Muon Flat Earth Project

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Theory

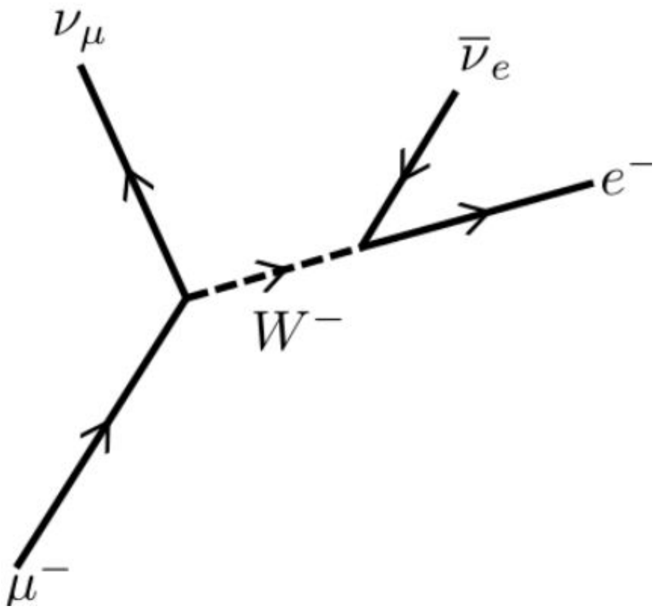
Bei Lu

What is Muon?

Muon decay

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

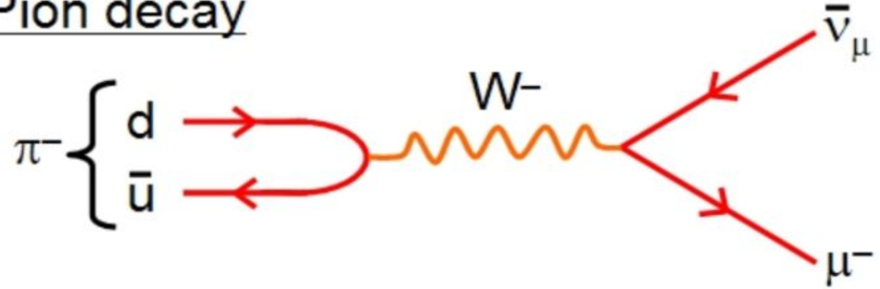


Cosmic Ray $\longrightarrow$

Kaon K^{+} K^{-} K^{0}

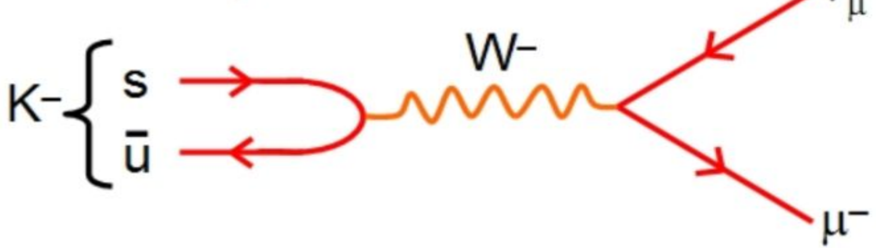
Pion π^{+} π^{-} π^{0}

Pion decay



$$\pi^-(d\bar{u}) \rightarrow \mu^- + \bar{\nu}_\mu$$

Kaon decay



$$K^-(s\bar{u}) \rightarrow \mu^- + \bar{\nu}_\mu$$

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$t' = change\ in\ \ time$ In our rest frame

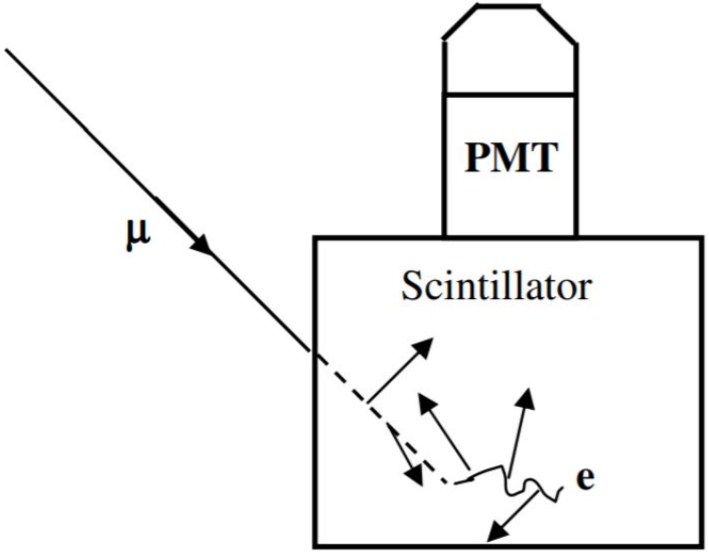
$t = rest\ \ time$ (time interval measured in Muon's proper frame)

