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Distributed algorithm for economic dispatch with transmission losses under digraph ☆

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ABSTRACT

By taking transmission losses and directed communication into considered, a more practical scenario of economic dispatch problem is studied in the paper. It is formulated as a non-convex optimization problem. Using convex relaxation, the non-convex optimization problem is transformed into convex optimization problem, and the conditions to ensure that they have the same solution are given. Then, a consensus-based distributed algorithm with time-varying feedback gains is presented, which can be used to the general directed communication networks. Especially, the convergence and optimality of the proposed algorithm are proved by using multi-parameter eigenvalue perturbation theory and graph theory. Finally, simulation results validate the theoretical results and illustrate the effectiveness and advantages of the proposed algorithm.

1. Introduction

Economic dispatch problem (EDP) is one key problem of energy management in power systems [1]. Many centralized methods have been proposed to solve the EDP [2–5]. But the centralized algorithms may face many challenges when applied to the future smart grid equipped with a large number of distributed renewable energies and energy storage devices [6], as they lack scalability in computing and communication issues, and are susceptible to single-point failures [7]. Compared with centralized methods, distributed algorithms are considered to be more scalable and robust for the future smart grid and have been studied by many scholars [8–34]. Although some distributed control and optimization algorithms are available, how to achieve the optimal dispatch in changeful environment is worthy of further study [29].

Recently, many distributed algorithms based on consensus theory of multi-agent systems have been widely studied. In general, by selecting incremental cost consensus (ICC) as consensus variable, they design distributed algorithms based on the information exchanges of neighbor nodes to solve the EDP. Authors in [9–11] proposed distributed incremental cost consensus (ICC) algorithms, [12,13] proposed "consensus + innovations" approaches, where the innovation term is added to ensure the supply–demand balance, and [14] proposed a distributed algorithm with time-varying feedback gains. Authors considered the cost-driven optimal energy management strategy in [15]. Distributed consensus-based alternating direction method of multipliers (ADMM) methods were presented for dynamic economic dispatch in [16,17]. Distributed nonconvex nonsmooth optimization problems were investigated in [18]. The distributed economic dispatch or distributed resource allocation have investigated in [19–22] over communication network with communication delay or information loss. According to statistics, transmission losses are estimated to account for about 6% of the total power generation. Especially, the network loss may reach 20%–30% of the total load for systems with large coverage and low load density [8]. Therefore, transmission losses should not be ignored and many authors take the transmission losses into their optimization models in [23–28].

Note that the communication networks are modeled undirected in the above-mentioned work. Nevertheless, undirected communication is rare in practice [29] because there are inevitable factors (such as the packet loss and communication interference) leading to directed communication [30]. Hence, it is meaning to consider the directed communication, and many authors investigated distributed optimization for directed communication [29–34]. Especially, transmission losses are considered in [30,34]. Considering the demand side, authors in [30] investigated distributed energy management under directed communication network. Authors in [34] presented a surplus based approach based on continuous-time consensus protocol for distributed optimal resource allocation over strongly connected digraph, and an auxiliary variable is needed.

Generally, the EDP with transmission losses is formulated as a non-convex optimization because the equality constraint includes nonlinear

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term [25,30]. Compared with convex optimization problems, it is more difficult to analyze the existence and optimality of solutions of non-convex optimization problems, especially for directed communication problems. As a result, the existing literature mostly ignores the transmission losses or are only applied to undirected communication, such as [9–22] and [29], [31–33]. Therefore, it is meaningful to study the distributed economic dispatch with transmission losses for directed communication.

Motivated by the above-mentioned work and results, we give a consensus-based distributed algorithm with time-varying feedback gains under directed communication in the paper. The proposed algorithm can solve the EDP with transmission losses under directed graph. Firstly, a more practical scenario model of the EDP with transmission losses is considered, and it is modeled as a non-convex optimization. Secondly, we give the conditions to ensure that the problem has an optimal solution by relaxing the non-convex optimization to a convex optimization. Then, a consensus-based distributed algorithm with time-varying feedback gains is presented to solve the EDP under an unbalanced directed communication network. Especially, we prove that the algorithm is convergent and the convergent value is the optimal solution of the EDP. Compared with [30], the feedback gains in our algorithm are different and time-varying, and the transmission loss model is more general. Unlike [34], the model studied in the paper is a non-convex optimization because Kron's modeled transmission losses are considered in the equality constraint (i.e., supply–demand balance constraint). Finally, simulation results with a five-generator and IEEE modified 118-bus system are given to demonstrate the effectiveness, robustness, and scalability of the presented algorithm. The results compared with related literature are given. In short, the contributions of the paper are as follows.

(1) The paper proposed a distributed algorithm for the EDP with transmission losses over a general unbalanced directed graph and provided provable convergence and optimality results. Compared with the related distributed algorithms over directed graphs (such as [29], [31–34]), Kron's modeled transmission losses are considered in the paper.
(2) Compared with the most existing work that can be applied to directed communication (such as [29–33]), the presented algorithm is more flexible in the choice of the control gains since it allows the control gains are time-varying and different.
(3) Simulation results further confirm the effectiveness and advantages of the algorithm proposed.

The paper is organized as follows. Problem statement are given in Section 2, including communication network model, EDP model with transmission losses, and transformation of the EDP model. A distributed algorithm is proposed for the EDP with transmission losses under an unbalanced directed graph in Section 3. Especially, the convergence and optimality of the algorithm is proved in the section. Several simulation and comparison results are included in Section 4. Section 5 concludes the paper.

2. Problem statement

2.1. Communication network model

Without loss of generality, suppose that the tested smart grid has n bus nodes. Generally, the communication network is described as a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. $\mathcal{V} = 1, 2, \dots, n$ is the node set and $\mathcal{E} = \{(i, j) | i, j \in \mathcal{V}\}$ is the edge set. $(i, j) \in \mathcal{E}$ means that there is a directed communication link from i to j. $\mathcal{N}_i^- = \{j | (i, j) \in \mathcal{E}\}$ and $\mathcal{N}_i^+ = \{j | (j, i) \in \mathcal{E}\}$ are called the out-neighbor set and the in-neighbor set of node i, respectively. In the paper, suppose that the generator can obtain its own state information, i.e., $i \in \mathcal{N}_i^+$ and $i \in \mathcal{N}_i^-$. In addition, suppose that graph $\mathcal{G}$ is strongly connected, i.e., for any two nodes, there is a directed path.

Two matrices $\boldsymbol{R} = [r_{ij}]_{n\times n}$ and $\boldsymbol{Q} = [q_{ij}]_{n\times n}$ associated with graph $\mathcal{G}$ are introduced as follows.

$$r_{ij} = \begin{cases} \dfrac{1}{|\mathcal{N}_i^+|}, & \text{if } j \in \mathcal{N}_i^+, \\ 0, & \text{otherwise.} \end{cases}$$

and

$$q_{ij} = \begin{cases} \dfrac{1}{|\mathcal{N}_j^-|}, & i \in \mathcal{N}_j^-, \\ 0, & \text{otherwise.} \end{cases}$$

It is easy to verify that $\boldsymbol{R}$ is row stochastic and $\boldsymbol{Q}$ is column stochastic.

