1901022038
Selen Erdoğan
2a.

a) Testere dalga için fonksiyon

$$x(t) = \frac{A}{T} \cdot t \quad \begin{pmatrix} A \text{ genliktir} \\ T \text{ ise periyottur} \end{pmatrix}$$

$$x(t) = \frac{A}{T} \cdot t \Rightarrow \frac{2}{2} \cdot t \Rightarrow \boxed{x(t) = t}$$

x(t)'nin tek mi çift mi fonk. olduğuna karar vermemiz lazım.

$x(t) = -x(-t) = t$
Tek fonksiyon

$$\frac{x(t) - x(-t)}{2} = \frac{t+t}{2} = t$$

→ x(t) fonksiyonu tek fonksiyon ise $\boxed{a_k = 0}$ $k \neq 0$

$$a_0 = \frac{1}{T} \int_0^T x(t)dt \longrightarrow \frac{1}{2} \int_0^2 t\,dt \rightarrow \frac{1}{2}\left(\frac{t^2}{2} \Big|_0^2\right) = \frac{1}{2}\left(\frac{4}{2} - 0\right)$$

$$\frac{1}{2} \cdot 2 = 1$$

$$\boxed{a_0 = 1}$$

$w = \frac{2\pi}{T} = \pi \quad a_k = 0$ ise b_k:

$$b_k = \frac{2}{T} \int_0^T x(t) \cdot \sin(kwt)\,dt \Rightarrow \frac{2}{2} \int_0^2 t \sin\left(\frac{k2\pi t}{T}\right) dt$$

$$b_k = \left[t \cdot \left(\frac{-T}{2k\pi} \cdot \cos\left(\frac{k2\pi t}{T}\right)\right)\Big|_0^2\right] - \int_0^2 \frac{T}{2k\pi} \cdot \cos\left(\frac{2\pi k t}{T}\right) dt$$

$$b_k = \left[2 \cdot \left(\frac{-2}{2k\pi} \cdot \cos\left(\frac{4\pi k}{2}\right)\right)\right] - \left[\frac{T}{2\pi k} \cdot \int_0^2 \cos\left(\frac{2\pi k t}{T}\right) dt\right]$$

$$b_k = \left[\frac{-2}{k\pi} \cdot \cos(2\pi k)\right] - \left[\frac{T}{2\pi k} \cdot \left(\frac{T}{2\pi k} \cdot \sin\left(\frac{2\pi k t}{T}\right) \Big|_0^2\right)\right.$$

$$b_k = \left(\frac{-2 \cdot \cos(2\pi k)}{k\pi}\right) = \left(\cos(2\pi k) = 1\right)$$

$$b_k = \frac{-2}{k\pi}$$

Elde edilen sonuçlara göre

$$a_0 = 1, \quad b_k = \frac{-2}{k\pi}, \quad a_k = 0$$

1

$\underline{b}$

$$x(t) = a_0 + \sum_{k=1}^{\infty} \left[a_k \cdot \cos(k w t) + b_k \cdot \sin(k w t) \right]$$

1901022038
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2Ö.

Fourier Serisi açılımı

↳ Fourier katsayıları biliniyor ise

$a_0 = 1 \quad b_k = -\frac{2}{k\pi} \quad a_k = 0$ ise $\quad w = \frac{2\pi}{T} = \pi$

$$x(t) = 1 + \sum_{k=1}^{\infty} \left[0 \cdot \cos(k\pi t) + \frac{2}{k\pi} \cdot \sin(k\pi t) \right]$$

$$\boxed{x(t) = 1 + \sum_{k=1}^{\infty} -\frac{2}{k\pi} \cdot \sin(k\pi t)}$$

$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ 'dan yola çıkılırsa

$$\cos\left(k\pi t + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2} - k\pi t - \frac{\pi}{2}\right) \Rightarrow \cos\left(k\pi t + \frac{\pi}{2}\right)$$

$$= \sin(-k\pi t)$$

$\sin(k\pi t) = -\cos\left(k\pi t + \frac{\pi}{2}\right)$

sinüs fonksiyonu tek fonksiyon olduğu için içerideki eksi dışarı çıkar.

