Homework 3

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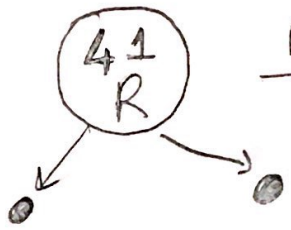
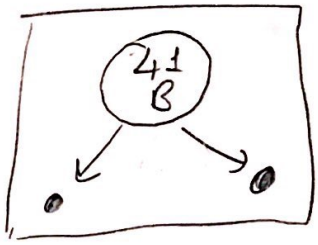
25331

1) B: "Black Node"
R: "Red Node"

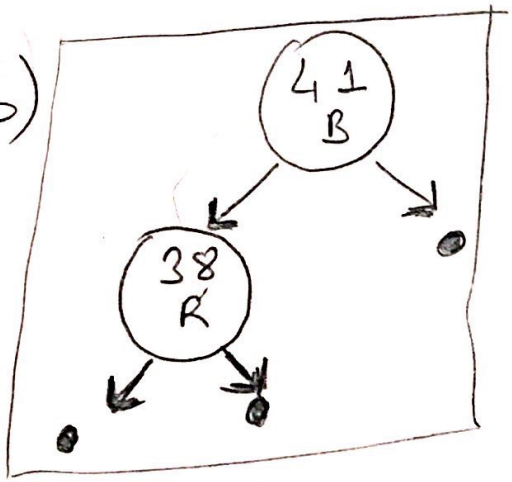
a)



RBT property violated
"root must be black"
color it

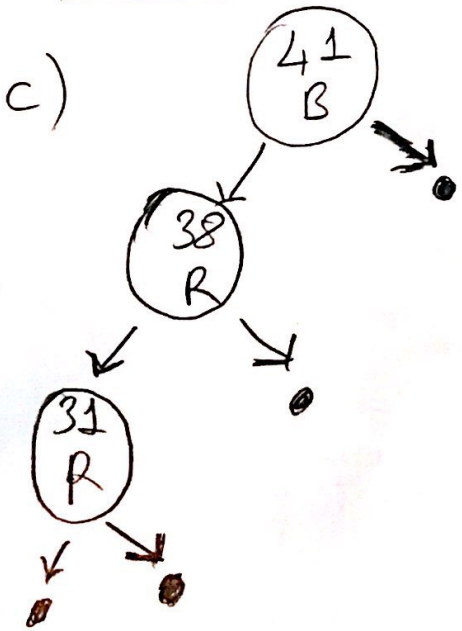


b)