2.2. Economic dispatch model with transmission losses

In the paper, the studied EDP with transmission losses is formulated as the following optimization problem.

$$\min \quad \sum_{i=1}^{n} f_i(p_i), \tag{1a}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} p_i - P_L(p_i) = D, \tag{1b}$$

$$p_i^m \le p_i \le p_i^M, i = 1, 2, \dots, n, \tag{1c}$$

where p_i is the generation output of generator i; D is the total demand; $P_L(\cdot)$ is the function of the total transmission losses; p_i^m and p_i^M are the lower and upper limits of the generation capability respectively; (1b) and (1c) are the supply–demand balance constraint and generation constraint, respectively; $f_i(\cdot)$ is the cost function of generator i, which is usually approximated as follows.

$$f_i(p_i) = a_i p_i^2 + b_i p_i + c_i,$$

$a_i > 0,\ b_i > 0$, and $c_i \ge 0$ are the cost coefficients.

Compared with the EDP model in our previous work [14,22], the transmission losses are considered in the supply–demand balance. According to Kron's equation [1], it can be modeled as the sum of the transmission losses of all generators, and the transmission loss of each generator can be seen as a quadratic function of the generation output p_i, i.e.,

$$P_L(p_i) = \sum_{i=1}^{n} p_i^L(p_i) = \sum_{i=1}^{n} [B_{i0} p_i^2 + B_{i1} p_i + B_{i2}],$$

where $B_{i0} \ge 0, B_{i1} \ge 0$, and $B_{i2} \ge 0$ are coefficients of the transmission losses. Generally, B_{i0}, B_{i1}, and B_{i2} are all very small.

The optimization problem (1) is non-convex, although the objective is a convex function. The reason is that the transmission losses are considered in the equality constraint (1b), which is a quadratic function, not an affine function. Following the definition in [35], it is a non-convex optimization. It is challenging to solve the non-convex optimization problem in a distributed way.

In the paper, we always assume that (1) is feasible. The following assumptions are needed.

Assumption 1 (*[25]*). $1 - \frac{dP_L}{dp_i} > 0$ always holds for all $i = 1, 2, \dots, n$.

Assumption 2 (*[25]*). $\sum_{i=1}^{n} p_i^m - P_L(p_i^m) \le D \le \sum_{i=1}^{n} p_i^M - P_L(p_i^M)$ always holds.

Assumption 3. For all i, $1 - B_{i1} - 2p_i^M B_{i0} > 0$ always hold.

Assumption 1 implies that the incremental loss of each generator cannot exceed the incremental generation [25]. Thus the incremental cost satisfies $\frac{df_i/dp_i}{1-dP_L/dp_i} > 0$. Assumption 2 guarantees that (1) is feasible. Assumption 3 is used in the proof of Theorem 2.

2.3. Transformation of the economic dispatch model

It is difficult to analyze the existence and optimality of solution for non-convex optimization problem (1). Transforming it into a convex optimization problem is necessary. Relaxing the equality constraint (1b) to inequality constraint, problem (1) can be transformed to the following optimization problem.

$$\min \quad \sum_{i=1}^{n} f_i(p_i), \tag{2a}$$

$$\text{s.t.} \quad D - \sum_{i=1}^{n} p_i + P_L(p_i) \le 0, \tag{2b}$$

$$p_i^m \le p_i \le p_i^M, i = 1, 2, \dots, n, \tag{2c}$$

Compared with problem (1), only the equality constraint (1b) is relaxed to inequality constraint (2b) in problem (2). Therefore, the solution of problem (1) is always the solution of problem (2). According to the definition of convex optimization in [35], problem (2) is a strictly convex optimization problem and it has a unique optimal solution because its objective function is strictly convex.

Based on the results of [30], the solution of (2) is as follows.

Theorem 1. *If the demand and the generation capacity satisfy*

$$\sum_{i=1}^{n} p_i^m - P_L(p_i^m) \le D \tag{3}$$

then the optimal solution of (2) denoted by $p^* = [p_1^*, \dots, p_n^*]$*, satisfies*

$$\sum_{i=1}^{n} p_i^* - P_L(p_i^*) = D. \tag{4}$$

Proof. Karush–Kuhn–Tucker (KKT) conditions of problem (2) are used to prove Theorem 1. The main reference is [35], and the detail proof can be found in Appendix A.

Remark 1. Theorem 1 means that problem (2) has a unique optimal solution $\boldsymbol{p}^*$ under condition $\sum_{i=1}^n p_i^m - P_L(p_i^m) \le D$, and the unique optimal solution $\boldsymbol{p}^*$ satisfies the equality constraint of problem (1). Therefore, the original non-convex problem (1) and the modified convex problem (2) have the same optimal solution under condition $\sum_{i=1}^n p_i^m - P_L(p_i^m) \le D$.

Remark 2. The sufficient condition of Theorem 1 is very weak, and it is a necessary condition to ensure that the problem (1) has a solution.

3. Distributed algorithm for the EDP under digraph

3.1. Distributed algorithm for the EDP

Note that equality (1b) is the global coupled constraint, and all inequalities in (1c) are local. We decouple (1b) by using Lagrangian multiplier method. The Lagrangian function of problem (1) only with equality constraint is

$$\mathcal{L}(\lambda, \boldsymbol{p}) = f(p_i) + \lambda(D - \sum_{i=1}^{n} p_i + P_L(p_i)). \tag{5}$$

The optimal solution p_i^* of problem (1) need to satisfy the KKT conditions $\left[\frac{df_i}{p_i} - \lambda(1 - \frac{dP_L}{p_i})\right]|_{p_i=p_i^*} = 0$ for all $i = 1, 2, \dots, n$. That is, each incremental cost $\lambda_i = \frac{df_i/dp_i}{1-dP_L/dp_i}$ should equal to a constant, which is the optimal incremental cost denoted by λ^*. Therefore, the average consensus method can be used to solve λ_i in a distributed manner. If the optimal incremental cost is solved, then the optimal generation output can be easily obtained. Our object is to solve problem (1) based on the information exchange between the neighbor agents in the section.

Select $x_i(k)$ be consensus variable representing the estimation of the optimal incremental cost. $p_i(k)$ is the estimation of the optimal generation output. For the sake of meeting the supply–demand balance, $y_i(k)$ is introduced to estimate the local power mismatch between demand and generation output. The distributed algorithm for problem (1) is designed as follows.

$$\begin{cases} x_i(k+1) = \sum_{j\in\mathcal{N}_i^+} r_{ij} x_j(k) + \epsilon_i(k) y_i(k), & \text{(a)} \\ p_i(k+1) = \underset{p_i^m \le p_i(k) \le p_i^M}{\arg\min} \ [f_i(p_i(k)) & \text{(b)} \\ \qquad -x_i(k+1)p_i(k)], & \\ y_i(k+1) = \sum_{j\in\mathcal{N}_i^-} q_{ij} y_j(k) + [p_i(k) - p_i^L(k)] & \text{(c)} \\ \qquad -[p_i(k+1) - p_i^L(k+1)]. & \end{cases} \tag{6}$$

Compared with [30,31], feedback gain $\epsilon_i(k) > 0$ is time-varying and ϵ_i may be different from ϵ_j for $i \ne j$. Following the results in [14], the time-varying feedback gain has advantage than the constant feedback gain in controlling the algorithms.