$v = velocity$ Of the muon

$c = speed\ \ of\ \ light$ = 299 792 458 m/s

How Is Muon Detected ?

The Detector



Equations:

- The energy distribution of primary cosmic rays follow power law

$$I(E, \theta = 0) = I_0 N (E_0 + E)^{-n}$$

$$I(E, \theta) = I_0 N (E_0 + E_\theta + E)^{-n}$$

The total muon flux (integrated over all energy) at angle θ is:

$$\Phi(\theta) = \int_0^{\infty} I(E,\theta)\, dE = I_0\,(E_0 + E_\theta + E)^{-n}$$

The ratio of vertical flux to flux at angle θ is:

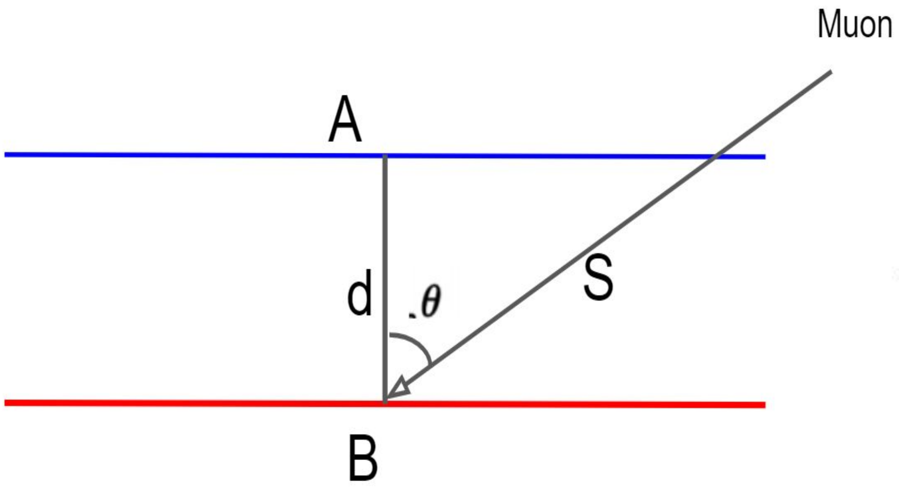
$$\frac{\Phi(\theta)}{\Phi(\theta=0)} = \frac{\int_0^{\infty} (E_0 + E_\theta + E)^{-n}\, dE}{\int_0^{\infty} (E_0 + E)^{-n}\, dE} = \left(\frac{E_0 + E_\theta}{E_0}\right)^{-(n-1)}$$

- Energy loss of Muon is proportional to the distance traveled

$$D(\theta) = \frac{\text{Distance traveled at vertical direction}}{\text{Distance traveled at angle}}$$

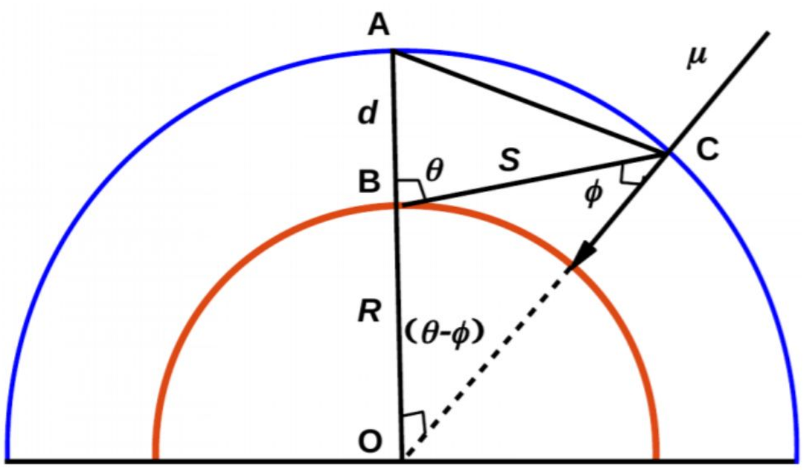
$$= \frac{E_0 + E_\theta}{E_0}$$

For a flat Earth:



$$D(\theta) = \frac{S}{d} = \cos^{-1}\theta$$

For a round Earth:



$$D(\theta) = \frac{S}{d}$$

$$= \sqrt{\left(\frac{R^2}{d^2}\cos^2\theta + 2\frac{R}{d} + 1\right)} - \frac{R}{d}\cos\theta.$$

- For a flat Earth:

$$\Phi(\theta) = I_0\, D(\theta)^{-(n-1)} = I_0 \cos^{n-1}\theta.$$

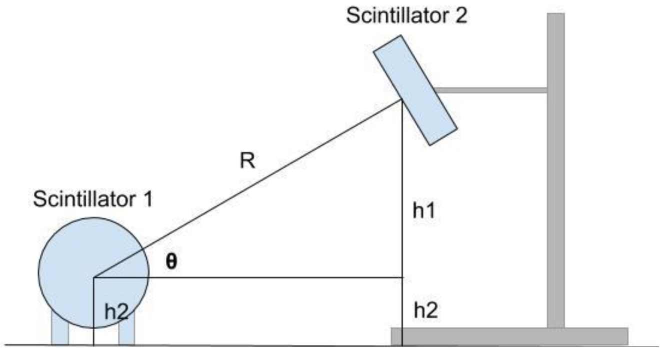
- For a round Earth:

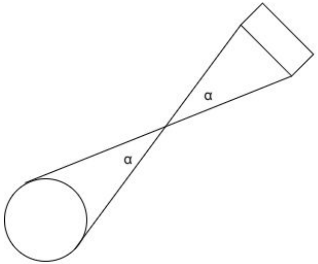
$$\Phi(\theta) = I_0\, D(\theta)^{-(n-1)} = I_0 \left(\sqrt{\left(\frac{R^2}{d^2}\cos^2\theta + 2\frac{R}{d} + 1\right)} - \frac{R}{d}\cos \right)^{-(n-1)}$$

