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Formation control of multi-agent systems with stochastic switching topology and time-varying communication delays

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Abstract: In this study, the authors formulate and study the distributed formation problem of multi-agent systems (MASs) with randomly switching topologies and time-varying delays. The non-linear dynamics of each agent at different time intervals corresponds to different switching mode, and the switching design reflects the changing of travelling path in practical systems. The communication topology of the system switches among finite modes that are governed by a finite-state Markov process. On the basis of artificial potential functions, a formation controller is designed in a general form. Sufficient conditions for stochastic formation stability of the MAS are obtained in terms of a Lyapunov functional and linear matrix inequalities. Some heuristic rules to design a formation controller for the MAS are then presented. Finally, specific potential functions are discussed and corresponding simulation results are provided to demonstrate the effectiveness of the proposed approach.

1 Introduction

In recent years, we have witnessed a growing recognition and attention on distributed coordination of multi-agent systems (MASs) across a wide range of disciplines, because of increasing technological advances in communication and computation. Coordination algorithms have been applied in cooperative control of unmanned air/underwater vehicles (UAVs) and spacecraft [1], formation control [2], distributed sensor networks [3] and spacecraft attitude alignment [4]. As one of the most important and fundamental issues in the coordination control of MASs, formation aims to achieve and maintain a desired structure that depends on the specific task. A formation algorithm (or strategy) is an interaction principle that specifies the information exchange between agents. Numerous methods have been applied to deal with these problems, such as leader–follower [1], virtual structure [5], artificial potential field [6–8], sampled control [9], etc.

Artificial potential functions have been widely developed in robot navigation and coordination control for MASs including formation, path-planning, collision and obstacle avoidance [8, 10]. Derived from the potential force laws between agents–agents, agents–targets and agents–obstacles, diverse potential functions are employed in MASs to achieve complicated behaviours. It is crucial to well design artificial potential functions (APFs) because different potentials, even involved in the same MAS, might result in unpredicted and undesired performances. In particular, the limitation of existing multiple local minima in the potential function leads to a non-reachable problem. By choosing an appropriate potential function in this paper, we show that the MASs will follow a prescribed trajectory and keep a desirable shape.

Many works on MASs have been devoted to investigate the formation problems with continuous- or discrete-time dynamics. In [11], a typical continuous-time consensus model was described, which considered the directed networks in fixed and switching cases, and the undirected networks with communication time delays. Consensus problem of discrete-time MASs with fixed topology is explored in [12]. However, many MASs are hybrid in the sense that they exhibit both discrete- and continuous-state dynamics. Note that dynamical behaviours of MASs are subject to not only agent dynamics but also communication topology. As an important class of hybrid systems, switching systems are used to describe the communication connections of MASs. For example, in [13, 14], the switching systems consist of a family of subsystems and are controlled by some logical rules. In addition, based on the graph theory and non-negative matrix theory, the asynchronous consensus problems of continuous-time MASs with time-dependent communication topology and time-varying delays are studied in [15]. However, most papers concerning switching topologies failed to illustrate the specific switching mechanism among the subsystems. In this paper, a finite-state Markov process is introduced to describe the jumping communication topologies.

Furthermore, the communication delays among agents frequently exist in most practical systems. It is well known that formation problems with switching topology and time-varying delays are more challenging than those with fixed topology and without delays. In [11], the consensus problem of continuous-time MASs with communication delays is discussed. One thing worth mentioning is that in prior works, it was often assumed that time-delays are constants [12]. Recently, consensus problems with time-varying delays can be found in [13, 15]. Moreover, the non-uniform time-varying delays are taken into account in a continuous- and discrete-time setting for consensus problems in [11], respectively. However, to the best of our knowledge, there are few works on stochastic consensus problem of MASs with ubiquitous communication delays. Since the MASs modelled in this paper are composed of homogenous agents which have the same communication capability, we assume that the time-varying delays are uniform for each agent.

In real applications of MASs, one may face the following issues:

1. The communication topology of agents is randomly switching, even the dynamics of each agent is switching.
2. It is inevitable to discuss the effect because of the existence of communication delays, which are commonly time-varying and even unknown.

Based on above consideration, a formation problem for MASs with stochastic switching topology and time-varying communication delays is discussed in this paper. Specifically, a switching non-linear function is presented to characterise the changes of navigation-track, and the switching communication topology is determined by a Markov chain taking values in a finite set. With respect to APFs and behaviour rules of agents, a distributed formation algorithm for MASs is proposed. Subsequently, the stochastic Lyapunov functional is employed to theoretically analyse this time-delay system, which is modelled by delayed differential equations. The sufficient conditions are provided in terms of a set of linear matrix inequalities (LMIs) and each LMI corresponds to one possible subsystem.

This paper is organised as follows. In Section 2, a model of MAS with switching communication topology and time-varying delays is presented. The stochastic formation-stability analysis is performed by a stochastic Lyapunov functional in Section 3. Section 4 contains some numerical examples under specific potential functions. Finally, in Section 5, concluding remarks are stated.

2 Problem formulation

Let $\mathbf{J} = [t_0, +\infty)$, $\mathbf{R}^+ = (0, +\infty)$, $\mathbf{R}_+ = [0, +\infty)$ and $\mathbf{R}^n$ denotes the n-dimensional Euclidean space. For $x = (x_1, \ldots, x_n) \in \mathbf{R}^n$, the norm of x is denoted as $\|x\| := \left(\sum_{i=1}^{n} x_i^2\right)^{(1/2)}$. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the maximum and minimum eigenvalues of corresponding matrix, respectively. In the following, if not explicitly stated, matrices are assumed to have compatible dimensions, and the identity matrix of order n is denoted as I_n (or simply as I if no confusion arises). $\mathbf{E}[\cdot]$ stands for the mathematical expectation. The asterisk * in a matrix is used to replace a term induced by symmetry.

Many real-world MASs have the following properties: every agent has its own dynamic behaviour, which may switch among different modes, and specifically, each corresponds to one navigating path in this paper; the agents can exchange their information, such as velocity and position in world coordinate system, through wired or wireless communications, but the interconnection structure of system is time-varying; information exchange is often with time-delays, which may be (randomly) time-varying and unknown. Consider a MAS consisting of N-identical nodes with the following n-dimensional dynamics

$$\begin{aligned}\dot{x}_i(t) = f^{\sigma_1(t)}(t, x_i) + \sum_{j=1}^{N} D_{ij}(\sigma_2(t))\\ [x_j(t-\tau(t)) - x_i(t-\tau(t))] + Bu_i(t)\end{aligned} \tag{2.1}$$