In the following, we give the conditions that the initial values of the algorithm should satisfy. Let $\Delta P(k) = D - (\sum_{i=1}^n p_i(k) - \sum_{i=1}^n p_i^L(k))$ denote the total power mismatch at the kth iteration. Summing all $y_i(k+1)$, one has

$$\begin{aligned} \sum_{i=1}^{n} y_i(k+1) &= \sum_{i=1}^{n} \sum_{j\in\mathcal{N}_i^+} q_{ij} y_i(k) + (D - \Delta P(k)) \\ &\quad - (D - \Delta P(k+1)) \\ &= \sum_{i=1}^{n} y_i(k) - \Delta P(k) + \Delta P(k+1). \end{aligned}$$

This is because $\boldsymbol{Q}$ is a column stochastic matrix. The above equation means that $\sum_{i=1}^n y_i(k+1) - \Delta P(k+1) = \sum_{i=1}^n y_i(k) - \Delta P(k)$ $(k = 1, 2, \dots)$ always holds. In other word, $\sum_{i=1}^n y_i(k) - \Delta P(k)$ will not change during the iteration process. Therefore, if $\sum_{i=1}^n y_i(0) - \Delta P(0) = 0$, then the supply–demand constraint is satisfied, and $\sum_{i=1}^n y_i(k) = \Delta P(k)$ i.e., $y_i(k)$ can track the local mismatch during each iteration. Thus, one can initialize the algorithm as follows.

$$\begin{cases} x_i(0) = \text{any valid value}, & \text{(a)} \\ y_i(0) = 0, & \text{(b)} \\ p_i(0), p_i^L(0) = \text{any valid values such that} & \\ \qquad \sum_{i=1}^{n} [p_i(0) - p_i^L(0)] = D. & \text{(c)} \end{cases} \tag{7}$$

For example, one can set $\sum_{i=1}^n p_i(0) = D$ and $p_i^L(0) = 0$, which satisfy the second equation of (7).

Fig. 1 shows the flowchart of the implementation process of the presented algorithm.

3.2. Global convergence of the proposed algorithm

we need transform algorithm (6) into a system in order to discuss the convergence. The following transformation is needed.

$$\begin{aligned} & y_i(k+1) \\ &= \sum_{i\in\mathcal{N}_i^-} q_{ij} y_j(k) + B_{i0}[p_i^2(k+1) - p_i^2(k)] \\ &\quad + B_{i1}[p_i(k+1) - p_i(k)] + [p_i(k) - p_i(k+1)] \\ &= \sum_{i\in\mathcal{N}_i^-} q_{ij} y_j(k) - [1 - B_{i1}][p_i(k+1) - p_i(k)] \\ &\quad + B_{i0}[p_i(k+1) + p_i(k)][p_i(k+1) - p_i(k)]. \end{aligned} \tag{8}$$

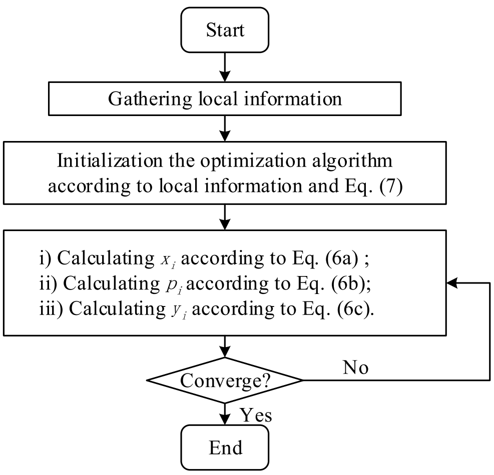

Fig. 1. The flowchart of the algorithm.

Let $\delta_i(k) := p_i(k+1) + p_i(k)$ and $\sigma_i(k) := 1 - B_{i1} - B_{i0}\delta_i(k)$. Eq. (8) can be reformed as follows.

$$\begin{aligned} &y_i(k+1) \\ &= \sum_{i\in\mathcal{N}_i^-} q_{ij} y_j(k) - [1 - B_{i1} - B_{i0}\delta_i(k)][p_i(k+1) \\ &\quad - p_i(k)] \\ &= \sum_{i\in\mathcal{N}_i^-} q_{ij} y_j(k) - \sigma_i(k)[p_i(k+1) - p_i(k)]. \end{aligned} \tag{9}$$

Following Assumption 3, $\sigma_i(k) = 1 - B_{i1} - B_{i0}\delta_i(k) > 0$ holds.

Then, if generator i does not reach its lower bound or upper bound, the matrix form of algorithm (6) can be written as

$$\begin{cases} \boldsymbol{x}(k+1) = \boldsymbol{R}\boldsymbol{\lambda}(k) + \boldsymbol{\epsilon}(k)\boldsymbol{y}(k), \\ \boldsymbol{p}(k+1) = \boldsymbol{B}\boldsymbol{\lambda}(k+1) - \boldsymbol{C}, \\ \boldsymbol{y}(k+1) = \boldsymbol{Q}\boldsymbol{s}(k) - \boldsymbol{D}(k)[\boldsymbol{p}(k+1) - \boldsymbol{p}(k)]. \end{cases} \tag{10}$$

Here, $\boldsymbol{I}$ represents identity matrix. $\boldsymbol{B} = \mathrm{diag}\left[1/(2a_1), \dots, 1/(2a_n)\right], \boldsymbol{C} = \left[b_1/(2a_1), \dots, b_n/(2a_n)\right]^{\mathrm{T}}$, $\boldsymbol{D}(k) = \mathrm{diag}[\sigma_1(k), \dots, \sigma_n(k)]$ and $\boldsymbol{\epsilon} = \mathrm{diag}[\epsilon_1, \dots, \epsilon_n]$.

Inserting $\boldsymbol{p}(\cdot)$ and $\boldsymbol{x}(\cdot)$ to the third equation in (10), algorithm (6) can be rewritten as the following system.

$$\begin{aligned} &\begin{bmatrix} \boldsymbol{x}(k+1) \\ \boldsymbol{y}(k+1) \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{R} & \boldsymbol{\epsilon}(k) \\ \boldsymbol{D}(k)\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{R}) & \boldsymbol{Q} - \boldsymbol{D}(k)\boldsymbol{B}\boldsymbol{\epsilon}(k) \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{y}(k) \end{bmatrix}. \end{aligned} \tag{11}$$

It is a nonlinear system of $\begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{y}(k) \end{bmatrix}$, because $\boldsymbol{D}(k)$ and $\boldsymbol{\epsilon}(k)$ is associated with k.

Denote $\boldsymbol{W}(k) := \begin{bmatrix} \boldsymbol{R} & \boldsymbol{\epsilon}(k) \\ \boldsymbol{D}(k)\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{R}) & \boldsymbol{Q} - \boldsymbol{D}(k)\boldsymbol{B}\boldsymbol{\epsilon}(k) \end{bmatrix}$. Rewriting it as $\boldsymbol{W}(k) = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{0} \\ \boldsymbol{D}(k)\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{R}) & \boldsymbol{Q} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{\epsilon}(k) \\ \boldsymbol{0} & -\boldsymbol{D}(k)\boldsymbol{B}\boldsymbol{\epsilon}(k) \end{bmatrix} := \boldsymbol{W}_0(k) + \boldsymbol{F}(k)$, the system matrix $\boldsymbol{W}(k)$ can be seen as $\boldsymbol{W}_0(k)$ perturbed by $\boldsymbol{F}(k)$. Note that $\boldsymbol{F}(k)$ includes n parameters $\epsilon_1(k), \epsilon_2(k), \dots, \epsilon_n(k)$.

It is difficult to obtain the convergence of the nonlinear system (11). Using multi-parameter eigenvalue perturbation approach, we obtain the following result.

