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Question 3.7 \Rightarrow Solution of (a)

$$\Rightarrow x[n] = u[-n-1] + \left(\frac{1}{2}\right)^n u[n]$$

$$\Rightarrow x(z) = \frac{-1}{1-z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} \quad \frac{1}{2} < |z| < 1$$

$$\Rightarrow H(z) = \frac{y(z)}{x(z)}$$

$$\Rightarrow \frac{y(z)}{x(z)} = \frac{\frac{-1}{2} z^{-1}}{(1-\frac{1}{2}z^{-1})(1+z^{-1})} \cdot \frac{(1-z^{-1})(1-\frac{1}{2}z^{-1})}{-\frac{1}{2} \cdot z^{-1}}$$

$$\Rightarrow \frac{1-z^{-1}}{1+z^{-1}}$$

$\rightarrow H(z)$ causal for $\text{ROC } |z| > 1$

Solution of (b)

$x(z)$ is constrained by poles that are less than 1 and one of these poles cancels out the zero of $H(z)$, which limits the region of ROC for $x(z)$ to a certain area. Therefore, the ROC of $y(z)$ satisfies the conditions of $|z| > 1/2$ and $|z| > 1$ on the z plane, in addition to other two constraints. As a result, $y(z)$ converges for $|z| > 1$. In other words, the ROC of $x(z)$ is limited by one of the poles of $x(z)$ that is less than 1, canceling out the zero of $H(z)$ in a specific region. The ROC of $y(z)$ satisfies the conditions of $|z| > 1/2$ and $|z| > 1$. Result, $y(z)$ converges for $|z| > 1$.

Solution of (c)

$$y(z) = \frac{-\frac{1}{3}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{1}{3}}{1+z^{-1}} \quad |z| > 1$$

$$y[n] = -\frac{1}{3} \cdot \left(\frac{1}{2}\right)^n \cdot u[n] + \frac{1}{3} (-1)^n \cdot u[n]$$

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3.17 Problem

$$y(z) \left(1 - \frac{5}{2}z^{-1} + z^{-2}\right) = x(z) \cdot (1 - z^{-1})$$

$$\hookrightarrow H(z) = \frac{y(z)}{x(z)} \longrightarrow \frac{1 - z^{-1}}{(1 - 2z^{-1}) \cdot (1 - \frac{1}{2}z^{-1})}$$

$$\hookrightarrow \frac{\frac{2}{3}}{1 - 2z^{-1}} + \frac{\frac{1}{3}}{1 - \frac{1}{2}z^{-1}}$$

$$\Rightarrow |z| < \frac{1}{2}$$

solution of (a)

$$h[n] = -\frac{2}{3} 2^n u[n-1] - \frac{1}{3} \left(\frac{1}{2}\right)^n u[-n-1]$$

$$h[0] = 0$$

$$\Rightarrow \frac{1}{2} < |z| < 2$$

solution of (b)

$$h[n] = -\frac{2}{3} 2^n u[n-1] + \frac{1}{3} \left(\frac{1}{2}\right)^n u[n]$$

$$h[0] = \frac{1}{3}$$

$$\Rightarrow |z| > 2$$

solution of (c)

$$h[n] = \frac{2}{3} 2^n u[n] + \frac{1}{3} \left(\frac{1}{2}\right)^n u[n]$$

$$h[0] = 1$$

$$\Rightarrow |z| > 2 \parallel |z| < \frac{1}{2}$$

solution of (d)

$$h[n] = \frac{2}{3} 2^n u[n] - \frac{1}{3} \left(\frac{1}{2}\right)^n u[n-1]$$

$$h[0] = \frac{2}{3}$$

Problem 3.45 (c)

The difference equation that characterizes the system

$$y[n] - \frac{3}{4} y[n-1] = x[n] - 2x[n-1]$$