Açılım şeklinde yazılırsa:

$$x(t) = 1 + \sum_{k=1}^{\infty} \frac{-2}{k\pi} \cdot \sin(k\pi t) = 1 + \sum_{k=1}^{\infty} \frac{2}{k\pi} \cdot \cos\left(k\pi t + \frac{\pi}{2}\right)$$

1901022038
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2Ö.

© $h(t) = e^{-t} \cdot u(t)$

$$H(jw) = \int_0^{\infty} e^{-\tau} \cdot e^{-jw\tau} d\tau$$

$$= -\frac{1}{1+jw} \cdot e^{-\tau} \cdot e^{-jw\tau} \Big|_0^{\infty} = \frac{1}{1+jw} = H(jw)$$

→ b_k, $x(t)$ sinyalinin fourier katsayıları ise c_k'da $y(t)$ sinyalinin katsayıları olsun. $w = (\pi)$ ise

$$c_k = b_k \cdot H(jk\pi) \longrightarrow y(t) = \sum_{k=1}^{\infty} c_k \cdot e^{-jk\pi t} \qquad \underline{\underline{a_0 = 1}}$$

$$b_k = \frac{2}{k\pi} \rightarrow b_1 = \frac{2}{\pi}, \quad b_2 = \frac{1}{\pi}, \quad b_3 = \frac{2}{3\pi}, \quad b_4 = \frac{1}{2\pi}, \quad b_5 = \frac{2}{5\pi}$$

Sırayla açarsak

$$c_k = b_k \cdot H(jk\pi) \longrightarrow c_k = \frac{2}{k\pi} \cdot \frac{1}{1+j\pi}$$

$$c_1 = \frac{2}{\pi} \cdot \frac{1}{1+j\pi} = \frac{2}{\pi + j\pi^2}$$

$$c_2 = \frac{1}{\pi} \cdot \frac{1}{1+j\pi} \Rightarrow \frac{1}{\pi + j\pi^2}$$

$$c_3 = \frac{2}{3\pi} \cdot \frac{1}{1+j\pi} \Rightarrow \frac{2}{3\pi + 3j\pi^2}$$

$$c_4 = \frac{1}{2\pi} \cdot \frac{1}{1+j\pi} \Rightarrow \frac{1}{2\pi + 2j\pi^2}$$

$$c_5 = \frac{2}{5\pi} \cdot \frac{1}{1+j\pi} \Rightarrow \frac{2}{5\pi + 5j\pi^2}$$

$$c_6 = \frac{1}{3\pi} \cdot \frac{1}{1+j\pi} \Rightarrow \frac{1}{3\pi + 3j\pi^2}$$

$$\rightarrow y(t) = \sum_{k=1}^{\infty} c_k \cdot e^{jk\pi t}$$

$$c_k = \frac{2}{k\pi + kj\pi^2}$$

$$y(t) = a_0 + \sum_{k=1}^{\infty} \frac{2}{k\pi + kj\pi^2} \cdot e^{jk\pi t}$$

$$= 1 + 2 \cdot \sum_{k=1}^{\infty} \frac{e^{jk\pi t}}{k\pi + kj\pi^2} = y(t) \text{ dır}$$

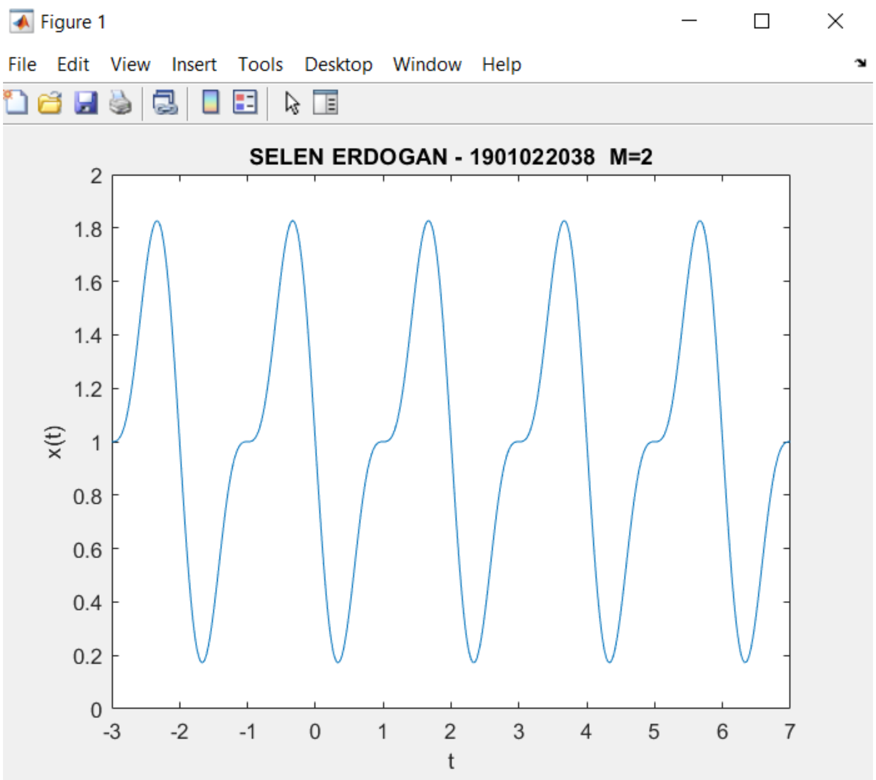
Figure 1
File Edit View Insert Tools Desktop Window Help
SELEN ERDOGAN - 1901022038 M=2
x(t)
t

