Selen ERDOĞAN
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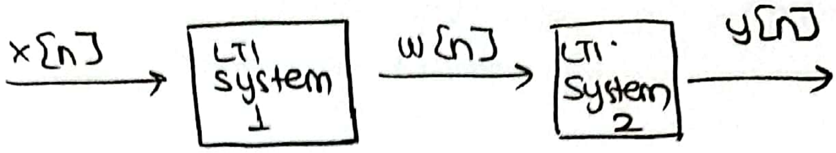
Ulaş.

2.37 Consider the cascade of LTI discrete time systems as shown



The first system described by:

$$H1(e^{jw}) = e^{-jw} \begin{cases} 0, & |w| \leq 0{,}25\pi \\ 1, & 0{,}25\pi \leq |w| \leq \pi \end{cases}$$

and the second system is described by:

$$h2[n] = \frac{2 \sin(0.5\pi n)}{\pi n}$$

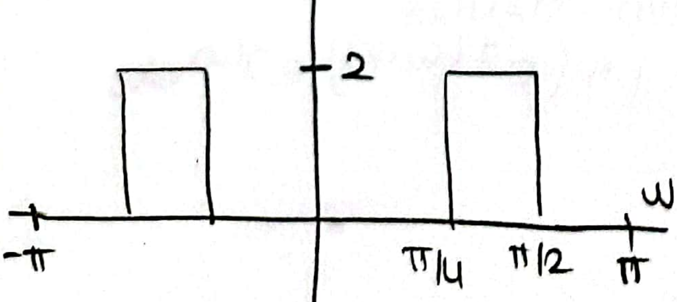
A Determine an equation that defines the frequency response. $H(e^{jw})$, of the overall system over the range $-\pi \leq w \leq \pi$

$$h2[n] = \frac{2 \sin(0.5\pi n)}{\pi n} \Rightarrow H2(e^{jw}) = \begin{cases} 2 & w < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} < w \leq \pi \end{cases}$$

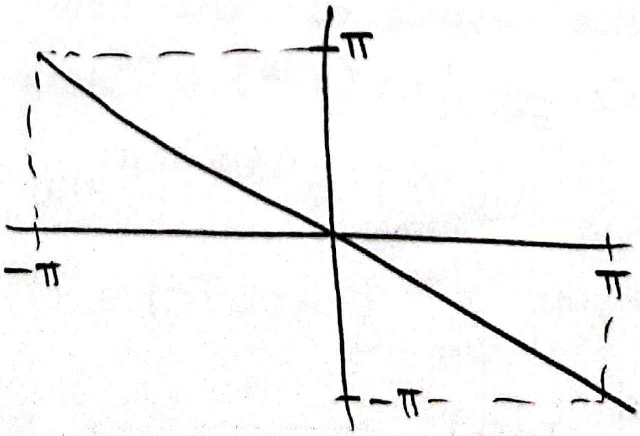
$$H(e^{jw}) = H1(e^{jw}) * H2(e^{jw}) \Rightarrow H(e^{jw}) = e^{-jw} \begin{cases} 0, & |w| < \pi/4 \\ 2, & \pi/4 \leq |w| < \pi/2 \\ 0, & \pi/2 < |w| \leq \pi \end{cases}$$

B Sketch the magnitude $|H(e^{jw})|$ and the phase $\angle H(e^{jw})$ of the overall frequency response over the range $-\pi \leq w \leq \pi$

$|H(e^{jw})|$



$\angle H(e^{jw})$



Selen ERDOĞAN
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2d.

c. Use any convenient means to determine the impulse response $h[n]$ of the overall cascade system.

$$h[n] = 2\frac{\sin\left(\frac{\pi}{8}(n-1)\right)}{\pi(n-1)} - 2\frac{\sin\left(\frac{\pi}{4}(n-1)\right)}{\pi(n-1)}$$

$\Rightarrow$ expressed as difference of 2 lowpass filters with one sample delay

2.58 An LTI system is described by input output relation

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

(a) Plot the magnitude and phase of the frequency response. This part is done in python and given in next page.