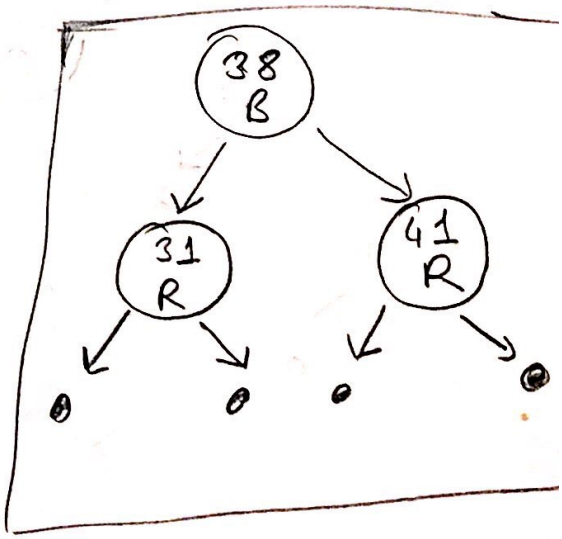


all RBT properties hold ✓

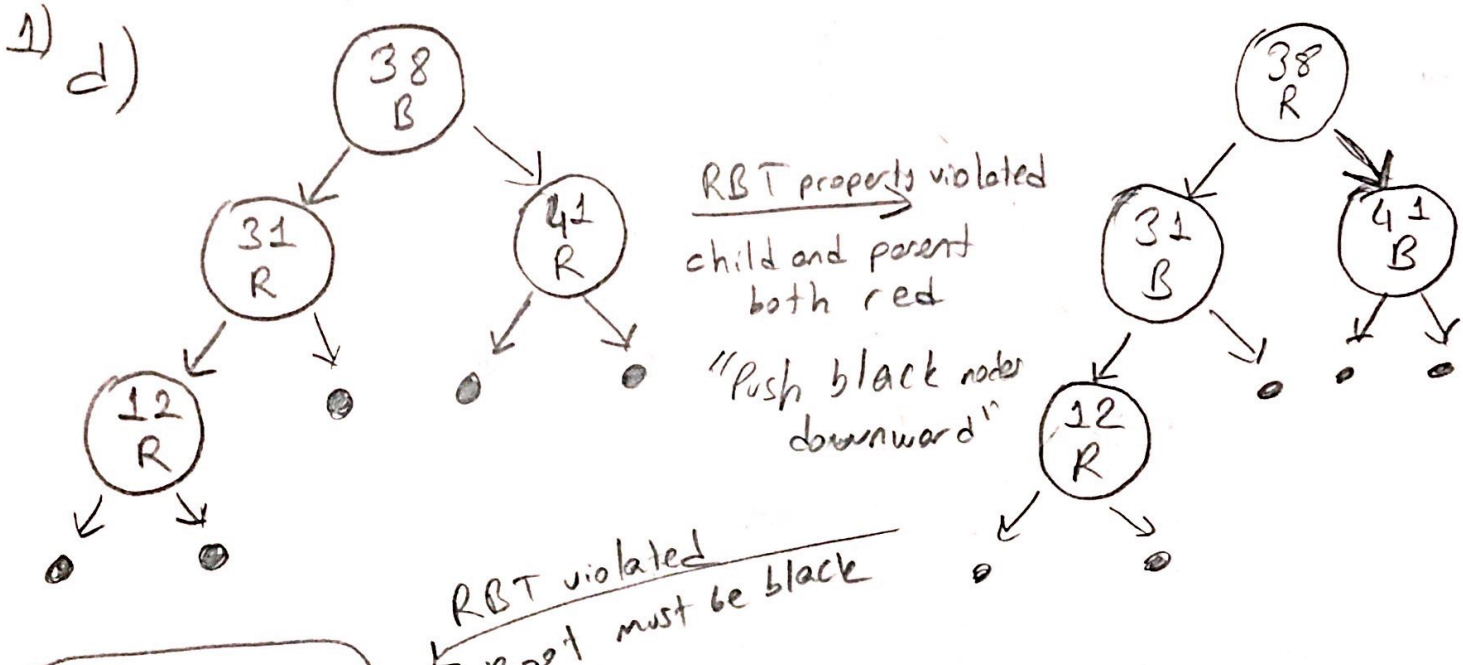
c)

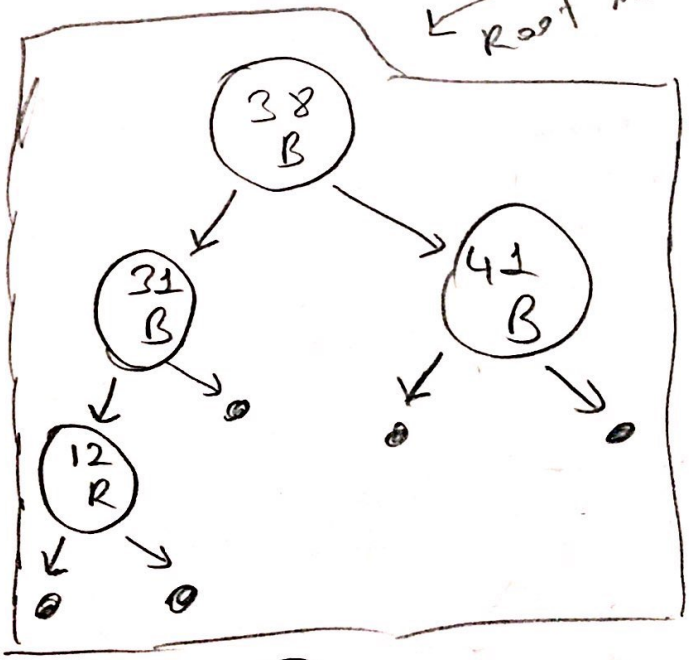


RBT property violated
Child and parent both red.
"Right Rotate
Fix Coloring
38 → Black
41 → Red