where $i = 1, 2, \ldots, N$, $x_i = (x_{i_1}, \ldots, x_{i_n})^\top \in \mathbf{R}^n$ are the state variables of agent i, and B is known to be positive-definite matrix. $u_i(t) \in \mathbf{R}^n$ denotes the external control law and will be derived from an APF at the end of this section. $D(\sigma_2(t)) = [D_{ij}(\sigma_2(t))]_{N\times N}$ are the switching coupling configuration matrix of MAS, describing the communication relationships of agents. $D_{ij}(\sigma_2(t))$ are the functions of the random jumping process $\sigma_2(t)$. $\sigma(t)$ is a continuous-time discrete-state Markov jump process, that is, $\sigma_2(t)$ takes discrete values in a predetermined finite set $\mathcal{M} = \{1, 2, \ldots, m_2\}$ with transition probability matrix $\Pi = [\pi_{rl}]_{m_2\times m_2}$ which is given by

$$\Pr\{\sigma_2(t+\Delta) = l | \sigma_2(t) = r\} = \begin{cases}\pi_{rl}\Delta + o(\Delta), & r \neq l\\ 1 + \pi_{rr}\Delta + o(\Delta), & r = l\end{cases} \tag{2.2}$$

where $\Delta > 0$, $\pi_{rl} \geq 0$ is the mode transition rate from r to l $(r \neq l)$, $\pi_r := \pi_{rr} = -\sum_{l=1, l\neq r}^{m_2} \pi_{rl}$ for each mode r $(r = 1, 2, \ldots, m_2)$, and $o(\Delta)/\Delta \to 0$ as $\Delta \to 0$. To simplify the notation, $D_{ij}(\sigma_2(t))$ will be replaced by $D_{ij}^{\sigma_2}$ in following analysis. If there is a connection between agent i and j $(j \neq i)$, then $D_{ij}^r = D_{ji}^r > 0$; otherwise, $D_{ij}^r = D_{ji}^r = 0$, and the diagonal elements of matrix D^r are defined as $D_{ii}^r = -\sum_{j=1, j\neq i}^{N} D_{ij}^r = -\sum_{j=1, j\neq i}^{N} D_{ji}^r$.

Vector-valued functions $f^{\sigma_1}(t, x_i) \in \mathbf{R}^n$ are continuously differentiable, representing the dynamic trajectories of MAS (2.1). It is worthy to mention that the non-linear term $f^{\sigma_1}(t, x_i)$ can be reduced to a time-function as presented in [10], where the dynamics of system only depends on the time, or state-dependent formula such as the synchronisation problem [16]. In engineering applications, the dynamical trajectories, which depend on current position and/or time, are created intelligently by the embedded processors based on sensor readings or image data. Without loss of generality, a general form which explicitly depends on the time and states of agents, is proposed in this paper.

The switching signal $\sigma_1(t)$:$\mathbf{R}_+ \to \{1, 2, \ldots, m_1\}$ is a piecewise constant function. In different time interval, each

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subsystem of MAS (2.1) corresponds to distinct switching modes. Similar to the complex spatiotemporal switching network [14], $f^{\sigma_1}(t, x_i) \in \{f^1, \ldots, f^{m_1}\}$ and $D^{\sigma_2} \in \{D^1, \ldots, D^{m_2}\}$ take constant mode at every time interval between two consecutive switching times. Assume that, for all $i = 1, \ldots, N$, $f^{\sigma_1}(t, x_i)$ satisfy Lipschitz condition with respect to x_i, that is, for any $x_i(t) \in \mathbf{R}^n$, $x_j(t) \in \mathbf{R}^n$ and $t \geq t_0$ there exists a positive constant $\phi < \infty$ such that

$$\|f^{\tilde{r}}(t, x_j) - f^{\tilde{r}}(t, x_i)\| \leq \phi\|x_j - x_i\|, \quad \tilde{r} = 1, \ldots, m_1 \quad (2.3)$$

Remark 2.1: In this paper, only the time-dependent switching signal $\sigma_1(t)$ is considered, but other switching signals can be found including state-dependent switching [17], logic-based switching [18], parameter-dependent switching [19], event-driven switching [20], etc. In reality, agents with collective computational abilities usually determine the following path based on the information from overall system and environment, when some emergent events are detected, for example, partial agents lost and obstacles approaching. Most of switching frameworks are determined by practical application in engineering, however, for convenience of theoretical analysis, only the time-dependent switching is considered in this paper (the main results could be extended to other switching signals).

Similar to the assumption in [21], the time-varying delay $\tau(t) > 0$ is a continuously differentiable function such that

$$0 \leq \tau(t) \leq p < \infty, \quad \dot{\tau}(t) \leq q < 1, \quad \forall t \geq t_0 \quad (2.4)$$

In the following discussion, time-dependent delay $\tau(t)$ is written as τ for short.

Remark 2.2: In most real engineering, time-delay does not change promptly. In particular, when the MASs obtain the predetermined formation shape, the agents will keep the fixed relative positions in the formation till another task is trigged. As a result, the communication delays between agents fluctuate smoothly, which is consistent to the assumption $\dot{\tau}(t) \leq q < 1$. Additionally, since the agents discussed in this paper are homogeneous, only the uniform delays are taken into account.

So far, the potential field theory has been widely used for swarm aggregation, formation control, and multi-agent coordination [6, 8]. The negative gradient of potential function is usually interpreted as an artificial force acting on the agents and manipulating their motions, that is, $F^{\mathrm{A}}(y) = -\nabla_y J^{\mathrm{A}}(\|y\|)$ and $F^{\mathrm{R}}(y) = -\nabla_y J^{\mathrm{R}}(\|y\|)$, where y is a relative position vector between agents, J^{A} and J^{R} are APFs of the attraction and repulsion between individuals, respectively. The formation controller $u_i(t)$ will be explored by combining the attractive force F^{A} and the repulsive force F^{R} in some ways. Commonly, the attractive term is used to keep the compactness of system, and the repulsive term acts on ensuring collision avoidance.

In this paper, the formation controller for agent i is given by

$$u_i(t) = -\nabla_{x_i} \sum_{j=1, j\neq i}^{N} J(\|x_j - x_i - w_{ij}\|)$$

where $J(\cdot)$ is potential function between agent i and j, and $w_{ij} \in \mathbf{R}^n$ is the desired formation vector for agent i and agent j with the properties

$$w_{ij} = -w_{ji} \quad \text{and} \quad w_{ii} = 0 \quad (2.5)$$

The norm $\|w_{ij}\|$ is the equilibrium distance, at which the attraction and the repulsion get balance. Subsequently, the entire potential J implemented between each couple of agents is derived from the interplay between the attractive and repulsive potentials, and J equals zero when agents obtain the desired formation vector. The potential function can be specified based on different structures and/or behaviours of the MAS. However, different potentials might result in different performances even for the same MAS [8]. Associated with the real-world formation and the characteristics of potential function, we assume $J(\|x_j - x_i - w_{ij}\|)$ $(i, j = 1,\ldots,N)$ possesses the following properties:

(A) $J(\|x_j - x_i - w_{ij}\|)$ has a unique minimum at the desired position. More explicitly, if and only if $x_j - x_i = w_{ij}$, the following equation holds

$$\nabla_{x_i} \sum_{j=1, j\neq i}^{N} J(\|x_j - x_i - w_{ij}\|) = 0.$$

Moreover, when $\|x_j - x_i\| > \|w_{ij}\|$, $F^{\mathrm{A}}(\|x_j - x_i - w_{ij}\|) > F^{\mathrm{R}}(\|x_j - x_i - w_{ij}\|)$; when $\|x_j - x_i\| < \|w_{ij}\|$, $F^{\mathrm{A}}(\|x_j - x_i - w_{ij}\|) < F^{\mathrm{R}}(\|x_j - x_i - w_{ij}\|)$.
(B) $J(\|x_j - x_i - w_{ij}\|)$ is differential and there exists a continuous function $g(\|x_j - x_i - w_{ij}\|)$:$\mathbf{R}^+ \to \mathbf{R}_+$ such that

$$-\nabla_{x_i} J(\|x_j - x_i - w_{ij}\|) = (x_j - x_i - w_{ij}) g(\|x_j - x_i - w_{ij}\|) \quad (2.6)$$

For simplicity, $g(\|x_j - x_i - w_{ij}\|)$ is written as g_{ij}, for all $i, j = 1, \ldots, N$. From the perspective of potential field theory, the direction of the potential force and the vector $x_j(t) - x_i(t) - w_{ij}$ should be the same. Owing to the continuity of the positive function g_{ij}, we can assume that there exists positive constant $\underline{g}$ such that $g_{ij} \geq \underline{g} > 0$.

Remark 2.3: It is worth mentioning that the existence of multiple local minima in the potential function results in achieving only local convergence and unexpected formation. Owing to the limitation of local minima, Assumption (A) is a necessary condition assuring the achievement of formation.

Remark 2.4: The term $-\nabla_{x_i} J(\|x_j - x_i - w_{ij}\|)$ in the Assumption (B) represents the potential force between individuals, which is a vector quantity involving the direction $(x_j - x_i - w_{ij})/(\|x_j - x_i - w_{ij}\|)$. The term g_{ij} determines the attraction–repulsion relationship between individuals. In view of the needs for proof and the fact that the direction of potential force should be consistent with the vector $x_j - x_i - w_{ij}$, the Assumption (B) is proposed. Compared with the conditions on potential function in [8, 16], the existence of lower bound on g_{ij} is necessary as the coupling term and non-linear term are considered in the MAS (2.1), which will be explicitly illustrated in the next section.

Similarly to our previous work in [10], for practical application, the MAS has the limited utilisation range, that is, $\max_{i,j=1,\ldots,N} \|x_j - x_i\| < \mathbb{L}$, where $\mathbb{L}$ represents the maximum utilisation range. For the given formation vectors, there exists $\overline{w} = \max_{i,j=1,\ldots,N} \|w_{ij}\|$, which commonly has $\overline{w} < \mathbb{L}$.

Initially, it may seem as if the lower bound $\underline{g}$ is a restrictive assumption, since g_{ij} must be known for all $i, j = 1, 2, \ldots, N$. However, note that once the knowledge of $J(\|x_j - x_i - w_{ij}\|)$ is available and associated with the constraints of limited utilisation range of MAS, computing $\underline{g}$ is straightforward. In practice, it is easy to find potential models satisfied Assumptions (A) and (B), such as the ones considered in [6, 8].

Thus, rewrite the formation protocol of MAS (2.1) as

$$u_i(t) = \sum_{\substack{j=1 \\ j \neq i}}^{N} (x_j - x_i - w_{ij}) g_{ij} \tag{2.7}$$

Now we are in a position to formally state the problem being discussed in this paper. First, derive a control law of the form (2.7), which guarantees the stochastic formation stability of the MAS (2.1). Especially, the MAS with stochastic switching topology and time-varying delays is considered in this paper. As an additional goal, define some heuristic rules to design a formation controller (2.7) for MAS, based on the prior stochastic stable conditions.

3 Analysis of formation stability

In this section, stochastic formation stability of the MAS with communication delays and switching topology is presented.

Before proceeding to the theoretical analysis, define a disagreement vector

$$e_{ij}(t) = X_{ij}(t) - w_{ij}$$

where $X_{ij}(t) = x_j(t) - x_i(t)$ is the relative position between agent i and j. According to (2.1), the time derivative of $e_{ij}(t)$ is

$$\begin{aligned} \dot{e}_{ij}(t) &= \dot{x}_j(t) - \dot{x}_i(t) \\ &= f^{\sigma_1}(t, x_j) - f^{\sigma_1}(t, x_i) \\ &\quad + \sum_{k=1}^{N} (D_{jk}^{\sigma_2} X_{jk}(t-\tau) - D_{ik}^{\sigma_2} X_{ik}(t-\tau)) \\ &\quad + B \sum_{k=1}^{N} (g_{jk} e_{jk}(t) - g_{ik} e_{ik}(t)) \end{aligned} \tag{3.1}$$

It is easy to see that $\dot{e}_{ij}(t) = \dot{X}_{ij}(t)$, for $i, j = 1, \ldots, N$. Obviously, if all the disagreement vectors $e_{ij}(t)$ $(i, j = 1, \ldots, N)$ uniformly asymptotically tend to zero, then the dynamical system (2.1) realises formation stability. Namely, the formation stability of system (2.1) is now equivalent to the problem of stabilising system (3.1) using a suitable choice of the control law, such that

$$\lim_{t \to \infty} \sum_{i=1}^{N} \sum_{j=1}^{N} \|e_{ij}(t)\| = 0$$

In the subsequent discussion, assume that for all $\delta \in [-\tau, 0]$, a finite scalar $\varepsilon > 0$ exists such that

$$\|e_{ij}(t+\delta)\| \leq \varepsilon \|e_{ij}(t)\| \tag{3.2}$$

As indicated by [22], this assumption does not bring the conservatism, since ε can be chosen arbitrarily.

The following definitions and Lemma are needed to facilitate the development of the main results in this paper.

Definition 3.1 [(Formation stabilisability) [10]]: If a formation controller $u(t)$ exists, which makes $\|x_j(t) - x_i(t) - \Delta_{ij}\| \to 0$ hold for every $(i, j) = \{(i, j) | i, j = 1, 2, \ldots, N\}$ when $t \to \infty$, then the MAS is said to be asymptotically formation stabilisable.

Lemma 3.1 [21]: For any vector $x, y \in \mathbf{R}^n$, the matrix inequality $2x^\top y \leq x^\top x + y^\top y$ holds.