Theorem 2. *If graph $\mathcal{G}$ is strongly connected and all feedback gains ϵ_i $(i = 1, 2, \dots, n)$ are sufficiently small, then system (11) satisfies*

$$\begin{bmatrix} \boldsymbol{x}(k) \\ \boldsymbol{y}(k) \end{bmatrix} \to span \begin{bmatrix} \boldsymbol{1} \\ \boldsymbol{0} \end{bmatrix} \; as \; k \to \infty.$$

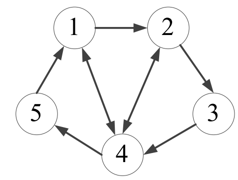

Fig. 2. The communication topology.

Table 1
Parameters of the tested generators in case 1 and 2.

DG_i	a_i	b_i	c_i	p_i^m	p_i^M	B_{i0}	B_{i1}	B_{i2}
1	0.094	1.22	51	10	80	0.00021	0	0
2	0.078	3.41	31	8	60	0.00031	0	0
3	0.105	2.53	78	3.8	40	0.00011	0	0
4	0.082	4.02	42	5.4	45	0.00022	0	0
5	0.074	3.17	62	4.2	18	0.00041	0	0

Proof. The multi-parameter eigenvalue perturbation theory is used to prove Theorem 2 by referring to [36] (chapter 2) or [14] (Proposition 1). The proof details can be found in Appendix A. □

According to Theorem 2, all $x_1, x_2, \dots, x_n$ converge to a constant (denoted by x^0), and $\lim_{k\to\infty} \boldsymbol{y}(k) = 0$ which means that the balance between demand and generation power can be guaranteed. Based on the $p_i(k)$'s iteration, $\boldsymbol{p}(k)$ can converge to a constant vector denoted by $\boldsymbol{p}^0$ because all $x_1, x_2, \dots, x_n$ converge to x^0. In the next subsection, we will prove that $\boldsymbol{p}^0$ is the optimal solution of problem (1).

3.3. The optimality of the distributed algorithm

Theorem 2 shows that algorithm (6) is convergent. The following theorem gives the optimality of the distributed algorithm.

Theorem 3. *If graph $\mathcal{G}$ is strongly connected, and all feedback gains ϵ_i $(i = 1, 2, \dots, n)$ are sufficiently small, then the proposed algorithm (6) with initialization (7) can solve problem (1), i.e.,*

$$\lim_{k\to\infty} x_i(k) = x^0 = x^*, \textit{for } i \in \mathcal{V}, \tag{12}$$

$$\lim_{k\to\infty} \boldsymbol{p}(k) = \boldsymbol{p}^0 = \boldsymbol{p}^*. \tag{13}$$

x^* and $\boldsymbol{p}^*$ represent the optimal incremental cost and the optimal solution of problem (1) respectively.

Proof. The proof details can be seen in Appendix A. □

4. Simulation studies

The effectiveness and robustness of the proposed algorithm are studied with five-generator system under a digraph in case studies 1 and 2,. Case study 3 investigates the scalability of the algorithm with the modified IEEE 118-bus system. All case studies have been tested in MATLAB 2018b, which operates on an Intel(R) Core(TM) i7-10700 CPU @ 2.90 GHz with 16 GB RAM. Note that the time consumed for each iteration is varied, which depends on the hardware used and software implementation [7]. Therefore, the number of iteration is used instead of time in case studies 1 and 2. But the time is used in case study 3 to test the scalability and the practicability for a large system.

In case studies 1 and 2, assume that the total load demand is $D = 120$kW. According to (7), the related variables initialize as follows. $\boldsymbol{x} = [2.55\ 3.075\ 3.725\ 3.55\ 2.9]^{\mathrm{T}}$, $\boldsymbol{p} = [35\ 20\ 25\ 30\ 10]^{\mathrm{T}}$, and $p_i^L = \boldsymbol{y} = [0\ 0\ 0\ 0\ 0]^{\mathrm{T}}$. The communication topology of the five-generator system described in Fig. 2. Parameters of the tested generators and coefficients of the transmission losses are given in Table 1.

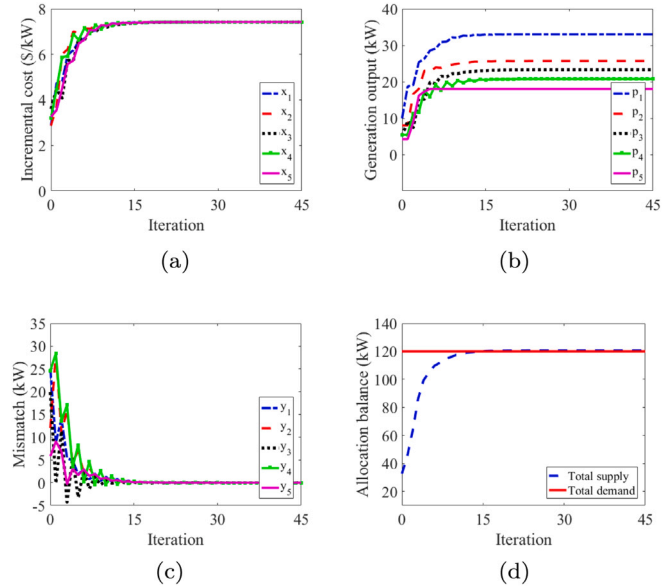

Fig. 3. Simulation results. (a) Optimal incremental cost $\boldsymbol{x}$. (b) Optimal generation $\boldsymbol{p}$. (c) Power mismatch $\boldsymbol{y}$. (d) Supply–demand balance.

4.1. Case study 1: The effectiveness of the algorithm

Assume that the error of the optimal power output is 0.0001, i.e., $|x_i(k+1) - x_i(k)| \leq 0.0001$ for all $i = 1, 2, \ldots, 5$. Let the feedback gains be $\boldsymbol{\epsilon} = [0.065 \quad 0.06 \quad 0.062 \quad 0.054 \quad 0.055]^{\mathrm{T}}$. The proposed algorithm can be converged and meet the accuracy requirements after 45 iterations, see Fig. 3. The optimal incremental cost is $x^* = 7.4208$\$/kW, see, Fig. 3(a). The optimal generation is $\boldsymbol{p}^* = [32.9832 \quad 25.7106 \quad 23.2898 \quad 20.7369 \quad 18]^{\mathrm{T}}$kW, see, Fig. 3(b). According to objective function, the total cost is $\sum_{i=1}^{5}[a_i(p^*)^2 + b_i p^* + c_i] = 861.2714$\$. The power mismatch converges to zero, see Fig. 3(c). The total transmission loss is $p_L = 0.7204$kW and the total generation output is 120.7204 kW, see Fig. 3(d). The total generation minus the total transmission loss is equal to the demand. Compared with the results in [14], the optimal incremental cost increases from 7.3890\$/kW to 7.4208\$/kW, because the transmission losses is considered in the EDP model.

4.2. Case study 2: The robustness of the algorithm

In this case, the robustness of the algorithm is tested based on case study 1. All generators are working properly until 50 iterations. Assumed that the 5th generator faults at the 51th iteration, and it returns to normal after the 100th iteration. Note that the others always work properly and the local load is still normal. The sensor in the 5th bus will perceive this change, and reset $p_5^m = p_5^M = c_5 = 0$ before the next iteration. When it returns to normal, the sensor in the 5th bus will again perceive the change, and reset p_5^m, p_5^M, and c_5 to their original values. It should be noted that the parameters and coefficients of the others remain unchanged. As the 5th generator faults, the optimal incremental cost increases from 7.4208\$/kW to 8.2217\$/kW to address the new supply–demand balance. The result can be seen in Fig. 4(a). The new optimal generation output is $\boldsymbol{p}^* = [37.2432 \quad 30.8444 \quad 27.1035 \quad 25.6203 \quad 0]^{\mathrm{T}}$kW, see, Fig. 4(b). The new total transmission loss is 0.8114 kW. The power mismatch converges to zero again. The total generation minus the total transmission loss is equal to the demand, see, Fig. 4(c) and (d). When the 5th generator returns to normal, the algorithm converges to the original values again.