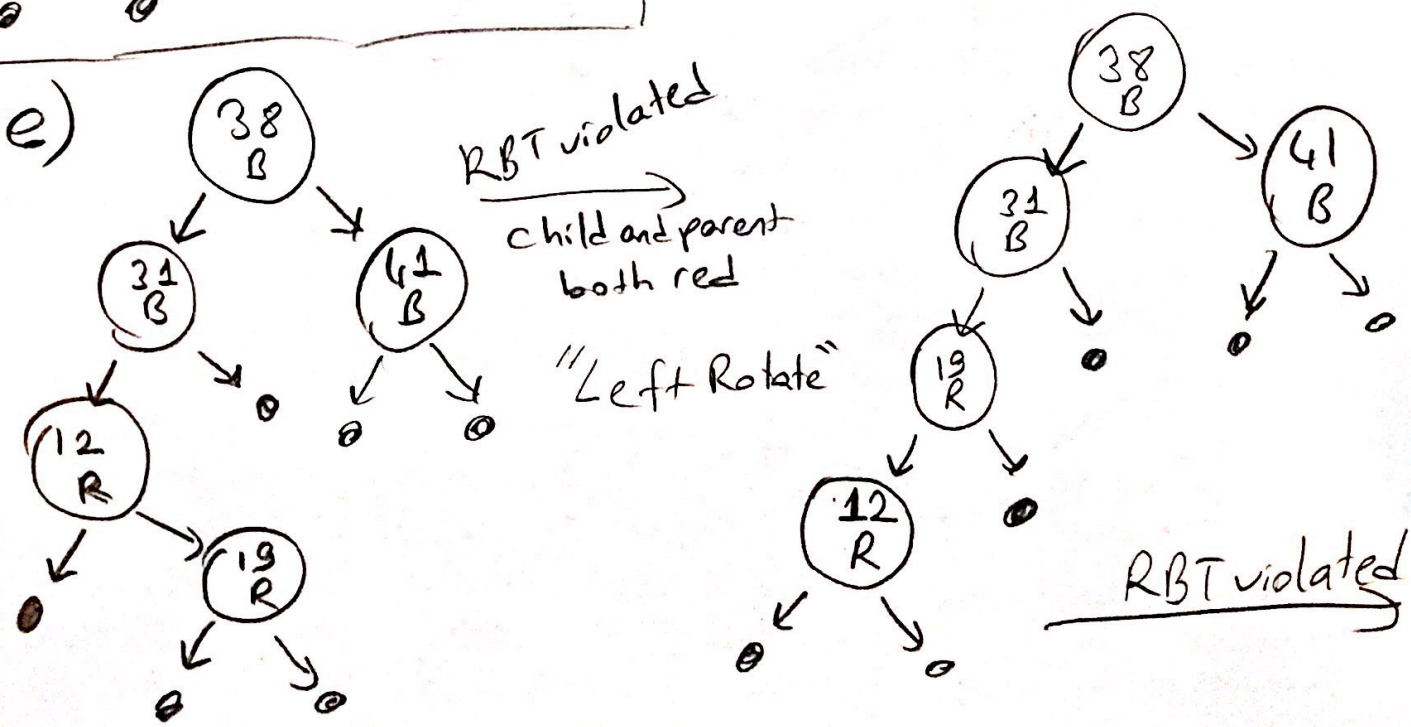


1) d)





e)

