Problem 3 Part (a)

Value of i in each iteration

$2, 2^2, 2^4, 2^8, 2^{16} \, \text{-----} \, 2^{2^m}$

where $m \in \mathbb{N} \cup \{0\}$

So, the loop can be written as (in terms of time complexity)

$\text{for} (\text{int} \;\; m = 0 ; \; 2^{2^m} < n \; ; \; m = m + 1) \{$

$\quad O(1) \;\; \text{action} \quad \}$

when $2^{2^m} = n$, loop stops

$\log(2^{2^m}) = \log n$

$2^m = \log(n)$

$m = \log(\log(n))$

So, the loop will iterate until $m = \log(\log(n$

which means $\log(\log(n))$ times

Therefore, $\Theta(\log(\log(n)))$

Problem 3 part (b)

Complexity of first for loop with the if condition $\Theta(n)$ (1)

The if condition is TRUE when "$\sqrt{n}$" is a coefficent of "i". This first happens when $i = \sqrt{n}$ and after that until $i = n$ there are $\sqrt{n} - 1$ more cases like this. Therefore this condition is TRUE $\sqrt{n}$ times. ($n/\sqrt{n} = \sqrt{n}$)

Each time the inner for loop will iterate $\underbrace{\sqrt{n}^3, (2\sqrt{n})^3, (3\sqrt{n})^3, \ldots, (\sqrt{n}\cdot\sqrt{n})^3}_{\sqrt{n}\text{ times}} = n^3$

# of times the inner loop will iterate:

$$\sum_{j=1}^{\sqrt{n}} j^3 \cdot (\sqrt{n})^3 = n^{3/2} \underbrace{\boxed{\sum_{j=1}^{\sqrt{n}} j^3}}_{\Theta(\sqrt{n}^{3+1}) = \Theta(n^2)} = n^{\frac{3}{2}} \cdot \Theta(n^2) =$$

$= \Theta(n^{7/2})$ (2)

Therefore if we add two time complexities (the outer and the inner) $\Theta(n) + \Theta(n^{7/2}) =$

$\boxed{\Theta(n^{7/2})}$

Problem 3 part c

The if condition is checked n^2 times. So the complexity of outer two loops is $\Theta(n^2)$

The inner most loop iterates $\log(n)+1$ times so the complexity is $\Theta(\log n)$

Now how many times the if condition can be TRUE in the worst given data will be considered:
The first n elements of the array are checked in the if condition n^2 times. So, in the worst case the if condition can be TRUE n times. So, the inner most loop will start n times making the complexity of the 2^{nd} part (if condition + inner most loop) $\Theta(n \log n)$
Therefore the total complexity is:

$$\Theta(n^2) + \Theta(n \log n) = \Theta(n^2)$$

Problem 3 part d.

The outer for loop iterates n times so it has a complexity of $\Theta(n)$. (1)

The if condition will be true for m times such that

$$10, \frac{3}{2}\cdot 10, \left(\frac{3}{2}\right)^2\cdot 10, \ldots, \left(\frac{3}{2}\right)^m\cdot 10 < n$$

$$< \left(\frac{3}{2}\right)^{m+1}\cdot 10 \quad \text{So, } \left(\frac{3}{2}\right)^m < \frac{n}{10} \rightarrow$$

$$\rightarrow \log_{3/2}\left({3/2}^m\right) < \log_{3/2}\left(\frac{n}{10}\right) \longrightarrow$$

$$m < \log_{3/2}\left(\frac{n}{10}\right)$$

Each time the if condition is true:

$$10\cdot\left(\frac{3}{2}\right)^k < n \quad \text{where, } k = 0, 1, 2, 3, \ldots$$

Therefore

$$\sum_{k=0}^{\log_{3/2}\left(\frac{n}{10}\right)} 10\left(\frac{3}{2}\right)^k = 10\sum_{k=0}^{\log_{3/2}\left(\frac{n}{10}\right)}\left(\frac{3}{2}\right)^k = \Theta\left(\frac{3}{2}^{\log_{3/2}\left(\frac{n}{10}\right)}\right)$$

$$= \Theta\left(\frac{n}{10}\right) = \frac{1}{10}\Theta(n) = \Theta(n) \quad (2)$$

If we add to two complexities $((1)+(2)) =$
$= \Theta(n) + \Theta(n) = \Theta(n)$