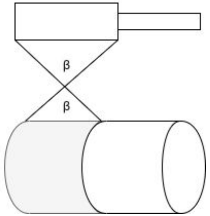
Materials and Methods

Adam Prieto

Materials



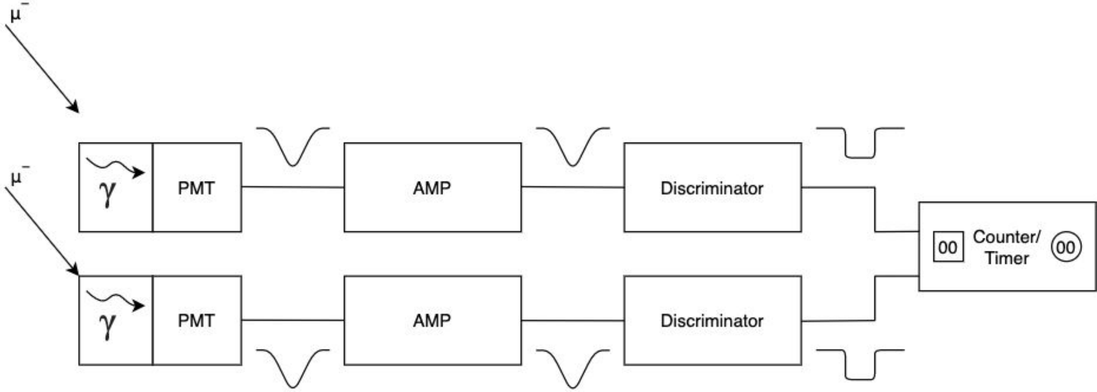


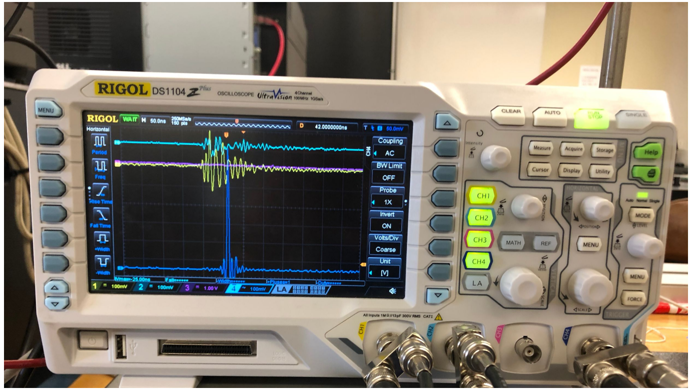


- Two scintillators (cylinder and rectangle)
    - Cylinder is shielded; rectangular was shielded with aluminum foil
- String to maintain a constant radius between the cylinder and rectangular scintillators
- PMTs in the scintillators go into the 5x amplifier to boost the signal
- Analog instruments(coincidence, discriminator) to coincide the discriminated voltages from the amplifier
- Counter/Timer to find the total counts for the coincident passing of muons
- Meter stick to measure a given height and used trigonometry to calculate the angles

Methods

- Measure the rate over different angles (10˚->85˚ in 15˚ increments)
- Minimizing uncertainty by calculating the rate of accidentals ($R_{acc}=R_1R_2\Delta t$) and lowering the threshold to 30mV for both discriminators





- Noise issues

Raw Data

- Weather: rainy and Sunny
- $R_{acc} = R_1 R_2 \Delta t$
  $R_{acc} = 1.5368 \times 10^{-5}$ Counts/s

Zenith Angle θ_z [°]	Counts [N]	Observation Time [s]	Flux [N m^{-2} s^{-1}]	δFlux
80	482	70,000	0.63	±0.03
65	652	70,000	0.85	±0.03
50	61	4,000	1.40	±0.2
35	85	4,000	1.95	±0.2
20	107	4,000	2.45	±0.2
5	172	4,000	3.94	±0.3

Results and Analysis

Evan Eastin

Relevant Equations

1) $$I_0\left(\sqrt{\left(\frac{R^2}{d^2}\right)\cos^2(\theta_z)+\frac{2R}{d}+1}-\left(\frac{R\cos(\theta_z)}{d}\right)\right)^{-n-1}$$

EQ 1: Muon flux distribution as a function of zenith angle as calculated for the round Earth theory. I_0 is the expected muon flux at zenith angle 0°, R is the radius of Earth in meters, d is the height above the surface of the Earth, in meters, at which muons enter the atmosphere, and n is the exponent number to be fit to the data.

2) $$I_0\cos^{n-1}(\theta_z)$$

EQ 2: Muon flux distribution as calculated for the flat Earth theory. Angle, I_0, and n are defined as above. The often quoted model for flux in a flat Earth theory results in a $\cos^2$ distribution.

Data

Zenith Angle θ_z [°]	Counts [N]	Observation Time [s]	Flux [N m^{-2} s^{-1}]	δFlux
80	482	70,000	0.63	±0.03
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TABLE 1: Summarization of observed values and calculated flux at each zenith angle. Angle uncertainty is ± 0.08 radians, or approximately ± 4.5°, and count uncertainty is given by $N^{-1/2}$.

- Flux is calculated as the number of counts per second per detector area. Detector area was measured to be A = (10.91 ± .04) × 10^{-3} m^2
- As seen in TABLE 1, flux increases as the zenith angle decreases, as we should expect.
- Flux uncertainty is calculated via standard error propagation techniques.