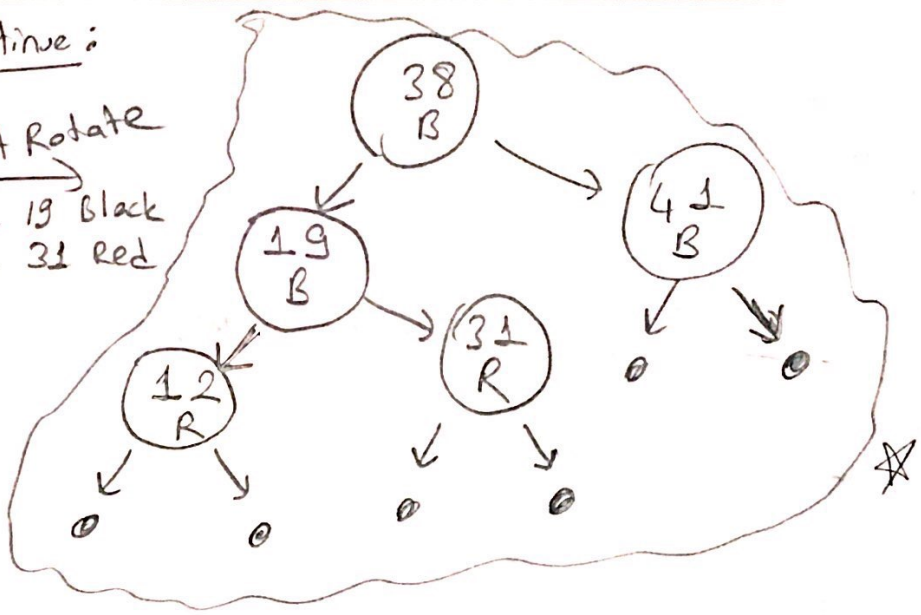


e) continue:

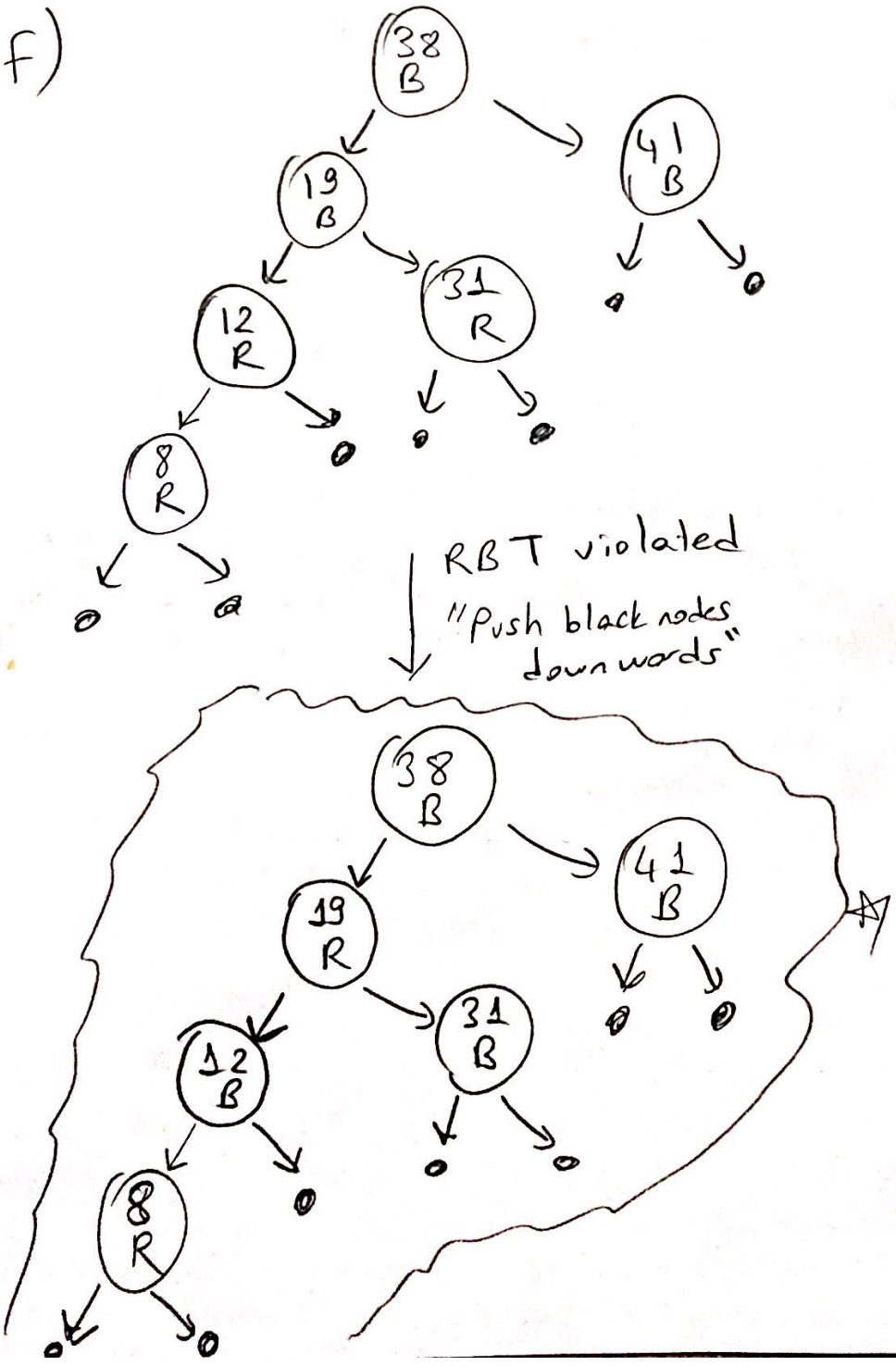
Right Rotate

make 19 Black

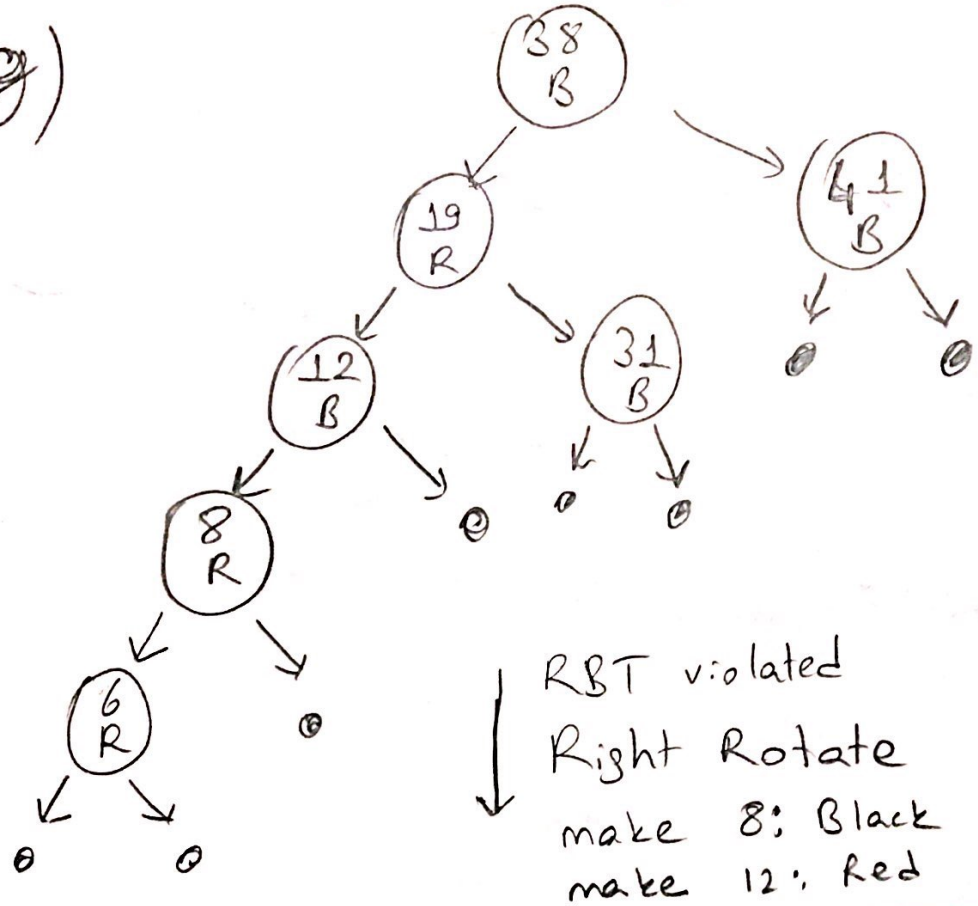
make 31 Red



f)



g)