Definition 3.2: The formation of MAS (2.1) is said to be stochastically stabilisable if, for all initial mode $\sigma_2(0) \in \mathcal{M}$, there exists a formation control law satisfying

$$\begin{aligned} &\lim_{T \to \infty} \mathbf{E}\left\{ \int_0^T \sum_{i=1}^{N-1} \sum_{j>i}^{N} e_{ij}^\top(t) e_{ij}(t)\, \mathrm{d}t \,|\, e(0), \sigma_2(0) \right\} \\ &\leq \sum_{i=1}^{N-1} \sum_{j>i}^{N} e_{ij}^\top(0) \widetilde{U} e_{ij}(0),\ i, j = 1, \ldots, N \end{aligned} \tag{3.3}$$

where $\widetilde{U}$ is a symmetric positive-definite matrix and $e(0)$ is an initial condition defined as

$$e(0) = [e_{11}^\top(t), \ldots, e_{1N}^\top(t), \ldots, e_{ij}^\top(t), \ldots, e_{NN}^\top(t)]^\top |_{t=0}.$$

This definition is similar to that of stochastic stabilisability of uncertain linear state-delay systems with Markovian jumping parameters [23]. Under the above definition, stochastic stabilisability of the formation in an MAS means that there exists a formation control protocol that drives the disagreement $e_{ij}(t)$ from any given initial $(e(0), \sigma_2(0))$ asymptotically to the origin in the mean square sense. Otherwise, if any of $e_{ij}(t)$ does not converge to zero, there exists a positive uniform lower bound $\underline{U}$ of $\mathbf{E}\{e_{ij}^\top(t) e_{ij}(t)\}$ for all t. Then the left-hand side of (3.3) is greater or equal to $\lim_{T \to \infty} \underline{U} T$, which goes to infinity.
In order to achieve stochastic formation stability, we design a formation controller to guarantee the MAS to form a desired shape asymptotically.

Theorem 3.1: The formation MAS (2.1) is stochastically stabilisable, for all $\tilde{r} = 1, \ldots, m_1$, $r = 1, \ldots, m_2$, $i, j = 1, \ldots, N$, if positive definite matrices $Q > 0$ and $P^r > 0$, and a constant $\underline{g} > 0$ exist, satisfying the coupling matrix inequalities

$$\begin{bmatrix} \Xi^r & -D_{ij}^r P^r & 0 & -D_{ij}^r P^r \\ * & -\tilde{q} Q & 0 & 0 \\ * & * & -\underline{g} B^\top P^r + \dfrac{\phi^2}{N} I & \underline{g} B^\top P^r \\ * & * & * & -\underline{g} B^\top P^r \end{bmatrix} < 0 \tag{3.4}$$

where

$$\Xi^r = Q + \frac{1}{N}\sum_{l}^{m_2} \pi_{rl} P^l - P^r\left(\underline{g}B - \frac{1}{N}P^r\right)$$

and $\widetilde{q} = 1 - q > 0$.

Proof: Let the topology mode at time t be D^r, that is $\sigma_2(t) = r \in \mathcal{M}$. Choose the stochastic Lyapunov functional in the form of

$$\begin{aligned} V(e(t), \sigma_2(t) = r) &\equiv V(e, r) \\ &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(e_{ij}^\top(t)\frac{P^r}{N}e_{ij}(t) + \int_{t-\tau}^{t} e_{ij}^\top(\theta)Qe_{ij}(\theta)\,\mathrm{d}\theta\right) \end{aligned}$$

where Q is a constant positive-definite matrix, and P^r is a constant positive-definite matrix for each r.

Consider the weak infinitesimal operator $\mathcal{A}$ [22] of the stochastic process $\{\sigma_2(t)\}$ $(t \geq 0)$, is give by (see (3.5)) where

$$\begin{aligned} V_1(e(t), r) = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} e_{ij}^\top(t)P^r(D_{jk}^r X_{jk}(t-\tau) \\ - D_{ik}^r X_{ik}(t-\tau)) \end{aligned}$$

and

$$\begin{aligned} V_2(e(t), r) = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} e_{ij}^\top(t)(P^rB + B^\mathrm{T}P^r) \\ (g_{jk}e_{jk}(t) - g_{ik}e_{ik}(t)) \end{aligned}$$

From Lemma 3.1 and inequality (2.3), the first term of inequality (3.5) becomes for all $\widetilde{r} = 1, \ldots, m_1$

$$\begin{aligned} & e_{ij}^\top(t)\frac{P^r}{N}(\widetilde{f^r}(t, x_j) - \widetilde{f^r}(t, x_i)) \\ & \leq e_{ij}^\top(t)\frac{P^r(P^r)^\top}{2N}e_{ij}(t) + \frac{1}{2N}\|\widetilde{f^r}(t, x_j) - \widetilde{f^r}(t, x_i)\|^2 \\ & \leq e_{ij}^\top(t)\frac{P^r(P^r)^\top}{2N}e_{ij}(t) + \frac{\phi^2}{2N}X_{ij}^\top(t)X_{ij}(t) \end{aligned} \tag{3.6}$$

Since $e_{ij}(t) = -e_{ji}(t)$, $e_{ii}(t) = 0$, the $V_1(e(t), r)$ can be

$$\begin{aligned} V_1(e(t), r) &= -\frac{2}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} D_{jk}^r e_{ji}^\top(t)P^r X_{jk}(t-\tau) \\ &= -\frac{2}{N}\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{j<k}^{N} D_{jk}^r e_{ji}^\top(t)P^r X_{jk}(t-\tau) \\ &\quad -\frac{2}{N}\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{k<j}^{N} D_{jk}^r e_{ji}^\top(t)P^r X_{jk}(t-\tau) \\ &= -\frac{2}{N}\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{j<k}^{N} D_{jk}^r e_{ji}^\top(t)P^r X_{jk}(t-\tau) \\ &\quad -\frac{2}{N}\sum_{i=1}^{N}\sum_{k=1}^{N-1}\sum_{j<k}^{N} D_{kj}^r e_{ki}^\top(t)P^r X_{kj}(t-\tau) \end{aligned}$$

Noted that $e_{ji}(t) + e_{ik}(t) = e_{jk}(t)$, thus

$$\begin{aligned} V_1(e(t), r) &= -\frac{2}{N}\sum_{i=1}^{N}\sum_{j=1}^{N-1}\sum_{j<k}^{N} D_{jk}^r e_{jk}^\top(t)P^r X_{jk}(t-\tau) \\ &= -2\sum_{i=1}^{N-1}\sum_{j>i}^{N} D_{ij}^r e_{ij}^\top(t)P^r X_{ij}(t-\tau) \\ &= -2\sum_{i=1}^{N-1}\sum_{j>i}^{N} D_{ij}^r e_{ij}^\top(t)P^r e_{ij}(t-\tau) \\ &\quad - 2\sum_{i=1}^{N-1}\sum_{j>i}^{N} D_{ij}^r e_{ij}^\top(t)P^r w_{ij} \end{aligned} \tag{3.7}$$

