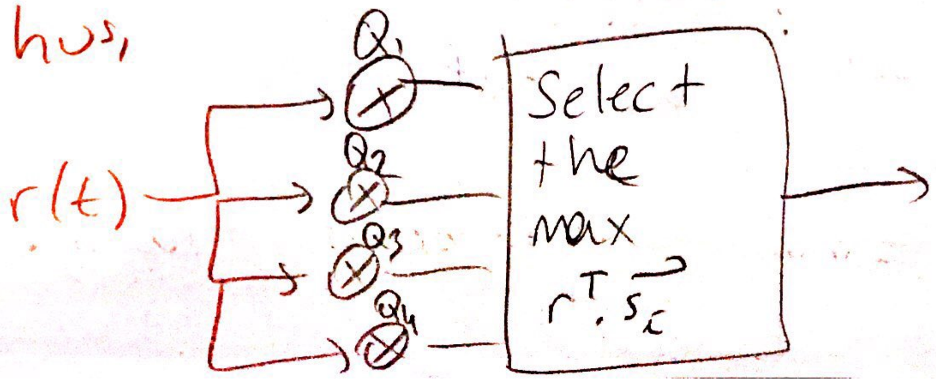


$$E_{Q_1} = E_{Q_2} = E_{Q_3} = E_{Q_4} = 1$$

$$\frac{N_0}{2} \ln(P(Q_1)) - \frac{E_{Q_1}}{2} = 10^{-8} \ln\left(\frac{1}{4}\right) - \frac{1}{2}$$

Since, this is an ML receiver, this part same for all of them.

Thus,



2) b) This transmission looks like MFSK because signals are frequency shifted versions of themselves.

Thus, (assuming MFSK)

$$P_E \approx (M-1)\, Q\left(\sqrt{\frac{E_s}{N_0}}\right) \approx \boxed{15\, Q\left(\sqrt{\frac{4}{2\times10^{-8}}}\right)}$$

$$E_s = E_b \cdot \log_2 M = 4E_b = \boxed{4 \text{ Joule}}$$

$$\boxed{E_b = 1}$$

c) Under Gray Coding;

$$P_B = \frac{P_E}{\log_2 M} = \frac{P_E}{4} = \frac{15}{4}\, Q\left(\sqrt{\frac{4}{2\times10^{-8}}}\right)$$

Gray Coding is not useful in MFSK because neighbors are equally seperated, so the way of assignment of bits to symbol does not matter.

d) For MFSK Bandwidth:

$$BW = \frac{M}{2T} = \frac{16}{2T} = \boxed{\frac{8}{T}}$$

T: symbol duration

$$\eta = \frac{T.R_s.\log M.\,2}{M} = \frac{8\, \overbrace{TR_s}^{1}}{16} = \boxed{\frac{1}{2}}$$

η: Bandwidth efficiency

CamScanner ile tarandı

3)

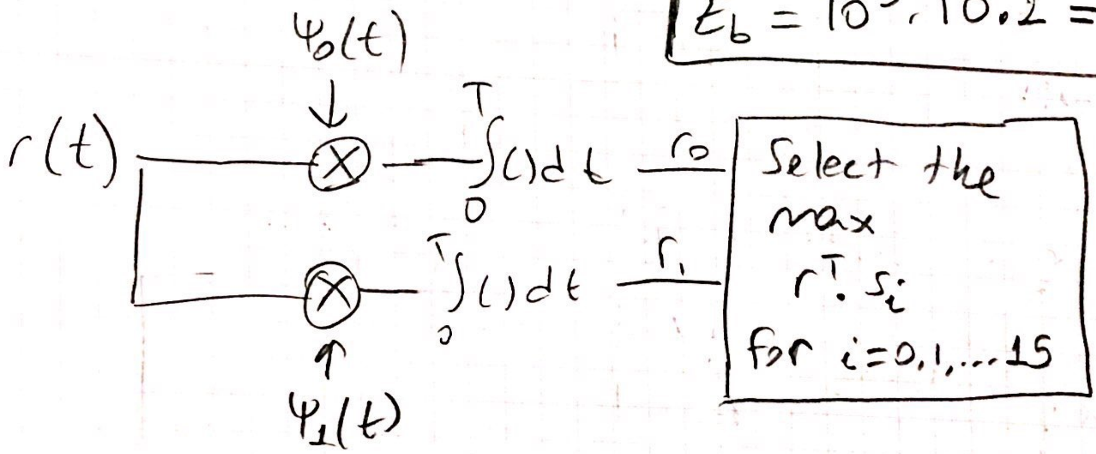
a) $\boxed{\begin{array}{l} \psi_0(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t), \quad 0 \le t \le T \\ \psi_1(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_0 t), \quad 0 \le t \le T \end{array}}$