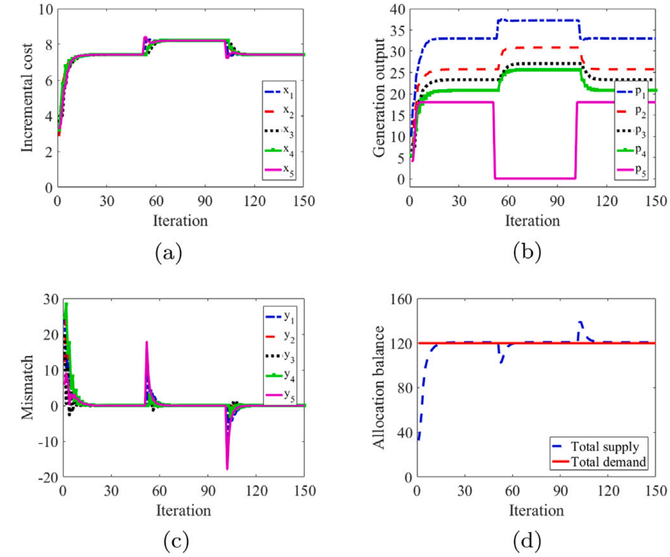

Fig. 4. Simulation results. (a) Optimal incremental cost $\boldsymbol{x}$. (b) Optimal generation $\boldsymbol{p}$. (c) Power mismatch $\boldsymbol{y}$. (d) Supply–demand balance.

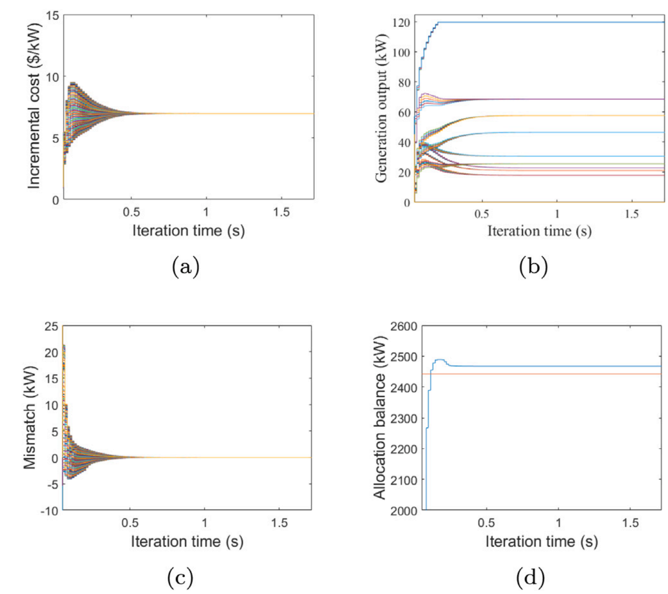

Fig. 5. Simulation results. (a) Optimal incremental cost $\boldsymbol{x}$. (b) Optimal generation $\boldsymbol{p}$. (c) Power mismatch $\boldsymbol{y}$. (d) Supply–demand balance.

4.3. Case study 3: The scalability of the algorithm

The modified IEEE 118-bus system is used to test the scalability and the practicability effectiveness of the algorithm. The system includes 108 generator/load bus and assume that there are 9 type generators. Supposed that each agent communicates with its adjacent two neighbors in the order of index number, i.e., the edge set of the communication network is $\mathcal{E} = \{(i,j) | j - i \leq 2\} \cup (107, 1) \cup (108, 1) \cup (108, 2)$. Select $\epsilon = 0.002$ and $D = 2442$kW. The generator parameters are can be found in Appendix B. Fig. 5 shows the results. The results illustrate the algorithm converge to the optimal solution of problem (1) less than 1 s. Therefore, even considering the communication time, the algorithm is fast enough for practical implementation because the scheduling time is usually from 5 to 15 min [22].

Table 2
Statistic results.

	Undirected communication ([23–28])	Directed communication ([30], [34], algorithm in the paper)
Convex optimization	–	[34]
Non-convex optimization	[23–28]	[30], algorithm in the paper
Time-varying feedback gains	–	[34], algorithm in the paper
Constant feedback gain	[23–28]	[30]
Convergence proof	[25,27,28]	[30,34], algorithm in the paper

Table 3
Comparison results.

	Bus systems	Convergence time (iterations)
Algorithm in [30]	5	51
Algorithm in the paper	5	45

4.4. Case study 4: Comparison results

Table 2 shows the results compared with the related work that considered transmission losses in their models or that can be applied to directed communication. Results show that the algorithm proposed in the paper can solve the non-convex optimization compared with [34], although the control gains are all time-varying. Table 3 shows results compared with [30] in terms of the convergence rate. Comparison results show that the algorithm proposed can meet the requirement of $|x_i(k+1)-x_i(k)| \leq 0.0001$ $(i=1,2,\ldots,5)$ after 45 iterations and the algorithm [30] need 51 iterations because the control gains can be time-varying and different in the paper.

5. Discussion

The distributed algorithm proposed can obtain the optimal solution of problem (1), and Theorem 3 shows the theoretical results. Simulation results and comparison results further confirm the effectiveness of the algorithm proposed. The algorithm proposed has strong robustness and scalability, and can meet the plug-and-play requirements of smart grids and the actual scheduling requirements. According to Theorems 2 and 3, as long as the initialization (7) holds, the proposed algorithm can solve the EDP with transmission losses. When some generator i faults, the sensor installed in the generator can perceive the emergency and reset the corresponding parameters, i.e., the bounds of the power generation (i.e. p_i^m and p_i^M) are reset to zeros to guarantee that the power generation is zero, and the coefficient (i.e. c_i) of the cost function is reset to zero to guarantee that the corresponding cost is zero. The functionalities of sensors installed in the buses can be found in [37]. For the same reason, the algorithm proposed can still solve the EDP when a new generator is added to the system, and can meet the requirements of the demand change or the other parameters' changes.

6. Conclusion

In the paper, we presented a consensus-based distributed algorithm with time-varying feedback gains for the EDP with Kron's modeled transmission losses. The proposed algorithm could solve the studied model over a general directed graph. We give the sufficient condition that ensure the model has a unique optimal solution. The convergence and optimality of the algorithm are proved by using multi-parameter eigenvalue perturbation theory and graph theory. Furthermore, simulation results validate the theoretical results and illustrate the effectiveness and advantages of the proposed algorithm. In fact, the algorithm proposed only handle the optimization problem with quadratic objective function, and many practical issues need to be further studied if the proposed distributed algorithm is applied to field implementation, such as the transmission line constraint, random communication errors, and power trading [38]. The authors would continue to refine the proposed distributed algorithm in the future work.