Similar to $V_1(e(t), r)$ and by the properties of w_{ij} $(i, j = 1, \ldots, N)$ showed in (2.5), $V_2(e(t), r)$ is treated as

$$\begin{aligned} V_2(e(t), r) &= -\sum_{i=1}^{N-1}\sum_{j>i}^{N} g_{ij}e_{ij}^\top(t)(P^rB + B^\top P^r)e_{ij}(t) \\ &\leq -\underline{g}\sum_{i=1}^{N-1}\sum_{j>i}^{N} e_{ij}^\top(t)(P^rB + B^\top P^r)e_{ij}(t) \\ &= -\underline{g}\sum_{i=1}^{N-1}\sum_{j>i}^{N}(e_{ij}^\top(t)P^rBe_{ij}(t) + w_{ij}^\mathrm{T}B^\top P^r w_{ij} \\ &\quad + X_{ij}^\top(t)B^\top P^r X_{ij}(t) - 2X_{ij}^\top(t)B^\top P^r w_{ij}) \end{aligned} \tag{3.8}$$

$$\begin{aligned} \mathcal{A}V(e(t), \sigma_2(t)) &= \lim_{\Delta\to\infty}\frac{1}{\Delta}[\mathbf{E}\{V(e(t+\Delta), \sigma(t+\Delta))|e(t),\sigma_2(t) = r\} - V(e(t), \sigma_2(t) = r)] \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} e_{ij}^\top\left(\frac{P^r}{N}\dot{e}_{ij} + \sum_{l=1}^{m_2}\left(\frac{\pi_{rl}P^l}{2N}\right)e_{ij}\right) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}[e_{ij}^\top(t)Qe_{ij}(t) - (1-\dot{\tau})e_{ij}^\top(t-\tau)Qe_{ij}(t-\tau)] \\ &\leq \sum_{i=1}^{N-1}\sum_{j>i}^{N}\left\{\left[e_{ij}^\top\frac{2P^r}{N}(\widetilde{f^r}(t, x_j) - \widetilde{f^r}(t, x_i))\right] + V_1(e(t), r) + V_2(e(t), r)\right. \\ &\quad \left. - \left[(1-q)e_{ij}^\top(t-\tau)Qe_{ij}(t-\tau) - e_{ij}^\top(t)\left(\frac{1}{N}\sum_{l=1}^{m_2}\pi_{rl}P^l + Q\right)e_{ij}(t)\right]\right\} \end{aligned} \tag{3.5}$$

Then associated (3.6), (3.7) and (3.8) with (3.5), the weak infinitesimal $\mathcal{A}V$ becomes

$$\begin{aligned}
&\mathcal{A}V(e(t),\sigma_2(t))\\
&\le \sum_{i=1}^{N-1}\sum_{j>i}^{N}\Bigg\{e_{ij}^{\top}(t)\bigg(Q+\frac{1}{N}\sum_{l=1}^{m_2}\pi_{rl}P^l - P^r(\underline{g}B-\frac{1}{N}P^r)\bigg)e_{ij}(t)\\
&\quad +2\underline{g}X_{ij}^{\top}(t)B^{\top}P^r w_{ij} - \underline{g}w_{ij}^{\top}B^{\top}P^r w_{ij} - 2D_{ij}^r e_{ij}^{\top}(t)P^r w_{ij}\\
&\quad -X_{ij}(t)\bigg(\underline{g}B^{\top}P^r - \frac{\phi^2}{N}I\bigg)X_{ij}(t) - \tilde{q}e_{ij}^{\top}(t-\tau)\\
&\quad \times Qe_{ij}(t-\tau) - 2D_{ij}^r e_{ij}^{\top}(t)P^r e_{ij}(t-\tau)\Bigg\}\\
&= \sum_{i=1}^{N-1}\sum_{j>i}^{N}\xi_{ij}^{\top}(t)\Omega^r\xi_{ij}(t) < 0
\end{aligned}$$

where

$$\xi_{ij}(t) = \begin{bmatrix} e_{ij}^{\top}(t) & e_{ij}^{\top}(t-\tau) & X_{ij}^{\top}(t) & w_{ij}^{\top}\end{bmatrix}^{\top}$$

and

$$\Omega^r = \begin{bmatrix} \Xi^r & -D_{ij}^r P^r & 0 & -D_{ij}^r P^r\\ * & -\tilde{q}Q & 0 & 0\\ * & * & -\underline{g}B^{\top}P^r + \frac{\phi^2}{N}I & \underline{g}B^{\top}P^r\\ * & * & * & -\underline{g}B^{\top}P^r\end{bmatrix}$$

Clearly, it is easy to prove that $\|e_{ij}\| < \|\xi_{ij}\|$, and note that $\Omega^r < 0$ and $P^r > 0$. Thus, for $t>0$

$$\begin{aligned}
\frac{\mathcal{A}V(e,r)}{V(e,r)} &\le \frac{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\xi_{ij}^{\top}(t)\Omega^r\xi_{ij}(t)}{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Big(e_{ij}^{\top}(t)\frac{P^r}{N}e_{ij}(t)+\int_{t-\tau}^{t}e_{ij}^{\top}(\theta)Qe_{ij}(\theta)\,\mathrm{d}\theta\Big)}\\
&= \frac{-\sum_{i=1}^{N-1}\sum_{j>i}^{N}\xi_{ij}^{\top}(t)(-\Omega^r)\xi_{ij}(t)}{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\Big(e_{ij}^{\top}(t)\frac{P^r}{N}e_{ij}(t)+\int_{t-\tau}^{t}e_{ij}^{\top}(\theta)Qe_{ij}(\theta)\,\mathrm{d}\theta\Big)}\\
&\le -\min_{r\in\mathcal{M}}\left\{\frac{\lambda_{\min}(-\Omega^r)}{\lambda_{\max}(P^r)/N+p\varepsilon^2\lambda_{\max}(Q)}\right\}
\end{aligned}$$

Define

$$\alpha := \min_{r\in\mathcal{M}}\left\{\frac{\lambda_{\min}(-\Omega^r)}{\lambda_{\max}(P^r)/N+p\varepsilon^2\lambda_{\max}(Q)}\right\}$$

Obviously, $\alpha>0$, so $\mathcal{A}V(e,r)\le -\alpha V(e,r)$. Then using Dynkin's formula [24], for all $\sigma_2(0)\in\mathcal{M}$, one has

$$\begin{aligned}
&\mathbf{E}\{V(e(t),\sigma_2(t))\} - V(e(0),\sigma_2(0))\\
&= \mathbf{E}\left\{\int_0^t \mathcal{A}V(e(s),\sigma_2(s))\,\mathrm{d}s\right\} \le -\alpha\int_0^t \mathbf{E}\{V(e(s),\sigma_2(s))\}\mathrm{d}s
\end{aligned}$$

The Gronwall–Bellman lemma [24] makes

$$\mathbf{E}\{V(e(t),\sigma_2(t))\} \le \exp(-\alpha t)V(e(0),\sigma_2(0))$$

Since $Q>0$, one obtains

$$\mathbf{E}\left\{\int_{t-\tau}^{t}\left(\sum_{i=1}^{N-1}\sum_{j>i}^{N}e_{ij}^{\top}(s)Qe_{ij}(s)\right)\mathrm{d}s\right\} > 0$$

Consequently

$$\begin{aligned}
&\mathbf{E}\left\{\frac{1}{N}\sum_{i=1}^{N-1}\sum_{j>i}^{N}e_{ij}^{\top}(t)P^r e_{ij}(t)|e(0),\sigma_2(0)\right\}\\
&= \mathbf{E}\{V(e,r)|e(0),\sigma_2(0)\}\\
&\quad -\mathbf{E}\left\{\int_{t-\tau}^{t}\left(\sum_{i=1}^{N-1}\sum_{j>i}^{N}e_{ij}^{\top}(s)Qe_{ij}(s)\right)\mathrm{d}s|e(0),\sigma_2(0)\right\}\\
&\le \exp(-\alpha t)V(e(0),r)
\end{aligned}$$