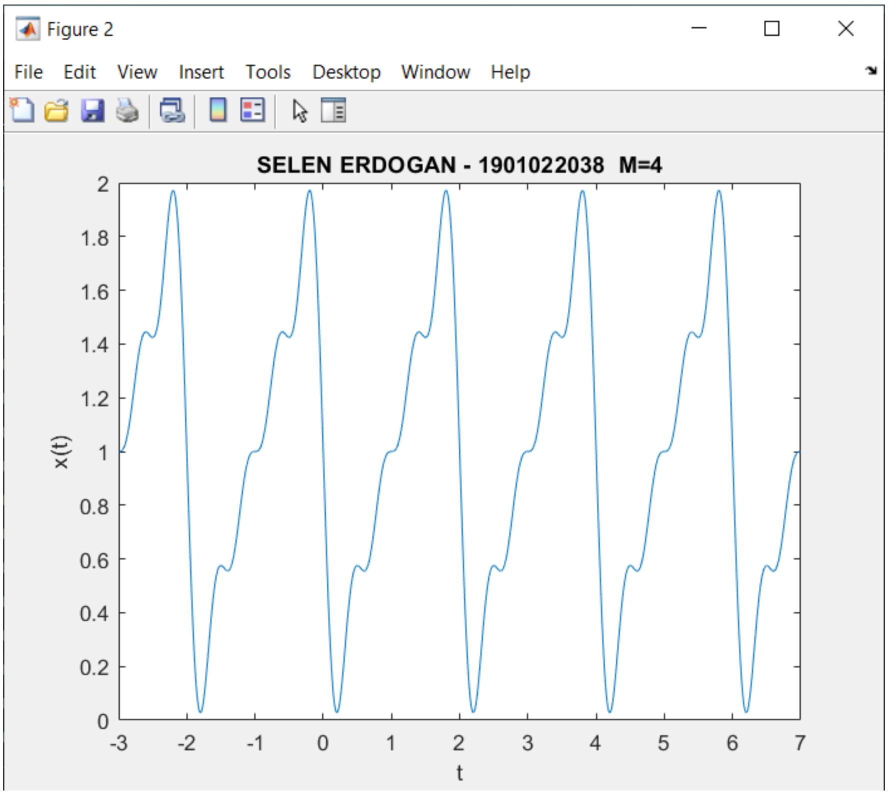
Figure 2
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SELEN ERDOGAN - 1901022038 M=4
x(t)
t

