Analysis of Noisy Gradient-Descent Bit Flipping (NGDBF

Using MATLAB and the PRISM Model Checking Tool

Eric Reiss
Utah State University

LDPC Codes and Trapping Sets

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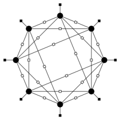

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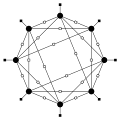

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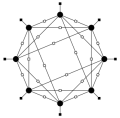

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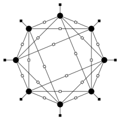
- Absorbing sets are a special case of a trapping sets that are stable in a bit flipping decoder [1]



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  - Given a threshold θ flip bit i if $E_i < \theta$

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- [4] NGDBF demo

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4. Build the presentation by running `make`

Including Figures

To include a figure using Markdown, use this syntax:

```
![Optional figure caption.](figures/example.png){width=60%}
```

Result:

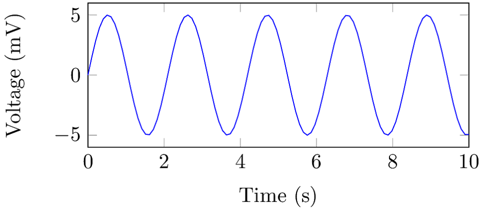


Figure 2: Optional figure caption.

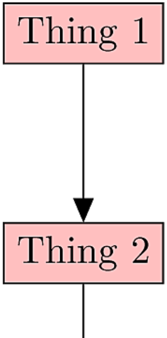
Two-Column Slides

Starting two-column mode:

Here is a column.

- Text.

And another column.



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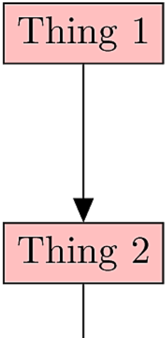

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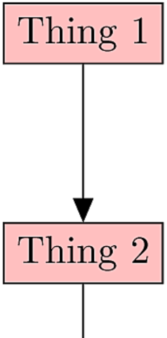

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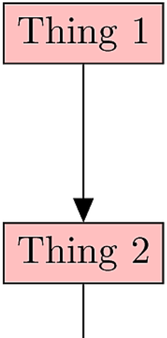

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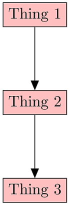

Figure 4: A tall figure.

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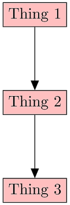

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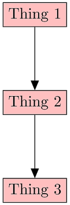

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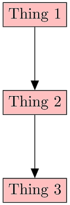

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