As a result, one can obtain

$$\begin{aligned}
&\mathbf{E}\left\{\int_0^{\mathrm{T}}\sum_{i=1}^{N-1}\sum_{j>i}^{N}e_{ij}^{\top}(t)P^r e_{ij}(t)\,\mathrm{d}t|e(0),\sigma_2(0)\right\}\\
&\le N\int_0^{\mathrm{T}}\exp(-\alpha t)\,\mathrm{d}tV(e(0),r)\\
&= -\frac{N}{\alpha}[\exp(-\alpha T)-1]V(e(0),r)
\end{aligned}\tag{3.9}$$

Taking limit as $T\to\infty$, matrix inequality (3.9) yields

$$\begin{aligned}
&\lim_{T\to\infty}\mathbf{E}\left\{\int_0^{\mathrm{T}}\sum_{i=1}^{N-1}\sum_{j>i}^{N}e_{ij}^{\top}(t)P^r e_{ij}(t)\,\mathrm{d}t|e(0),\sigma_2(0)\right\}\\
&\le \lim_{T\to\infty}\left\{-\frac{N}{\alpha}[\exp(-\alpha T)-1]V(e(0),r)\right\}\\
&\le \frac{N}{\alpha}\sum_{i=1}^{N-1}\sum_{j>i}^{N}\left[e_{ij}^{\top}(0)\left(\frac{\lambda_{\max}(P^r)}{N}+\varepsilon^2 p\lambda_{\max}(Q)\right)Ie_{ij}(0)\right]
\end{aligned}$$

Since $P^r>0$, for each $r\in\mathcal{M}$, one has

$$\begin{aligned}
&\lim_{T\to\infty}\mathbf{E}\left\{\int_0^{\mathrm{T}}\sum_{i=1}^{N-1}\sum_{j>i}^{N}e_{ij}^{\top}(t)e_{ij}(t)\,\mathrm{d}t|e(0),\sigma_2(0)\right\}\\
&\le \sum_{i=1}^{N-1}\sum_{j>i}^{N}e_{ij}^{\top}(0)\tilde{U}e_{ij}(0)
\end{aligned}$$

where

$$\tilde{U} = \max_{r\in\mathcal{M}}\left\{\frac{\lambda_{\max}(P^r)+\varepsilon^2 pN\lambda_{\max}(Q)}{\alpha\lambda_{\min}(P^r)}\right\}I$$

Based on Definition (3.2), one can prove that the formation of MAS (2.1) under control law (2.7) is stochastically stable. □

Remark 3.1: The coupling term in (2.1) may help for the group cohesion. However, in order to achieve the desired formation, the controller (2.7) is designed to make the attraction and repulsion forces balance. In view of the influence of the non-linear term, we introduce the

Assumption (B), which can guarantee the diagonal elements in inequality (3.4) to be negative.

Remark 3.2: It should be noted that the proposed conditions (3.4) are formulated in terms of LMIs. Therefore by using MATLAB LMI Toolbox, for a given MAS, the lower bound $\underline{g}$ can be efficiently calculated by optimising a generalised eigenvalue problem from LMIs (3.4).

When the MAS (2.1) achieves desired formation, all the agents move into a stable group along certain trajectory which is dependent on $f^{\sigma_1}(t, x_i) + \sum_{j=1}^{N} D_{ij}^r w_{ij}$ $(i, j = 1, \ldots, N, r = 1, \ldots, m_2)$.

Regrading to Theorem 3.1, the LMIs (3.4) hold under a priori $\underline{g}$, which depends on the specific form of controller. For the sake of generality, we develop the following theorem to drop the explicit dependence of $\underline{g}$.

Theorem 3.2: The formation of MAS (2.1) is stochastically stabilisable, for $\tilde{r} = 1, \ldots, m_1$, $r = 1, \ldots, m_2$, $i, j = 1, \ldots, N$, if there exist positive-definite matrices R, Z^r and Y^r, such that the coupled LMIs

$$\begin{bmatrix} -BY^r + \dfrac{\pi_r}{N} Z^r & 0 & -D_{ij}^r Z^r & \chi^r \\ * & -Y^r B^\top & Y^r B^\top & 0 \\ * & * & -Y^r B^\top & 0 \\ * & * & * & -\Upsilon^r \end{bmatrix} < 0 \quad (3.10)$$

hold, where

$$\chi^r = \Big[\sqrt{\pi_{r1}/N} Z^r, \ \ldots \ \sqrt{\pi_{r,r-1}/N} Z^r, \sqrt{\pi_{r,r+1}/N} Z^r, \ \ldots \ \sqrt{\pi_{rm_2}/N} Z^r, Z^r\Big] \quad (3.11)$$

and

$$\Upsilon^r = \operatorname{diag}(Z^1 \ \cdots \ Z^{r-1} \ Z^{r+1} \ \cdots \ Z^{m_2} \ R) \quad (3.12)$$

Moreover, the minimum of functionals g_{ij} can be calculated from $\underline{g} = \inf_{r \in \mathcal{M}} \{[\|Y^r\|_2]/[\|(Z^r)\|_2]\}$.

Proof: From the Schur complement, one can find the matrix inequalities (3.4) are equivalent to

$$\begin{bmatrix} \varpi^r & 0 & -D_{ij}^r P^r \\ * & -\underline{g} B^\top P^r + \dfrac{\phi^2}{N} I & \underline{g} B^\top P^r \\ * & * & -\underline{g} B^\top P^r \end{bmatrix}$$

$$= \begin{bmatrix} Q + \sum_{l=1}^{m_2} \dfrac{\pi_{rl}}{N} P^l - \underline{g} P^r B & 0 & -D_{ij}^r P^r \\ * & -\underline{g} B^\top P^r & \underline{g} B^\top P^r \\ * & * & -\underline{g} B^\top P^r \end{bmatrix}$$

$$+ \underbrace{\begin{bmatrix} P^r \left(\dfrac{(D_{ij}^r)^2 Q^{-1}}{\tilde{q}} + \dfrac{1}{N} I \right) P^r & 0 & 0 \\ * & \dfrac{\phi^2}{N} I & 0 \\ * & 0 & 0 \end{bmatrix}}_{\tilde{A}} < 0$$

where $r \in \mathcal{M}$ and

$$\varpi^r = Q + \frac{1}{N} \sum_{l=1}^{m_2} \pi_{rl} P^l - \underline{g} P^r B + P^r \left(\frac{(D_{ij}^r)^2 Q^{-1}}{\tilde{q}} + \frac{1}{N} I \right) P^r$$