* full code used for analysis is copied in the speaker notes on this slide

Analysis

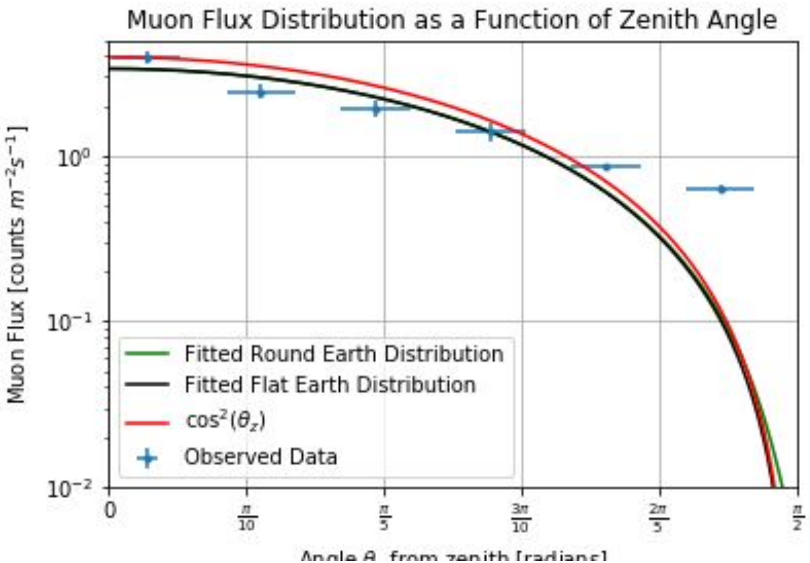


FIG 1: Observed distribution of muon flux on a log10 scale as a function of the zenith angle, in radians. The rate of accidental counts was subtracted from each calculated count rate.

- The calculated rate of accidental counts for our equipment is $R_{acc} = 0.000015\ N$ □ s^{-1}

- All fit functions are done using the scipy.optimize.curve_fit() method. Each fit was made by optimizing the peak flux (I_0), which we expect to see at a zenith angle of 0°, and the exponent value (n).

- From FIG 1, the fitted flat and round Earth distributions appear as a flattened $\cos^2$ curve. The round Earth flux distribution slightly diverges from the other two present curves at zenith angles approaching 90°.

Analysis

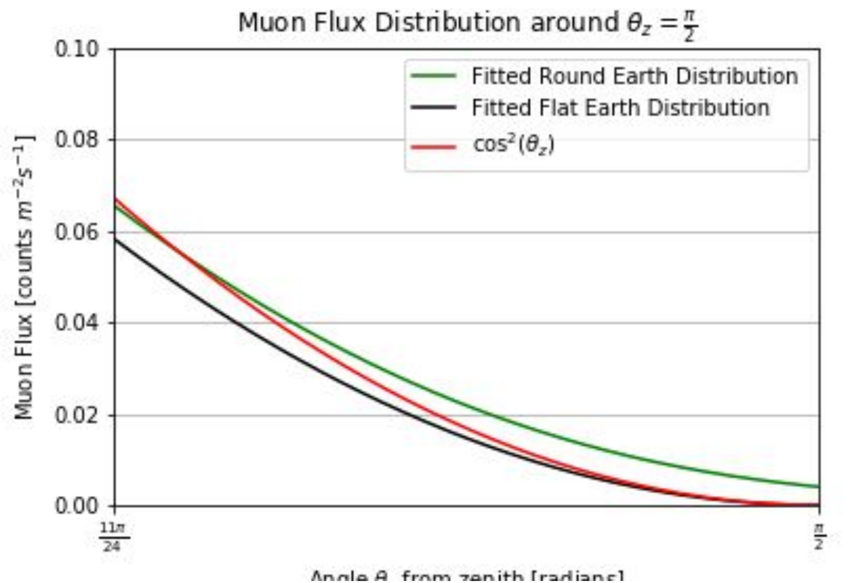


FIG 2: Detailed view of the distributions around a zenith angle of 90°. These large zenith angles are the most salient, as a flat Earth theory predicts no flux at 90°, while a round Earth does not.

- FIG 2 shows the respective distributions at zenith angles close to 90°. The separation between the fit function for a round Earth and that of a flat Earth are clearly visible at this resolution.

- This result is the defining characteristic of a round Earth theory as compared to the flat Earth.

Results

Fit Function $\Phi(\theta_z)$	I_0	n	$\frac{\chi^2}{ndf}$	p-value
$I_0 \cos^{n-1}(\theta_z)$	3.4 ± 0.4	3.0 ± 0.7	3.06	0.38
$I_0(\sqrt{(\frac{R^2}{d^2})\cos^2(\theta_z) + \frac{2R}{d} + 1} - (\frac{R\cos(\theta_z)}{d}))^{-n-1}$	3.4 ± 0.4	3.0 ± 0.6	2.81	0.42

TABLE 2: Description of flux fit functions, optimized quantities, chi-squared test value per degree of freedom, and the resulting p-value. I_0 is the expected muon flux in N □ s^{-1} □ m^{-2} at 0°, and n is the exponent value in each fit function. The first entry is for the flat Earth model, and the second for the round Earth model.