CRediT authorship contribution statement

Rui Wang: Gives the idea of the manuscript, designs the distributed algorithm and gives the convergence and optimality of the algorithm proposed, and compiles the manuscript. **Qi Li:** Gives the simulation results of Case Study 3 and 4, and checks the manuscript. **Juan Zou:** Give the simulation results of Case Study 1 and 2, and check the manuscript. **Cuixia Miao:** Give the simulation results of Case Study 1 and 2, and check the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Appendix A

A.1. Proof of Theorem 1

Proof. The Lagrange function of problem (2) is as follows.

$$\begin{aligned}\widetilde{\mathcal{L}}(p,\lambda,\boldsymbol{u},\boldsymbol{v}) &= f(p_i)+\lambda(D-\sum_{i=1}^{n}p_i+P_L(p_i)) \\ &\quad +\sum_{i=1}^{n}u_i(p_i-p_i^M)+\sum_{i=1}^{n}v_i(p_i^m-p_i),\end{aligned} \tag{14}$$

where $\lambda \geq 0, \boldsymbol{u}=[u_1,\ldots,u_n]^{\mathrm{T}} \geq \mathbf{0}$, and $\boldsymbol{v}=[v_1,\ldots,v_n]^{\mathrm{T}} \geq \mathbf{0}$ are Lagrange multipliers.

The optimal solution of (2) needs to satisfy the following KKT conditions.

$$\begin{cases} \dfrac{d\mathcal{L}}{dp_i}|_{p_i=p_i^*} = \left[\dfrac{df_i}{dp_i}-\lambda(1-\dfrac{dP_L}{dp_i})\right]|_{p_i=p_i^*} \\ \qquad +u_i-v_i=0; \qquad\qquad (a) \\ \lambda\left[D-\displaystyle\sum_{i=1}^{n}p_i^*+P_L(p^*)\right]=0, \lambda\geq 0, \\ \qquad D-\displaystyle\sum_{i=1}^{n}p_i^*+P_L(p^*)\leq 0; \qquad (b) \\ u_i(p_i^*-p_i^M)=0, u_i\geq 0, p_i^*-p_i^M\leq 0, \\ \qquad \text{for all } i\in\mathcal{V}; \\ v_i(p_i^m-p_i^*)=0, v_i\geq 0, p_i^m-p_i^*\leq 0, \\ \qquad \text{for all } i\in\mathcal{V}. \qquad\qquad (c) \end{cases} \tag{15}$$

We will prove the result by contradiction. Suppose that the optimal solution does not satisfy (4), i.e., $\sum_{i=1}^{n}p_i^*-P_L(p_i^*)\neq D$. According to (15)(b), if $\sum_{i=1}^{n}p_i^*-P_L(p_i^*)\neq D$, then $\lambda=0$ and the following inequality holds.

$$D-\sum_{i=1}^{n}p_i^*+P_L(p_i^*)<0.$$

According to $a_i>0$ and $b_i>0$, $\frac{df_i}{p_i}|_{p_i=p_i^*}=2a_ip_i^*+b_i>0$ holds for all $i=1,2....n$. Therefore, taking the fact that all Lagrange multipliers $u_i\geq 0$ and $v_i\geq 0$, and according to (15)(a), $v_i>0$ holds for all $i=1,2....n$.

It follows from (15)(c) that $p_i^* = p_i^m$. Therefore, one has $D - \sum_{i=1}^n p_i^* + P_L(p_i^*) = D - \sum_{i=1}^n p_i^m + P_L(p_i^m) < 0$, which contradicts inequality (3) (the sufficient condition of Theorem 1). Therefore, $\sum_{i=1}^n p_i^* - P_L(p_i^*) = D$ holds. The proof is complete. □

A.2. Proof of Theorem 2

Proof. The system is similar to the one in our previous work [14]. Therefore, the proof is based on the proof of Theorem 1 in [14]. It should be noted that $\boldsymbol{W}(k)$ is a discrete function of iteration time k. For the sake of using Proposition 1 in [14], let $\boldsymbol{W}(k)$ be the value of continuous function denoted as $\boldsymbol{W}(t)$ at $t = k$. Note that $\boldsymbol{W}(t)$ includes n perturbation parameters, denoted as a vector $\boldsymbol{p} = [\epsilon_1(t), \epsilon_2(t), \dots, \epsilon_n(t)]$.

We conclude that all eigenvalues of $\boldsymbol{W}(t)$ meet the following inequality.

$$|\mu_{2n}(t)| \le |\mu_{2n-1}(t)| \le \cdots \le |\mu_2(t)| < \mu_1(t) = 1. \tag{16}$$

Based on the fact that $\boldsymbol{W}_0(t)$ is a lower block triangular matrix, the spectrum of $\boldsymbol{W}_0(t)$ satisfies $\sigma(\boldsymbol{W}_0(t)) = \sigma(\boldsymbol{R}) \cup \sigma(\boldsymbol{Q})$. Therefore, the eigenvalues of $\boldsymbol{W}_0(t)$ are constants because $\boldsymbol{R}$ and $\boldsymbol{Q}$ are constant matrices. Considering that $\boldsymbol{R}$ and $\boldsymbol{Q}$ are row and column stochastic matrices, 1 is their eigenvalue, and the others are smaller than 1. Therefore, the eigenvalues of $\boldsymbol{W}_0(t)$ satisfy $1 = \xi_1 = \xi_2 \ge |\xi_3| \ge \cdots \ge |\xi_{2n}|$.

Suppose that $\boldsymbol{u}$ is the unit right eigenvector of $\boldsymbol{Q}$ and $\boldsymbol{v}^{\mathrm{T}}$ is the unit left eigenvector of $\boldsymbol{R}$ corresponding to 1. Following the fact that $\boldsymbol{Q}$ and $\boldsymbol{R}$ are column and row stochastic matrices, $\boldsymbol{u}$ and $\boldsymbol{v}^{\mathrm{T}}$ are positive vectors. Construct vectors

$$\boldsymbol{U} = \begin{bmatrix}\boldsymbol{U}_1 & \boldsymbol{U}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{1}\\ \boldsymbol{u} & -\phi(t)\boldsymbol{u}\end{bmatrix}$$

and

$$\boldsymbol{V}^{\mathrm{T}} = \begin{bmatrix}\boldsymbol{V}_1^{\mathrm{T}}\\ \boldsymbol{V}_2^{\mathrm{T}}\end{bmatrix} = \begin{bmatrix}\boldsymbol{1}^{\mathrm{T}}D(t)B & \boldsymbol{1}^{\mathrm{T}}\\ \boldsymbol{v}^{\mathrm{T}} & \boldsymbol{0}^{\mathrm{T}}\end{bmatrix},$$

where $\boldsymbol{0} = [0, 0, \dots, 0]^{\mathrm{T}}$, $\boldsymbol{1} = [1, 1, \dots, 1]^{\mathrm{T}}$, and $\phi(t) = \boldsymbol{1}^{\mathrm{T}}\boldsymbol{D}(t)\boldsymbol{B}\boldsymbol{1} = \sum_{i=1}^n \frac{\sigma_i(t)}{2a_i}$. One can verify that $\boldsymbol{W}_0(t)\boldsymbol{U} = \boldsymbol{U}$, $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{W}_0(t) = \boldsymbol{V}^{\mathrm{T}}$, and $\boldsymbol{V}^{\mathrm{T}}\boldsymbol{U} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$. Based on Assumption 3 and the definition of $\sigma_i(t)$, $\phi(t) > 0$ holds.