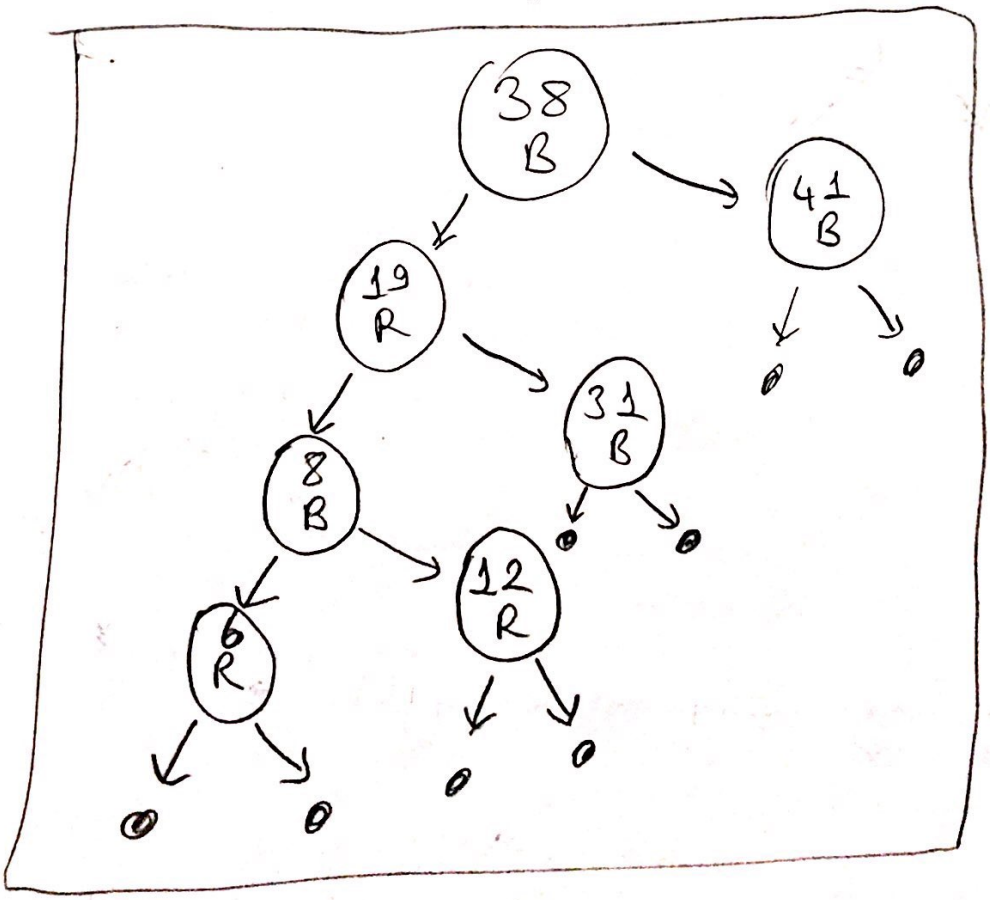


RBT violated

Right Rotate

make 8: Black

make 12: Red



2) a) **True**

We know that we have **maximum 2 rotations** in the worst case which is case 2 in our lecture notes.
We will transform case 2 into case 3 with 1 rotation.
Then, we will solve case 3 with + 1 rotation.

2 rotation

b) **False**

If we ended up with case 1 where we push black nodes downwards, if the re-coloring continues until root we may re-coloring as much as the height of tree.
Thus, it is $O(\log n)$ in the worst case.

c) **False**, we may have equal number of in both left and right subtrees of root in worst.

Then, $T(n) = 2T(\frac{n}{2}) + 1$

$a \geq 1$, $b > 1$, $f(n) = 1 = c$ asymptotically positive.

Apply Master Theorem

Case 1 applies, Then **$T(n) = \Theta(n)$**