- While the plotted result supports a round Earth theory, statistical analysis is inconclusive. The p-values listed in TABLE 2, clearly larger than 0.05, fail to reject the null hypothesis and thus do not confirm a good fit to the data.

- Our optimized n values and respective projected error include the expected values as calculated by Shukla and Sankrith (3.01±0.03 for a flat Earth and 3.09±0.03 for a round Earth) [1]. However, our uncertainty is too large to precisely confirm this value.

Results

- Using statistical analysis, we fail to confirm that the Earth is round due to large p-values from the chi-squared tests.

- While we did confirm the expected n values as calculated by Shukla and Sankrith [1], the projected 1 standard deviation error is not precise enough to give a valuable result.

- Our defining result in this study comes from the fit function values near normal incidence from the zenith, returning a predicted flux of 0.00399 N □ s^{-1} □ m^{-2} for the round Earth theory, as compared to an expected flux of 0 for the flat Earth theory.

Reducing Error

Lukas Keeling

- Round earth vs Flat earth conclusion reliant upon small difference between:

$$I_0\left(\sqrt{\left(\frac{R^2}{d^2}\right)\cos^2(\theta_z) + \frac{2R}{d} + 1} - \left(\frac{R\cos(\theta_z)}{d}\right)\right)^{-n-1}$$ and $$I_0 \cos^{n-1}(\theta_z)$$

- Difference is most evident at steeper angles
- Requires very high angular certainty
- Wide error bars result in inconclusive data

Ideas

- Better equipment vs better setup/approach
- Better equipment
  - 3 scintillators instead of 2
  - Compact Muon Solenoid
- Better approach
  - more/longer data points
  - Take weather into account

Changing methods

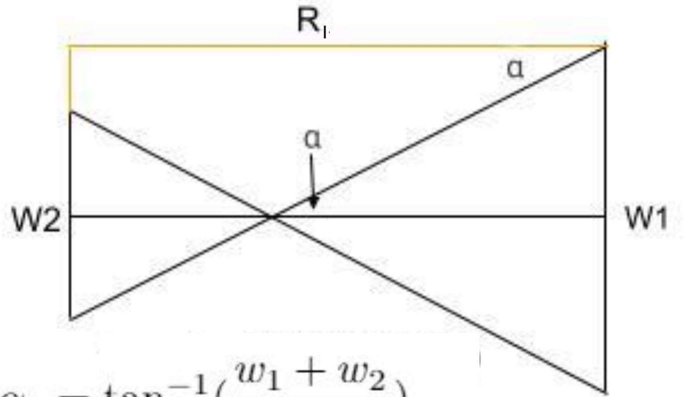
$$\delta r = \sqrt{((\frac{\partial r}{\partial N}\delta n)^2 + (\frac{\partial r}{\partial t}\delta t)^2)}$$

$$\delta r = \sqrt{((\frac{\sqrt{N}}{t})^2 + (\frac{N}{t^2}\delta t)^2)}$$

$$\delta r = \sqrt{\frac{N}{t^2} + \frac{N^2}{t^4}\delta t^2}$$

- Increased sampling time
  - More data points across longer periods reduces effects of systematic errors
  - Reduces the uncertainty in rate (counts/time)
- Increased distance between scintillators
  - Increased R
  - Requires moving lab set up
  - Reduces uncertainty in angle