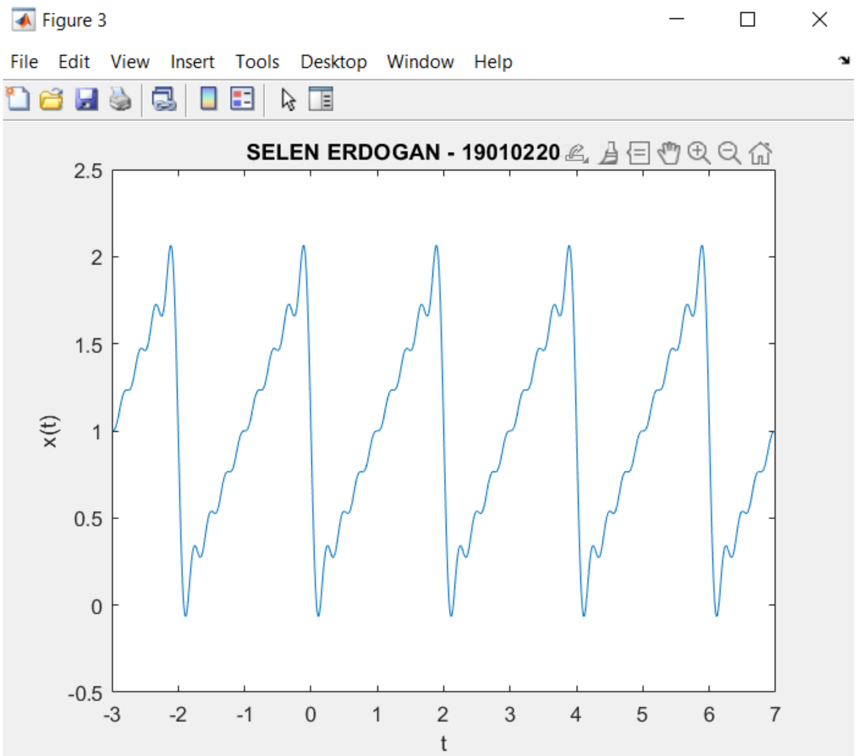
Figure 3
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SELEN ERDOGAN - 19010220
x(t)
t
2.5
2
1.5
1
0.5
0
-0.5
-3 -2 -1 0 1 2 3 4 5 6 7

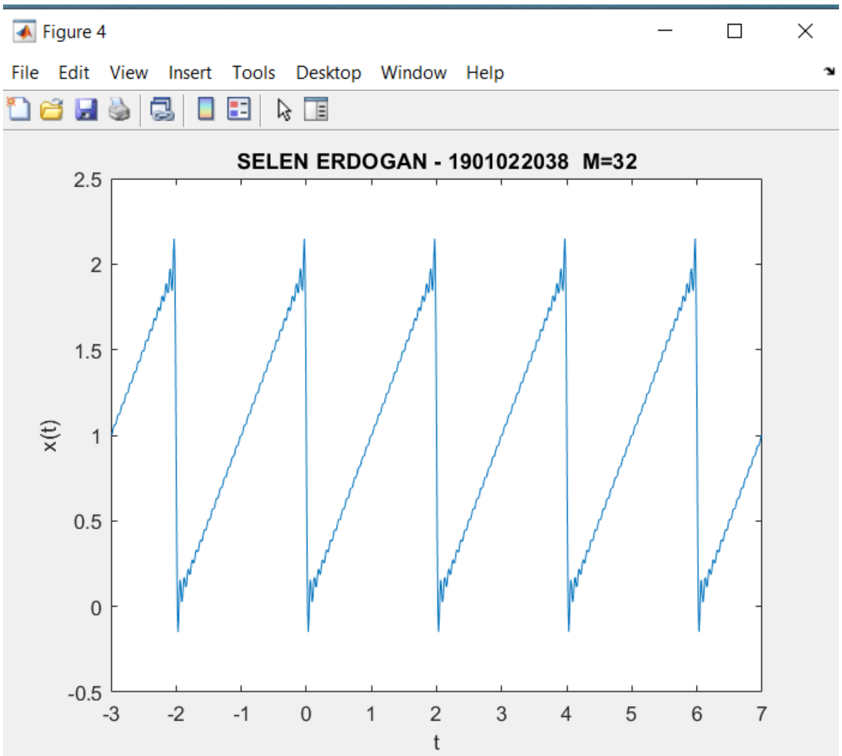
Figure 4
File Edit View Insert Tools Desktop Window Help
SELEN ERDOGAN - 1901022038 M=32
x(t)
t
2.5
2
1.5
1
0.5
0
-0.5
-3 -2 -1 0 1 2 3 4 5 6 7