(e) Now consider a new system whose frequency response is $H_1(e^{jw}) = H(e^{j(w+\pi)})$ determine $h_1[n]$ the impulse response of of the new system.

old system $h[n]$ =

$$y[n] = x[n] + 2x[n-1] + x[n-2] = x[n].h[n]$$

$$= x[n].(\delta[n] + 2\delta[n-1] + \delta[n-2])$$

$$h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

impulse response of the new system $h_1[n]$ =

$$h_1[n] = \frac{1}{2\pi}\int H_1(e^{jw}).e^{jw}dw = \frac{1}{2\pi}\int H(e^{j(w+\pi)})e^{jwn}dw$$

$$= \frac{1}{2\pi}\int H(e^{jw}).e^{j(w-\pi)n}dw$$

$$= e^{-jwn}\frac{1}{2\pi}\int H(e^{jw})e^{jwn}dw$$

$$= -1^n . h[n] \longrightarrow \delta[n] - 2\delta[n-1] + \delta[n-2]$$

1901022038-ODEV4

March 26, 2023

1 Selen Erdoğan - 1901022038

1.1 An LTI system is described by the input-output relation

1.1.1 y[n] = x[n] + 2x[n1] + x[n - 2].

1.1.2 (d) Plot the magnitude and phase of the frequency response

impulse response found h[n] = [n] + 2 [n-1] + [n-2]

The frequency response of an LTI system is given by the Discrete Fourier Transform (DFT) of the impulse response, which is the Fourier transform of the system's unit impulse response.

Let's start by finding the Fourier transform of h[n]:

H(e^(j)) = h[n] e^(-j n)

= ([n] + 2 [n-1] + [n-2]) e^(-j n)

= e^(-j 0) + 2e^(-j 1) + e^(-j 2)

where 0 = 0, 1 = , and 2 = 2 .

H(e^(j)) = e^(-j) (1 + 2e^(-j) + e^(-2j))

= e^(-j) [(e$^{(j\ )+1)}$2]/(e$^{(j\ ))}$2

= (e$^{(-j\ )+1)}$2/e^(j)

= (cos( )-j sin( )+1)$^{2/e}$(j)

= [(cos( )+1)-j sin( )]$^{2/e}$(j)

Magnitude = |H(e^(j))| = |(cos( )+1)-j sin( )|$^{2/|e}$(j)|

= |cos( )+1-j sin( )|^2

$= (\cos(\ )+1)^{2+\sin}2(\ )$

$= 2(\cos(\ )+1)$

Phase = $H(e\hat{}(j\ )) = -\arctan[Im(H(e^{(j\ )))/Re(H(e}(j\ )))]$

$= -\arctan[-\sin(\ )/(\cos(\ )+1)]$

$= -$

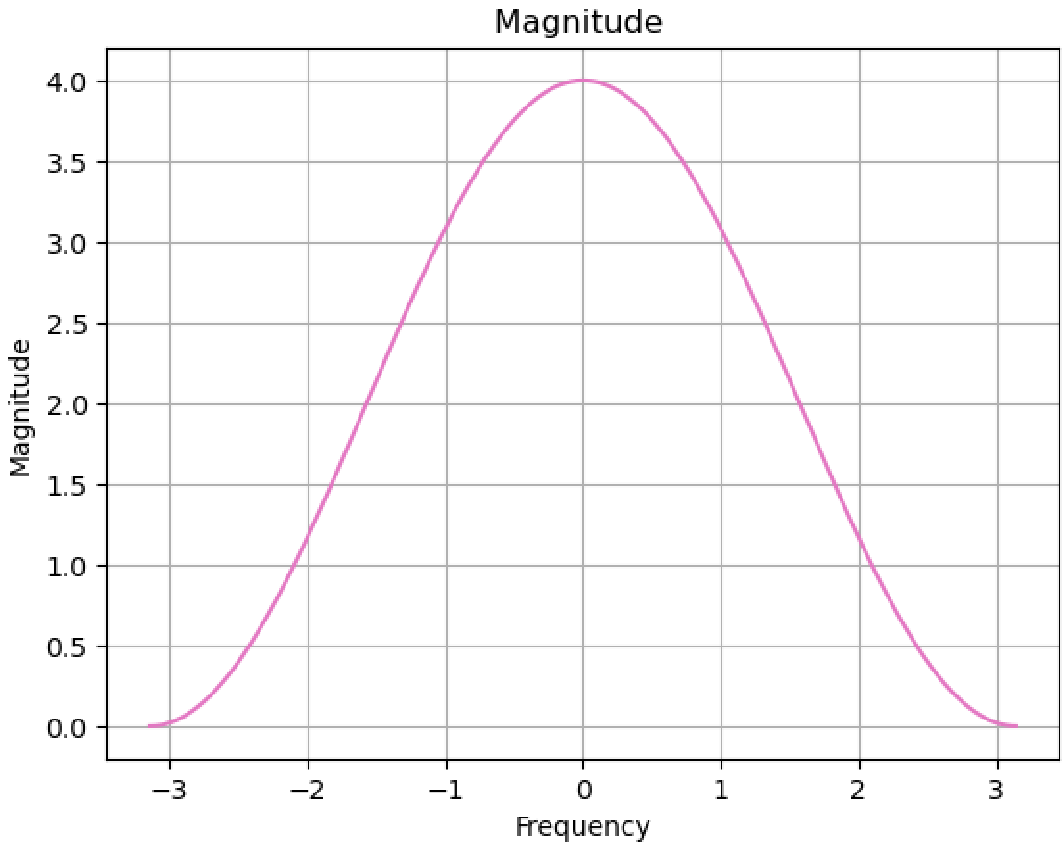
1.1.3 Therefore, we have:

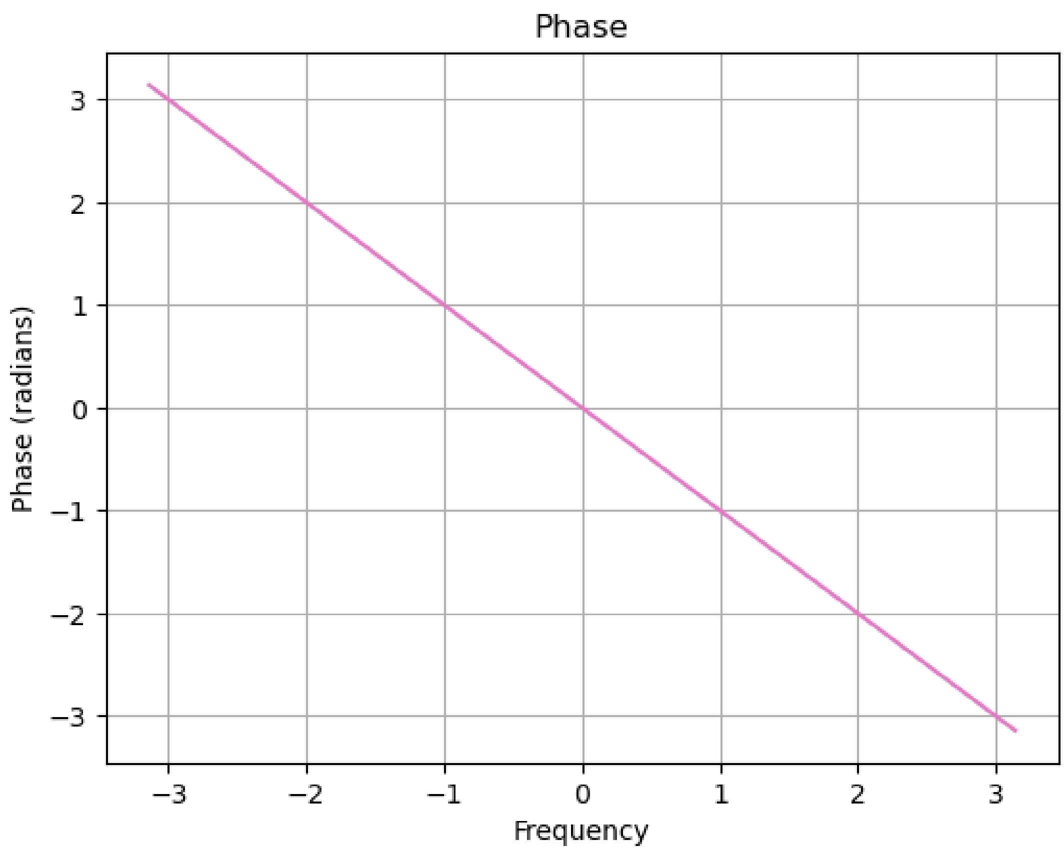
1.1.4 $|H(e\hat{}(j\ ))| = 2(\cos(\ )+1)$

1.1.5 $H(e\hat{}(j\ ))= -$ which is the desired result.

[22]:

```
import numpy as np
import matplotlib.pyplot as plt


def H(w):
    return 2*(np.cos(w) + 1)

w = np.linspace(-np.pi, np.pi, 1000)

H_mag = np.abs(H(w))
H_phase = -w

plt.figure()
plt.plot(w, H_mag, 'C6-', label='Magnitude')
plt.xlabel('Frequency')
plt.ylabel('Magnitude')
plt.title('Magnitude ')
plt.grid()

plt.figure()
plt.plot(w, H_phase, 'C6-')
plt.xlabel('Frequency')
plt.ylabel('Phase (radians)')
plt.title('Phase')
plt.grid()

plt.show()
```


Magnitude
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Magnitude
−3
−2
−1
0
1
2
3
Frequency


Phase
Phase (radians)
Frequency
3
2
1
0
−1
−2
−3