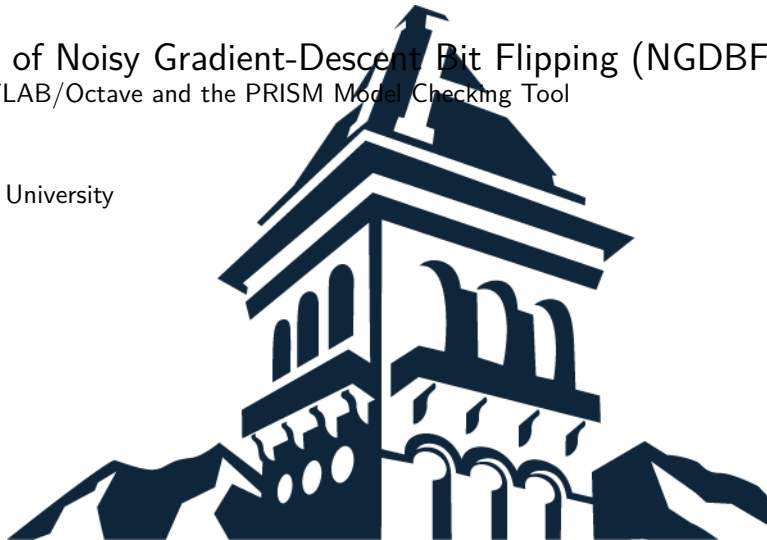


Analysis of Noisy Gradient-Descent Bit Flipping (NGDBF)

Using MATLAB/Octave and the PRISM Model Checking Tool

Eric Reiss
Utah State University



Overview

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LDPC Codes and Trapping Sets

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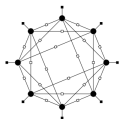


Figure 1: (8,8) Absorbing set that is dominant in the 802.3an 10GBASE-T LDPC Code [2].

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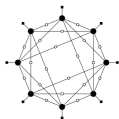


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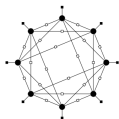


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- ▶ Absorbing sets are a special case of a trapping sets that are stable in a bit flipping decoder [1]

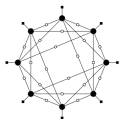


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- ▶ Given a threshold θ flip bit i if $E_i < \theta$

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Sample Generation

- ▶ Samples are pulled from Gaussian distribution with a mean of 1 and a standard deviation of $\sigma = \sqrt{\frac{1}{R \cdot 10^{SNR/10}}}$, where R is the code rate

```
loop_check = get_error_sample_size(sym_size);
valid_samples = zeros(1,loop_check); % initialize valid samples
valid_idx = 1;
error_samples = zeros(1,loop_check); % initialize error samples
error_idx = 1;
while valid_idx <= loop_check || error_idx <= loop_check
    temp = normrnd(1,sigma); % generate samples
    if temp > 0 % sort valid samples
        valid_samples(valid_idx) = temp;
        valid_idx = valid_idx + 1;
    else % sort error samples
        error_samples(error_idx) = temp;
        error_idx = error_idx + 1;
    end
end
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- ▶ Currently the tool generates a list of samples and sorts into valid and error sample bins
- ▶ An error sample is one that comes from the negative tail of the Gaussian distribution
- ▶ There is probably a better way to do this, and finding that is on the to-do list

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```

Energy Calculation

- To create the model, all possible energy values must be calculated

```
%initialize Energy and check node matrices
E = zeros(2^sym_size,sym_size);
chk_nodes = ones(1, check_size);
chk_sum = zeros(1,sym_size);
% Calculate all possible energy values for each state
for row = 1:2^sym_size
    % Calculate all check nodes
    for adj_row = 1:check_size
        for adj_col = 1:sym_size
            if adj_mat(adj_row,adj_col) == 1
                chk_nodes(adj_row) = chk_nodes(adj_row)*x(row,adj_col);
                chk_sum(adj_col) = chk_sum(adj_col)+chk_nodes(adj_row);
            end
        end
    end
end
% Calculate energy values
for E_idx = 1:sym_size
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Energy Calculation

- ▶ To create the model, all possible energy values must be calculated
- ▶ The syndrome, called `chk_nodes` here, is also calculated

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        end
    end
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Transition Probabilities

- Using the energy calculations, the transition matrix is generated

```
p = ones(2^sym_size,2^sym_size);  
    % Flip probabilities calculated according to Eq 3.13 in  
    % dissertation (pg. 26)  
    for row = 1:2^sym_size  
        px = zeros(1,sym_size);  
        for p_idx = 1:sym_size  
            px(p_idx) = normcdf(theta,E(row,p_idx),sigma);  
        end  
        rowbin = dec2bin(row-1,sym_size);  
        for col = 1:2^sym_size  
            colbin = dec2bin(col-1,sym_size);  
            for p_idx = 1:sym_size  
                if rowbin(p_idx) == colbin(p_idx)  
                    p(row,col) = p(row,col)*(1-px(p_idx));  
                else  
                    p(row,col) = p(row,col)*px(p_idx);  
                end  
            end  
        end  
    end
```

Write Files and Process Outputs

- The model is written using the helper functions `write_model.m` and `write_explicit_model.m`

```
% Process Output for transient and steady state
if (tag(3) == 't' && tag(4) == 'r') || (tag(3) == 's' && tag(4)
    str_idx = regexp(output,regexptranslate('wildcard','0:\'\\(\\*\\
    output = substr(output,str_idx);
    split_output = strsplit(output,\'\\n\');
    for out_idx = 1:2^sym_size
        str_to_parse = char(split_output(out_idx));
        if (str_to_parse(1) >= "0") && (str_to_parse(1) <= "9")
            temp = textscan(str_to_parse,"%d:(%d)=%f");
```

Write Files and Process Outputs

- ▶ The model is written using the helper functions `write_model.m` and `write_explicit_model.m`
- ▶ A system call to PRISM runs either a transient analysis, steady-state analysis, or a user-defined property analysis

```
% Process Output for transient and steady state
if (tag(3) == 't' && tag(4) == 'r') || (tag(3) == 's' && tag(4)
    str_idx = regexp(output,regexptranslate('wildcard','0:\'\\
    output = substr(output,str_idx);
    split_output = strsplit(output,\'\\n\');
    for out_idx = 1:2^sym_size
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▶ It works with some properties, but this needs to be expanded

```
% Process Output for transient and steady state
if (tag(3) == 't' && tag(4) == 'r') || (tag(3) == 's' && tag(4) == 'r')
    str_idx = regexp(output,regexptranslate('wildcard','0:.*\n'));
    output = substr(output,str_idx);
    split_output = strsplit(output,'\n');
    for out_idx = 1:length(split_output)
        str_to_parse = char(split_output{out_idx});
        if (str_to_parse(1) >= "0" && (str_to_parse(1) <= "9"))
            temp = textscan(str_to_parse,"%d:(%d)=%f");
```


Sources

- ▶ [1] T. Tithi, “Error-Floors of the 802.3an LDPC Code for Noise Assisted Decoding”, *All Graduate Theses and Dissertations*, pp. 7465, 2019.

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- ▶ [4] C. Winstead and E. Boutillon, “Hardware Demonstration of Noisy Gradient Descent Bit Flipping (NGDBF) for IEEE 802.3 Standard Code”.

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