With respect to the non-negative of $\tilde{A}$, one can obtain

$$\begin{bmatrix} Q + \sum_{l}^{m_2} \dfrac{\pi_{rl}}{N} P^l - \underline{g} P^r B & 0 & -D_{ij}^r P^r \\ * & -\underline{g} B^\top P^r & \underline{g} B^\top P^r \\ * & * & -\underline{g} B^\top P^r \end{bmatrix} < 0 \quad (3.13)$$

Let $Z^r = (P^r)^{-1}$, $Y^r = \underline{g} Z^r$, $R = Q^{-1}$ and define $K^r = \operatorname{diag}(Z^r, Z^r, Z^r)$. Pre- and post-multiplying (3.13) by K^r, one can see that the coupled matrix inequalities (3.13) are equivalent to the following matrix inequalities

$$\begin{bmatrix} \Pi^r & 0 & -D_{ij}^r Z^r \\ * & -Y^r B^\top & Y^r B^\top \\ * & * & -Y^r B^\top \end{bmatrix} < 0 \quad (3.14)$$

where

$$\Pi^r = Z^r R^{-1} Z^r + \sum_{l}^{m_2} \frac{\pi_{rl}}{N} Z^r (Z^l)^{-1} Z^r - BY^r$$

Then the inequalities (3.14) are in turn equivalent to (3.10). This immediately completes the proof. □

Note that if the communication topology has only one fixed form, the MAS (2.1) reduces to a deterministic one. In the subsequent corollary, we present the formation stability property for the deterministic MAS (2.1).

Corollary 3.1: For the MAS (2.1) with fixed topology and time-varying delays, if there exist matrices $P > 0$, $Q > 0$ and a constant $\underline{g}$, such that the following LMIs:

$$\begin{bmatrix} \tilde{\Xi} & -D_{ij} P & 0 & -D_{ij} P \\ * & -\tilde{q} Q & 0 & 0 \\ * & * & -\underline{g} B^\top P + \dfrac{\phi^2}{N} I & \underline{g} B^\top P \\ * & * & * & -\underline{g} B^\top P \end{bmatrix} < 0 \quad (3.15)$$

hold, for all $i, j = 1, \ldots, N$, where

$$\tilde{\Xi} = Q - P\left(\underline{g} B - \frac{1}{N} P\right),$$

then the formation of MAS (2.1) is asymptotically stabilisable.

From Theorem 3.1, one can derive an algorithm of formation controller design for MAS with stochastic switching topology and time-varying communication delays.

(i) According to physical law, choose a differential potential function under the structural constraints Assumption (A) and (B).

(ii) For all $i, j = 1,\ldots,N$, validate that the lower bound of g_{ij} exists and satisfies the constraint $\underline{g} > 0$. If the lower bound $\underline{g} > 0$ does not exist, then regulate the parameters of potential function and go to (*i*).
(iii) Solve (3.4) and, verify the positive definiteness of Q and P^r for all $r = 1, \ldots, m_2$. If (3.4) does not have positive definite solutions P^r and Q, regulate the parameters of potential functions and go to (*i*).

Remark 3.3: In coordination control of MAS, some classic modes derived from physics can be found to construct the potential function, such as Newtonian potential [25], sigmoid function [7], harmonic function [26] and Morse potential [27]. It is challenging to give a conclusion to address the optimal design from above methods. The potential can be obtained by the MAS designers based on the collection behaviours of MAS and practical physical law. With respect to step (ii), how to adjust the parameters is dependent on specific form of potential function. After that, some boundary conditions of parameters in this potential function can be obtained from the constraint $g_{ij} \geq \underline{g} > 0$. Then the consequent adjustment of parameters can be achieved by these generated conditions.

4 Numerical examples

In this section, two examples are conducted to show the effectiveness of the proposed theoretical results. As discussed above the formation of MASs may have distinct performance based on different potential function J.

4.1 Fixed path

Firstly, according to the assumption of a potential function, the formation controller $u_i(t)$ is chosen as in [10]

$$\begin{aligned} u_i(t) = 2\sum_{\substack{j=1\\ j\neq i}}^{N} e_{ij}(t)\Bigg(\frac{C_a}{L_a^2}\exp\left(-\frac{\|e_{ij}(t)\|^2}{L_a^2}\right) \\ -\frac{C_r}{L_r^2}\exp\left(-\frac{\|e_{ij}(t)\|^2}{L_r^2}\right) + C_r\left(\frac{1}{L_r^2}+\frac{1}{L_a^2}\right) \\ \times \exp\left(-\left(\frac{1}{L_r^2}+\frac{1}{L_a^2}\right)(\|e_{ij}(t)\|^2)\right)\Bigg) \end{aligned} \tag{4.1}$$

with constraints $L_a > L_r$ and

$$\begin{aligned} \frac{C_a}{C_r} > \frac{L_a^2}{L_r^2}\exp\left(-\left(\frac{1}{L_r^2}-\frac{1}{L_a^2}\right)\|e_{ij}(t)\|^2\right) \\ -\left(1+\frac{L_a^2}{L_r^2}\right)\exp\left(-\frac{\|e_{ij}(t)\|^2}{L_r^2}\right) \end{aligned} \tag{4.2}$$

which guarantees $g_{ij} > 0$. It is worthy to note that conditions (4.2) associated with $\mathbb{L} > L_a > L_r > 0$ are boundary constraints of parameters mentioned in Remark 3.3. Obviously, the potential function developed in (4.1) has a unique minimum at a desired value, when $X_{ij}(t) = w_{ij}$ for all $i, j = 1, 2, \ldots, N$. L_a L_r C_a, and C_r are positive parameters representing ranges and strengths of attraction and repulsion, respectively. Let $L_a = 0.5$, $L_r = 0.32$, $C_a = 21$, $C_r = 0.8$, $N = 4$, $n = 2$, $B = I$ and consider the time-varying delay $\tau(t) = 0.5\sin t$ in the MAS (2.1). The coupling configuration matrix is designed to be stochastically switching with equal probability between two modes

$$D^1 = \begin{bmatrix} -0.64 & 0.32 & 0 & 0.32 \\ * & -0.64 & 0.32 & 0 \\ * & * & -0.64 & 0.32 \\ * & * & * & -0.64 \end{bmatrix}$$

and

$$D^2 = \begin{bmatrix} -0.64 & 0 & 0.32 & 0.32 \\ * & -0.32 & 0 & 0.32 \\ * & * & -0.64 & 0.32 \\ * & * & * & -0.96 \end{bmatrix}$$

Then, we consider the formation of MAS (2.1) with above chosen controller (4.1) under a constant desired path w.r.t. σ_1 in the first case, where this path is set as

$$f(t, x_i) = 0.3 * [\cos(0.5 * x_{i_1}(t)),\ \sin(0.25 * x_{i_2}(t))]^{\top}$$

One can obtain $\phi = 0.13$ and $\underline{g} = 0.4021$ in this example. The vector-valued functions $f^{\sigma_1}(t, x_i)$ in this paper can be easily extended to arbitrary cases in practice. The goal of this task is to drive this MAS to keep a formation of square. From Theorem 3.1, the formation of MAS with switching topology and time delays (2.1) is stochastically stable. Fig. 1 shows four agents achieve the predefined formation from the initial positions $(1.1,3.0)^T$, $(3.0,4.0)^T$, $(3.7,3.2)^T$ and $(2.6,2.0)^T$.

