Sabancı University
Faculty of Engineering and Natural Sciences

CS301 – Algorithms

Homework 4

Due: January 23, 2021 @ 23:55

Notes

- No homework will be accepted if it is not submitted using SUCourse.
- Write your solutions on a piece of paper and then upload your solutions to SUCourse.
- The files you upload must be the scanned form of your papers (not photographs). You can use scanner apps available for phones.
- LATE SUBMISSION POLICY:
  Late submission is allowed subject to the following conditions:
  - Your homework grade will be decided by multiplying what your normal grade (i.e. what you get from your answers) by a "submission time factor (STF)".
  - If you submit on time (i.e. before the deadline), your STF is 1. So, you don't lose anything.
  - If you submit late, you will lose 0.01 of your STF for every 5 mins of delay.
  - We will not accept any homework later than 500 mins after the deadline.
  - SUCourse's timestamp will be used for STF computation.
  - If you submit multiple times, the last submission time will be used.

Consider the following directed graph $G = (V, E)$, where

- $V = \{1, 2, 3, 4, 5\}$
- $E = \{(1, 2), (2, 5), (3, 2), (3, 4), (4, 1), (4, 5), (5, 1), (5, 3)\}$

Suppose that we are given the following weight function $w : E \to R$ for the edges:

$w(1, 2) = -5$
$w(2, 5) = 8$
$w(3, 2) = 1$
$w(3, 4) = -4$
$w(4, 1) = 7$
$w(4, 5) = 6$
$w(5, 1) = 0$
$w(5, 3) = 2$

For all pairs shortest path problem, if we would like to apply Dijkstra's algorithm, we have to reweight the edges, as suggested by Johnson's algorithm. As you know, Johnson's algorithm is based on first finding some suitable weights for the vertices of the graph. Let us use x_1, x_2, x_3, x_4, x_5 as the weight of the nodes $1, 2, 3, 4, 5$ respectively. After we find a suitable weight for each node, we will define a new weight function $\hat{w} : E \to R$ for the edges, such that all edges will have non–negative weights when we use

$$\hat{w}(i,j) = w(i,j) - x_i + x_j \quad \forall (i,j) \in E \tag{1}$$

(a) For each edge $(i,j) \in E$, write down the constraint on the difference of x_i and x_j based on the way $\hat{w}$ is defined in Equation (1) above.

$$\begin{array}{lcccc}
\text{for } (1,2) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots \\
\text{for } (2,5) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots \\
\text{for } (3,2) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots \\
\text{for } (3,4) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots \\
\text{for } (4,1) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots \\
\text{for } (4,5) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots \\
\text{for } (5,1) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots \\
\text{for } (5,3) \in E: & x_{\dots} & - & x_{\dots} & \leq & \dots
\end{array}$$

(b) The constraints you have for part (a) is a system of difference constraints. Give a solution for this system constraints which makes sure $\hat{w}(i,j) \geq 0$, for all edges $(i,j) \in E$.

$x_1 = \dots$
$x_2 = \dots$
$x_3 = \dots$
$x_4 = \dots$
$x_5 = \dots$

(c) For each $(i,j) \in E$, give $\hat{w}(i,j)$.

$$\begin{array}{lccc}
\text{for } (1,2) \in E: & \hat{w}(1,2) & = & \dots \\
\text{for } (2,5) \in E: & \hat{w}(2,5) & = & \dots \\
\text{for } (3,2) \in E: & \hat{w}(3,2) & = & \dots \\
\text{for } (3,4) \in E: & \hat{w}(3,4) & = & \dots \\
\text{for } (4,1) \in E: & \hat{w}(4,1) & = & \dots \\
\text{for } (4,5) \in E: & \hat{w}(4,5) & = & \dots \\
\text{for } (5,1) \in E: & \hat{w}(5,1) & = & \dots \\
\text{for } (5,3) \in E: & \hat{w}(5,3) & = & \dots
\end{array}$$