$P(s_i) = \frac{1}{16} \quad \frac{N_0}{2} = 10^{-8} \quad \frac{E_b}{N_0} = 20\,dB$

$$10 \log\left(\frac{E_b}{N_0}\right) = 20\,dB$$

$$\frac{E_b}{N_0} = 10^{19}$$

$$\boxed{E_b = 10^{19} \cdot 10^{-8} \cdot 2 = 2 \times 10^{11}}$$



$\boxed{M = 16}$

b) $P_E \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{16}\right) \approx \boxed{2Q\left(\sqrt{\frac{16 \times 10^{11}}{2 \times 10^{-8}}} \sin\frac{\pi}{16}\right)}$

$$\boxed{E_s = E_b \cdot \log_2 16 = 8 \times 10^{11}}$$

c) Utilizing Gray code;

$$P_B = \frac{P_E}{\log_2 M} = \frac{2Q\left(\sqrt{\frac{16 \times 10^{11}}{2 \times 10^{-8}}}\right) \sin\frac{\pi}{16}}{\log_2 16}$$

Yes, Grey could should be used. Occurence of a multibit error, for a given symbol error is reduced.

3) d) for MPSK Bandwidth;

$$\boxed{BW \approx \frac{2}{T}}$$

$T \rightarrow$ symbol duration

$$\eta = \frac{\overbrace{R_b \cdot T_b}^{1} \cdot \log_2 M}{2} = \frac{4}{2} = \boxed{2}$$

$\eta \rightarrow$ Bandwidth efficiency

4) $BW = 5k\,Hz$

a) $f_s = 2 \times BW = 10k\,Hz$

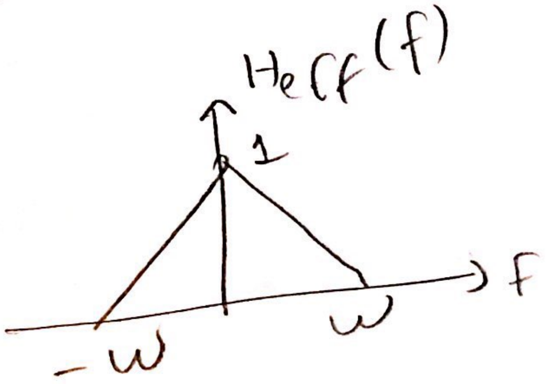
$l = \log_2 64 = 6$

$\boxed{R_b = l \times f_s = 6 \times 10k = 60k \text{ bits/sec}}$

b) $\boxed{\text{Symbol Rate } R_s = \frac{60k}{\log_2 16} = 15k \text{ symbols/sec}}$

c) $\boxed{\text{Minimum Bandwidth} = \frac{R_s}{2} = 7.5k\,Hz}$

d)



For no ISI, as we state upper parts we need and we need to satisfy Nyquist rate.

$W = \frac{R_s}{2}$

Thus, $\boxed{W = 7.5k\,Hz}$ mini

5) a) $$\begin{bmatrix} h_{eff}[0] & h_{eff}[-1] & h_{eff}[-2] \\ h_{eff}[1] & h_{eff}[0] & h_{eff}[-1] \\ h_{eff}[2] & h_{eff}[1] & h_{eff}[0] \end{bmatrix} \begin{bmatrix} heq[-1] \\ heq[0] \\ heq[1] \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.1 & -0.5 \\ -0.2 & 1 & 0.1 \\ 0.05 & -0.2 & 1 \end{bmatrix} \begin{bmatrix} heq[-1] \\ heq[0] \\ heq[1] \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

↓ from Matrix calculator ;

$$\boxed{\begin{aligned} heq[-1] &= 0 \\ heq[0] &= \frac{50}{51} \\ heq[1] &= \frac{10}{51} \end{aligned}}$$

b) ?