An error function

$$d(t) = \sum_{i=1}^{N-1}\sum_{j>i}^{N}(\|x_j(t) - x_i(t)\| - \|w_{ij}\|)$$

is introduced to visualise the effectiveness of formation which is showed in Fig. 2. From Figs. 1 and 2, one can see that the MAS obtains the desired formation shape in a short period of time.

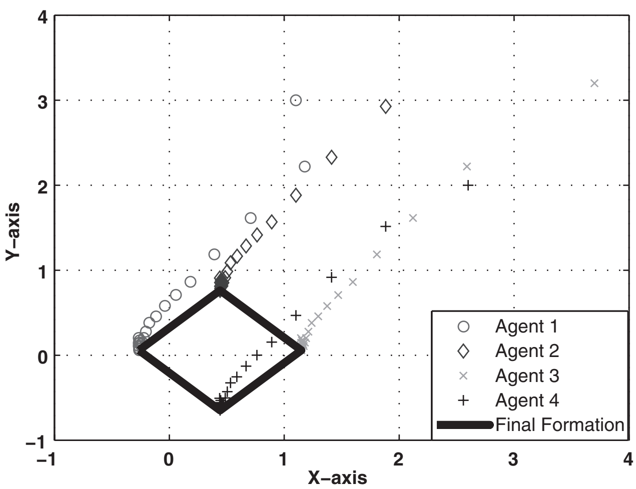

Fig. 1 *Formation of MAS with stochastic switching topology and time-varying delays*

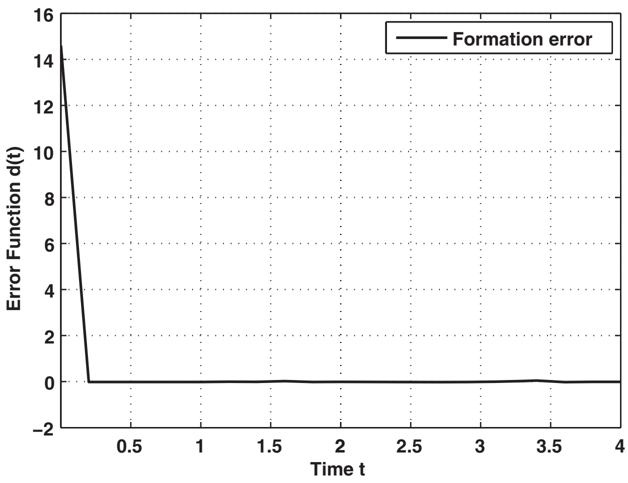

Fig. 2 *Formation error of MAS with stochastic switching topology and time-varying delays*

Table 1 Track table

Period of time t/(s)	$f^{\sigma_1}(t, x_i)$
0–10	$[0.03*t, 0.1]^T$
10–15	$[0.3, 0.01*t]^T$
15–29	$[0.01*t+0.15, -0.002*t+0.18]^T$
29–31	$[0.44, -0.1*t+0.25]^T$
31–36	$[0.03*t-0.49, -0.06]^T$
36–38	$[0.59, -0.008*t+0.224]^T$
38–42	$[-0.005*t+0.78, -0.03*t+0.034]^T$

4.2 Switching paths

In the second example, the MAS will perform more complex tasks following the switching track scheme showed in Table 1 with a square formation. The switching paths $f^{\sigma_1}(t, x_i)$ chosen in this example guarantee the Lipschitz continuity. The initial position of agents are given as $(1.0, 1.2)^T$, $(2.0, 4.0)^T$, $(6.7, 3.0)^T$ and $(4.2, 2.0)^T$. It should be mentioned that the predesigned tracks in this example could be derived from embedded microprocessor in each agent, which are responsible for collecting information from environment and providing the accessible path. The trajectories of the system (2.1) exploring in a cave-like scenario (the coloured part in Fig. 3) and the corresponding formation error are depicted in Figs. 3 and 4, which demonstrates the effectiveness of the proposed formation protocol (2.7) in this paper.

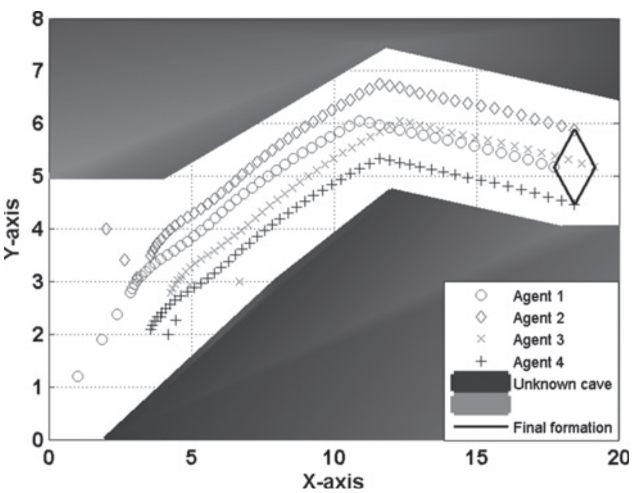

Fig. 3 *Formation of MAS with switching trajectories*

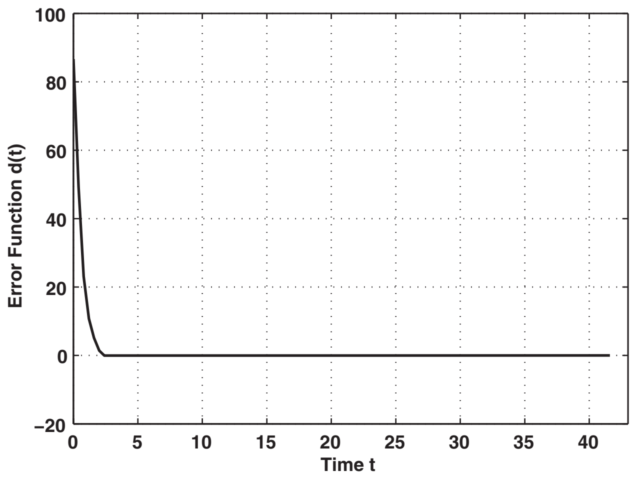

Fig. 4 *Formation error of MAS with switching trajectories*

5 Conclusions

The formation protocol of a MAS with stochastic switching topology and time-varying delays is addressed in this paper. The formation controller based on APFs has been designed in a general form. By introducing a disagreement function, the formation problem of MAS is translated into the stochastic stability of an error system. Then by employing stochastic Lyapunov functional approach and LMIs, the sufficient conditions for formation keeping of the MAS are obtained. The main contribution of this paper is to provide a valid distributed formation algorithm that overcomes the difficulties caused by unreliable communication channels, such as stochastic information transmission, switching communication topology, and time-varying communication delays. Therefore this approach possesses great potential in practical applications. Finally, examples have been provided to verify the effectiveness of the proposed approach.

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