Analysis of Noisy Gradient-Descent Bit Flipping (NGDBF Using MATLAB/Octave and the PRISM Model Checking Tool

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- ▶ Absorbing sets are a special case of a trapping sets that are stable in a bit flipping decoder [1]



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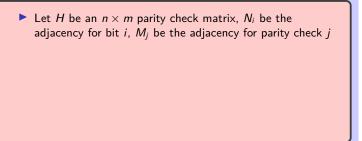
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 - Given a threshold θ flip bit i if $E_i < \theta$

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Samples are pulled from Gaussian distribution with a mean of 1 and a standard deviation of $\sigma = \sqrt{\frac{1}{R*10^{SNR/10}}}$, where R is the code rate

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loop_check = get_error_sample_size(sym_size);
valid_samples = zeros(1,loop_check); % initialize valid samples
valid_idx = 1;
error_samples = zeros(1,loop_check); % initialize error samples
error_idx = 1;
while valid_idx <= loop_check || error_idx <= loop_check
    temp = normrnd(1,sigma); % generate samples
    if temp > 0 % sort valid samples
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- ► There is probably a better way to do this, and finding that is on the to-do list

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Energy Calculation

```
%initialize Energy and check node matrices
E = zeros(2^sym_size,sym_size);
chk_nodes = ones(1, check_size);
chk_sum = zeros(1,sym_size);
% Calculate all possible energy values for each state
for row = 1:2^sym size
   % Calculate all check nodes
   for adj row = 1:check size
         for adj col = 1:sym size
            if adj mat(adj row,adj col) == 1
               chk nodes(adj row) = chk nodes(adj row)*x(row,adj
               chk sum(adj col) = chk sum(adj col)+chk nodes(adj
            end
         end
   end
   % Calculate energy values
   for E_idx = 1:sym_size
         E(row, E_idx) = y(E_idx)*x(row, E_idx)+w*chk_sum(E_idx);
   end
end
```

Transition Probabilities

```
p = ones(2^sym size,2^sym size);
        % Flip probabilities calculated according to Eq 3.13 in
        % dissertation (pg. 26)
        for row = 1:2^sym size
            px = zeros(1, sym size);
            for p idx = 1:sym size
                px(p_idx) = normcdf(theta, E(row, p_idx), sigma);
            end
            rowbin = dec2bin(row-1,sym_size);
            for col = 1:2^sym_size
                colbin = dec2bin(col-1,sym_size);
                for p_idx = 1:sym_size
                    if rowbin(p_idx) == colbin(p_idx)
                        p(row,col) = p(row,col)*(1-px(p idx));
                    else
                        p(row,col) = p(row,col)*px(p idx);
                    end
                end
            end
        end
        9 Sanity chack
```

Write Files and Process Outputs % Process Output for transient and steady state

```
if (tag(3) == 't' \&\& tag(4) == 'r') || (tag(3) == 's' \&\& tag(4))
      str_idx = regexp(output,regexptranslate('wildcard','0:\(*\)
      output = substr(output,str idx);
      split output = strsplit(output, "\n");
      for out idx = 1:2^sym size
         str_to_parse = char(split_output(out_idx));
         if (str_to_parse(1) >= "0") && (str_to_parse(1) <= "9")
            temp = textscan(str_to_parse, "%d:(%d)=%f");
            state_temp = temp{1,2};
            p out(idx,state_temp+1) = temp{1,3};
         else
            break;
         end
      end
elseif tag(3) == 'p'
      str idx = strfind(output, "Result");
      output = substr(output,str idx);
      p_temp = textscan(output, "Result: %f (exact floating point
      p \text{ out(idx,1)} = p \text{ temp}\{1,1\};
```

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