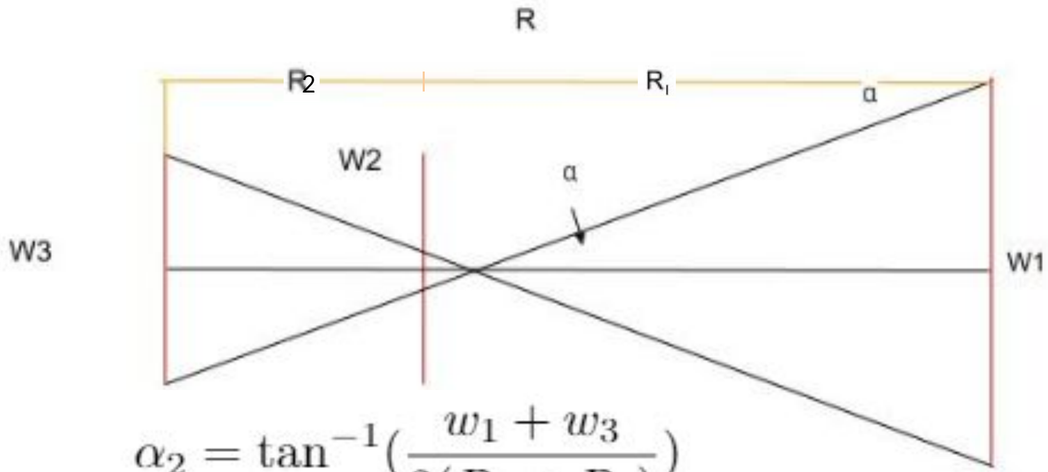
2 scintillators



$$\alpha_1 = \tan^{-1}(\frac{w_1 + w_2}{2R_1})$$

- Angle dependent on width of both scintillators as well as the distance between them.
- Lab environment made using a large R difficult
- This means difficulty in obtaining small alpha

3 scintillators



$$\alpha_2 = \tan^{-1}\left(\frac{w_1 + w_3}{2(R_1 + R_2)}\right)$$

- Adding a second square scintitattor means W2 and W3 are the same
  W2=W3
- Reduces alpha in the same way that increasing R would
  R=R1+R2
- decreases rate of accidentals

Analysis

$$\alpha_1 = \tan^{-1}(\frac{w_1 + w_2}{2R_1}) \qquad \alpha_2 = \tan^{-1}(\frac{w_1 + w_3}{2(R_1 + R_2)})$$

The difference in alpha between the 2 and 3 scintillator setups is:

Assuming $w_2 = w_3$: $\Delta\alpha_= \tan^{-1}(\frac{w_1 + w_3}{2(R_1 + R_2)}) - \tan^{-1}(\frac{w_1 + w_3}{2R_1})$

$$\Delta\alpha_= \tan^{-1}(\frac{2R_1(w_1 + w_3)}{4R_1(R_1 + R_2) + (w_1 + w_3)^2})$$

Assuming $w_2 = w_3$ and that $R_2 = 30$ cm this would have resulted in $|\Delta\alpha| = 2.478°$

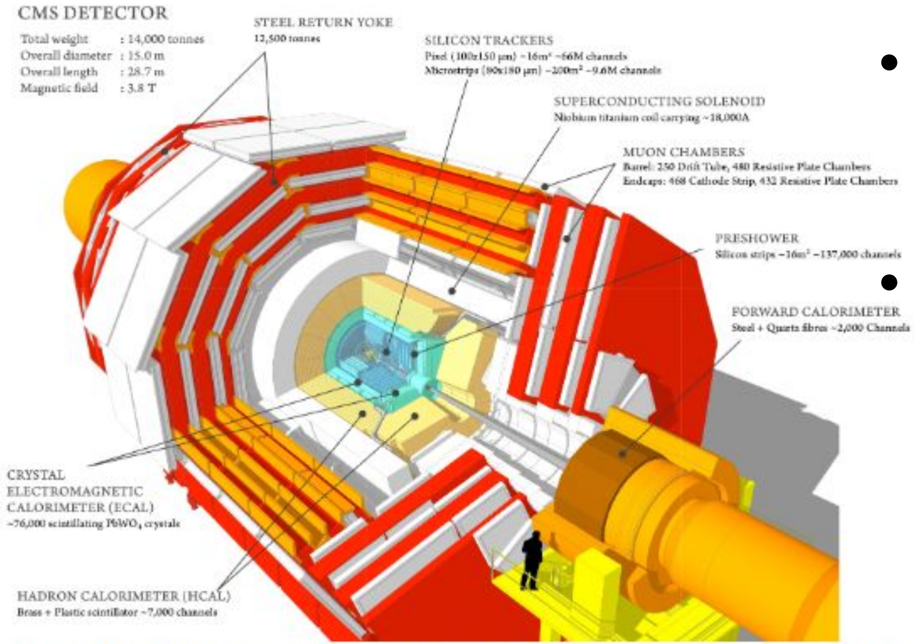
$w_1 = 78.8$cm $R_1 = 78.8$ cm

Reduction in rate of accidentals

$$R'_{acc} = R_{acc} R_3 \Delta t = R_1 R_2 R_3 (\Delta t)^2$$

- Adding a third scintillator decreases the likelihood of accidental counts significantly
- Δt on the order of 98 ns
- $(\Delta t)^2$ on the order of 9.6^(-15) s
- Reducing rate of accidentals reduces offset at larger angles, making data more distinguishable