Denote the eigenvalue of $\boldsymbol{W}(t)$ corresponding to $\xi_1 = \xi_2 = 1$ of $\boldsymbol{W}_0(t)$ by μ_1 and μ_2. Using multi-parameter eigenvalue perturbation theory (more details can be found in Chapter 2 of [36] or Proposition 1 in [14]), the derivatives of μ_1 and μ_2 exist and can be calculated by the eigenvalues of matrix

$$\begin{bmatrix}\sum_{i=1}^n\left(\mathbf{V}_1^{\mathrm{T}}\frac{\partial \boldsymbol{F}}{\partial p_i}\boldsymbol{U}_1\right) & \sum_{i=1}^n\left(\mathbf{V}_1^{\mathrm{T}}\frac{\partial \boldsymbol{F}}{\partial p_i}\boldsymbol{U}_2\right)\\ \sum_{i=1}^n\left(\boldsymbol{V}_2^{\mathrm{T}}\frac{\partial \boldsymbol{F}}{\partial p_i}\boldsymbol{U}_1\right) & \sum_{i=1}^n\left(\boldsymbol{V}_2^{\mathrm{T}}\frac{\partial \boldsymbol{F}}{\partial p_i}\boldsymbol{U}_2\right)\end{bmatrix} = \begin{bmatrix}0 & 0\\ \boldsymbol{v}^{\mathrm{T}}\boldsymbol{u} & -\phi(t)\boldsymbol{v}^{\mathrm{T}}\boldsymbol{u}\end{bmatrix}.$$

Therefore, 0 and $-\phi(t)\boldsymbol{v}^{\mathrm{T}}\boldsymbol{u}$ are the two eigenvalues of the above matrix. Since $\phi(t) > 0$, $\boldsymbol{u} > 0$, and $\boldsymbol{v}^{\mathrm{T}} > 0$, $-\phi(t)\boldsymbol{v}^{\mathrm{T}}\boldsymbol{u} < 0$ holds. It follows from eigenvalue perturbation theory that $\frac{d\mu_1(t)}{dt} = 0$ and $\frac{d\mu_2(t)}{dt} = -\phi(t)\boldsymbol{v}^{\mathrm{T}}\boldsymbol{u} < 0$. In other words, there exists a constant $t_1 > 0$, such that $\mu_1(t) = \xi_1 = 1$ and $\mu_2(t) < \xi_2 = 1$ for all $t < t_1$. In addition, there exists a constant $t_2 > 0$, such that all eigenvalues $|\mu_i(t)| < 1, i = 3, \dots, 2n$, for all $t < t_2$. The reason is that eigenvalues are continuous functions of matrix entries. In summary, there exists $t_0 = \max\{t_1, t_2\}$ such that the eigenvalues of the system matrix $\boldsymbol{W}(t)$ meet inequality (16) for all $t < t_0$.

Meanwhile, $\begin{bmatrix}\boldsymbol{1}\\ \boldsymbol{0}\end{bmatrix}$ is the right eigenvector of the system matrix $\boldsymbol{W}(t)$ corresponding to $\mu_1(t) = 1$, and all the rest eigenvalues are smaller than 1 in modulus. Thus, one has

$$\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{y}(k)\end{bmatrix} \to span\begin{bmatrix}\boldsymbol{1}\\ \boldsymbol{0}\end{bmatrix} \text{ as } t \to \infty.$$

The proof is complete. □

Table 4
Parameters of the 9 type generators.

DG_i	a_i	b_i	p_i^m	p_i^M	B_{i0}	B_{i1}	B_{i2}
1	0.0031	8.71	0	113.23	0.00021	0	0
2	0.0074	3.53	0	179.10	0.00031	0	0
3	0.0066	7.58	0	90.03	0.00011	0	0
4	0.0063	2.24	0	106.41	0.00022	0	0
5	0.0069	8.53	0	193.80	0.00041	0	0
6	0.0014	2.25	0	37.19	0.0001	0	0
7	0.0041	6.29	0	195.40	0.00015	0	0
8	0.0051	4.30	0	62.17	0.00001	0	0
9	0.0032	8.26	0	143.41	0.0002	0	0

Remark 3. From the proof of Theorem 2, the algorithm is convergent if and only if $\mu_2 < 1$, where μ_2 is the second largest eigenvalue of system matrix $\boldsymbol{W}(k)$. To found the best convergence rate is equal to solve the optimization problem $\min_{\epsilon_i} \mu_2^2(\boldsymbol{W})$ in the constraint $\mu_2 < 1$ [22]. It is difficult to obtain the theoretical solution of the problem. For the especial case that all ϵ_i are equal to a constant ϵ, [31] gave a method to found the solution. In general, the heuristic candidate is $\epsilon \le \frac{1}{k*n}$ [24].

A.3. Proof of Theorem 3

Proof. From iteration $p_i(k+1) = \arg\min_{p_i^m \le p_i(k) \le p_i^M}[f_i(p_i(k)) - x_i(k+1)p_i(k)]$ of algorithm (6), $p_i(k+1)$ needs to satisfy

$$p_i(k+1) = \frac{x_i(k+1) - b_i}{2a_i}, \text{ if } p_i^m \le p_i(k) \le p_i^M.$$

Let $\lambda_i^m := 2a_ip_i^m + b_i$ and $\lambda_i^M := 2a_ip_i^M + b_i$. The above equation means that, when $\lambda_i^m < \lambda_i < \lambda_i^M$, p_is always satisfy $p_i^0 = \frac{\lambda^0 - b_i}{2a_i}$ as $k \to \infty$; when $\lambda_i \le \lambda_i^m$ or $\lambda_i \ge \lambda_i^M$, p_is satisfy $p_i = p_i^m$ or p_i^M as $k \to \infty$, because $p_i(\cdot)$ is a monotone increasing function of λ_i.

In order to facilitate the following analysis, it may be assumed that all generators do not exceed the generation capabilities, i.e., p_is satisfy $p_i^m < p_i < p_i^M$. According to Theorem 2, the supply–demand balance constraint holds, i.e.,

$$\sum_{i=1}^n p_i^0 = \sum_{i=1}^n\left(\frac{\lambda^0 - b_i}{2a_i}\right) = D + P_L(p_i^0).$$

On the other hand, λ^* and $\boldsymbol{p}^*$ are the optimal incremental cost and the optimal generation power of problem (1), which also satisfy the supply–demand balance constraint, i.e.,

$$\sum_{i=1}^n p_i^* = \sum_{i=1}^n\left(\frac{\lambda^* - b_i}{2a_i}\right) = D + P_L(p_i^*).$$

Using the definition of P_L and the above two equations, we have

$$(\lambda^0)^2\sum_{i=1}^n \frac{B_{i0}}{4a_i^2} + \lambda^0\left[\sum_{i=1}^n \frac{1 - B_{i1}}{2a_i}\right] = (\lambda^*)^2\sum_{i=1}^n \frac{B_{i0}}{4a_i^2} + \lambda^*\left[\sum_{i=1}^n \frac{1 - B_{i1}}{2a_i}\right]$$

That is,

$$\left[\lambda^* - \lambda^0\right]\left[(\lambda^* + \lambda^0)\sum_{i=1}^n \frac{B_{i0}}{4a_i^2} + \sum_{i=1}^n \frac{1 - B_{i1}}{2a_i}\right] = 0.$$

Note that $(\lambda^* + \lambda^0)\sum_{i=1}^n \frac{B_{i0}}{4a_i^2} + \sum_{i=1}^n \frac{1 - B_{i1}}{2a_i} > 0$ because λ^*, λ^0, B_{i0}, a_i, and $1 - B_{i1}$ are all positive. Therefore, it follows the above equation, $\lambda^0 = \lambda^*$ holds. Accordingly, $\boldsymbol{p}^0 = \boldsymbol{p}^*$ holds. The proof is complete. □

Appendix B

The IEEE 118-bus system involves 54 generators and 54 loads. In the case study, In case study 4, there are 9 types of generators, and each type has 6 same generators. Parameters of the 9 type generators are from [10]. The coefficients of transmission losses are from [30]. The above parameters are listed in Table 4.