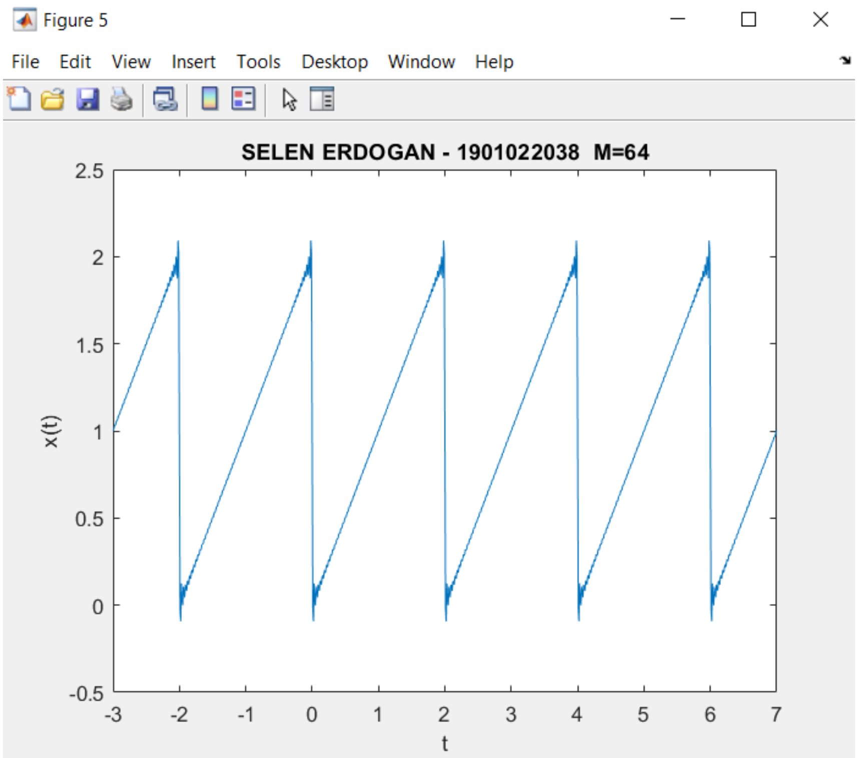
Figure 5
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SELEN ERDOGAN - 1901022038 M=64
x(t)
t

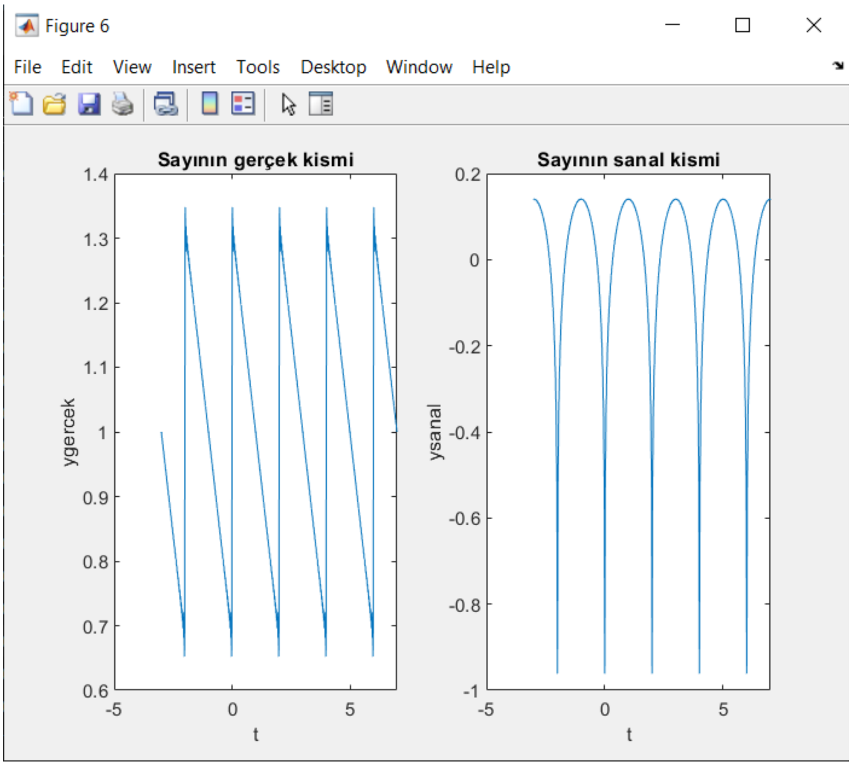
Figure 6
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Sayının gerçek kismi
ygercek
t
Sayının sanal kismi
ysanal
t