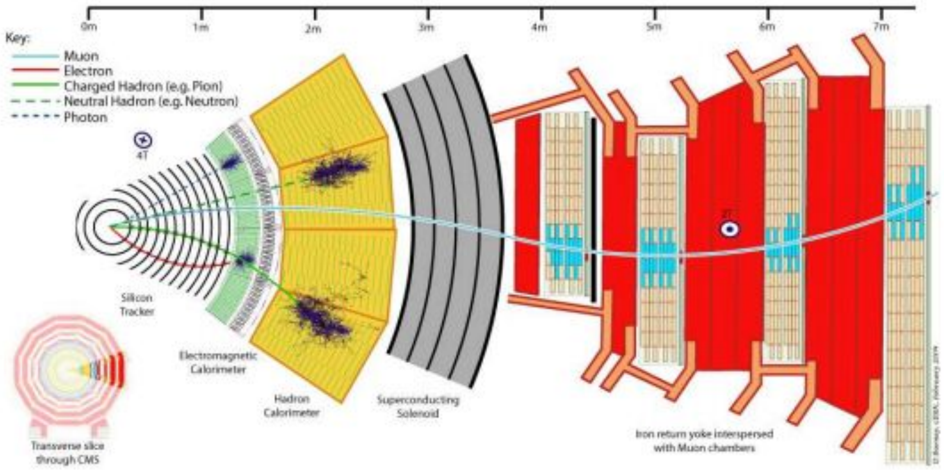
Compact Muon Solenoid (CMS)

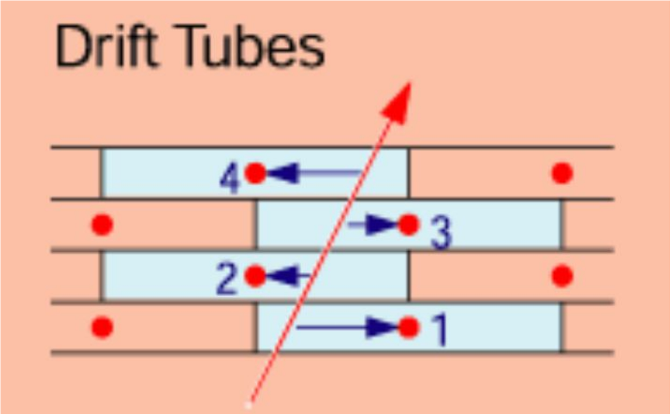


Sectional view of the CMS detector. The LHC beams travel in opposite directions along the central axis of the CMS cylinder colliding in the middle of the CMS detector. Click on the image to enlarge

http://cms.web.cern.ch/news/cms-detector-design

- About
  - 13m x 7m solenoid
  - Can accurately characterize a particle's path into and out of the solenoid
  - See next slide for Muon tracking
- Our application
  - Use only the path tracking capabilities
  - Precisely measure the flux for various angles
  - Allows for flux measurements of various angles simultaneously





- CMS uses drift tubes and cathode strips to detect incoming particles' position to 10 um certainty.
- Can detect up to a billion particles a second
- Costs $15.9 million [3]

Bibliography

[1] Shukla, Prashant, and Sundaresh Sankrith. “Energy and Angular Distributions of Atmospheric Muons at the Earth.” *International Journal of Modern Physics A*, vol. 33, no. 30, 2018, p. 1850175., doi:10.1142/s0217751x18501750.

[2]“CMS Detector Design: CMS Experiment.” CMS Detector Design | CMS Experiment, cms.web.cern.ch/news/cms-detector-design.

[3]Major Research Equipment.” Major Research Equipment, 2000, www.nsf.gov/about/budget/fy2000/00MRE.htm.