References

[1] Wood AJ, Wollenberg BF, Sheblé GB. Power generation, operation, and control. John Wiley & Sons; 2013.

[2] Bakirtzis A, Petridis V, Kazarlis S. Genetic algorithm solution to the economic dispatch problem. IEE Proc Gener Transm Distrib 1994;141(4):377–82.

[3] Park JB, Lee KS, Shin JR, et al. A particle swarm optimization for economic dispatch with nonsmooth cost functions. IEEE Trans Power Syst 2005;20(1):34–42.

[4] Park JB, Jeong YW, Shin JR, Lee KY. An improved particle swarm optimization for nonconvex economic dispatch problems. IEEE Trans Power Syst 2010;25(1):156–66.

[5] Gaing ZL. Particle swarm optimization to solving the economic dispatch considering the generator constraints. IEEE Trans Power Syst 2003;18(3):1187–95.

[6] Huang AQ, Crow ML, Heydt GT, Zheng JP, Dale SJ. The future renewable electric energy delivery and management (FREEDM) system: the energy internet. Proc IEEE 2011;99(1):133–48.

[7] Xu Y, Zhang W, Liu W. Distributed dynamic programming-based approach for economic dispatch in smart grids. IEEE Trans Ind Inf 2015;11(1):166–75.

[8] Jian L, Qian A, Liangang Z, Mengkai Y. Distributed economic dispatch method for power system based on consensus. IET Renew Power Gener 2020;14(9):1424–32.

[9] Zhang Z, Chow MY. Incremental cost consensus algorithm in a smart gridenvironment. In: Proc. power energy soc. gen. meeting. IEEE; 2011, p. 1–6.

[10] Rahbari-Asr N, Ojha U, Zhang Z, Chow M-Y. Incremental welfare consensus algorithm for cooperative distributed generation/demand response in smart grid. IEEE Trans Smart Grid 2014;5(6):2836–45.

[11] Zhang Z, Chow MY. Convergence analysis of the incremental cost consensus algorithm under different communication network topologies in a smart grid. IEEE Trans Power Syst 2012;27(4):1761–8.

[12] Kar S, Hug G. Distributed robust economic dispatch in power systems: a consensus innovations approach. In: Proc. power energy soc. gen. meeting. IEEE; 2012, p. 1–8.

[13] Hug G, Kar S, Wu C. Consensus + innovations approach for distributed multiagent coordination in a microgrid. IEEE Trans Smart Grid 2015;6(4):1893–903.

[14] Wang R, Li Q, Zhang B, Wang L. Distributed consensus based algorithm for economic dispatch in a microgrid. IEEE Trans Smart Grid 2019;10(4):3630–40.

[15] He X, Zhao Y, Huang T. Optimizing the dynamic economic dispatch problem by the distributed consensus-based ADMM approach. IEEE Trans Ind Inf 2019;16(5):3210–21.

[16] Duan Y, He X, Zhao Y. Distributed algorithm based on consensus control strategy for dynamic economic dispatch problem. Int J Electr Power Energy Syst 2021;129:106833.

[17] He X, Fang X, Yu J. Distributed energy management strategy for reaching cost-driven optimal operation integrated with wind forecasting in multimicrogrids system. IEEE Trans Syst Man Cybern C 2019;49(8):1643–51.

[18] Yang J, He X. A continuous-time neurodynamic algorithm for distributed nonconvex nonsmooth optimization problems with affine equality and nonsmooth convex inequality constraints. Neurocomputing 2022;507:383–96.

[19] Chen G, Zhao Z. Delay effects on consensus-based distributed economic dispatch algorithm in a microgrid. IEEE Trans Power Syst 2018;33(1):602–12.

[20] Huang B, Liu L, Zhang H, Li Y, Sun Q. Distributed optimal economic dispatch for microgrids considering communication delays. IEEE Trans Syst Man Cybern Syst 2019;49(8):1634–42.

[21] Zhang Y, Rahbari-Asr N, Chow MY. A robust distributed system incremental cost estimation algorithm for smart grid economic dispatch with communications information losses. J Netw Comput Appl 2016;59:315–24.

[22] Wang R, Li Q, Li G, Liu H. A gossip-based distributed algorithm for economic dispatch in smart grids with random communication link failures. IEEE Trans Ind Electr 2020;67(6):4635–45.

[23] Loia V, Vaccaro A. Decentralized economic dispatch in smart grids by self-organizing dynamic agents. IEEE Trans Syst Man Cybern C Appl Rev 2014;44:397–408.

[24] Binetti G, Davoudi A, Lewis FL, Naso D, Turchiano B. Distributed consensus-based economic dispatch with transmission losses. IEEE Trans Power Syst 2014;29(4):1711–20.

[25] Lee S, Hyungbo S. Distributed algorithm for economic dispatch problem with separable losses. IEEE Control Syst Lett 2019;3(3):685–90.

[26] Wang R, Li Q, Shi Y, Wang L. A gossip-based asynchronous distributed algorithm for economic dispatch problem with transmission losses, in: 2019 IEEE innovative smart grid technologies - asia (ISGT Asia), Chengdu, China, 2019, pp. 770–775.

[27] Tripathi S, Jain A, Behera AK. A partially distributed fixed-time economic dispatch algorithm with Kron's modeled power transmission losses. 2022, arXiv preprint arXiv 2201.09461.

[28] Wei B, He X. A finite-time projection neural network to solve the joint optimal dispatching problem of CHP and wind power. Neural Comput Appl 2022;34(10):7405–17.

[29] Li H, Wang Z, Chen G, Dong ZY. Distributed robust algorithm for economic dispatch in smart grids over general unbalanced directed networks. IEEE Trans Ind Inform 2020;16(7):4322–32.

[30] Zhao C, He J, Cheng P, Chen J. Consensus-based energy management in smart grid with transmission losses and directed communication. IEEE Trans Smart Grid 2017;8(5):2049–61.

[31] Yang S, Tan S, Xu J-X. Consensus based approach for economic dispatch problem in a smart grid. IEEE Trans Power Syst 2013;28(4):4416–23.

[32] Dai H, Jia J, Yan L, Fang X, Chen W. Distributed fixed-time optimization in economic dispatch over directed networks. IEEE Trans Ind Inf 2020;17(5):3011–9.

[33] Lü Q, Liao X, Li H, Huang T. Achieving acceleration for distributed economic dispatch in smart grids over directed networks. IEEE Trans Netw Sci Eng 2020;7(3):1988–99.

[34] Chen G, Li Z. Distributed optimal resource allocation over strongly connected digraphs: A surplus-based approach. Automatica 2021;125.

[35] Boyd SP, Vandenberghe L. Convex optimization. Cambridge, UK: Cambridge Univ. Press; 2004.

[36] Seyranian AP, Mailybaev AA. Multiparameter stability theory with mechanical applications. World Scienrific; 2004.

[37] Zhang W, Liu W, Wang X, Liu L, Ferrese F. Online optimal generation control based on constrained distributed gradient algorithm. IEEE Trans Power Syst 2015;30(1):35–45.

[38] Liu Z, Xu Y, Zhang C, et al. A blockchain-based trustworthy collaborative power trading scheme for 5G-enabled social internet of vehicles. Digit Commun Netw 2022.