7)

Total number of ways the Professor can ask questions is $= 15^8$

Total number of ways Professor choosing distinct 8 students is $= \binom{15}{8}$

So, probability is $=$

$$= \frac{\binom{15}{8}}{15^8}$$

2)

2 odd digits to start
5 unique digits
last digit even

Total ways to obtain 8 numbers =

$$= \binom{100\,000}{8}$$

number of numbers meeting the criteria:

$$= \underbrace{\binom{5}{2} \times 2!}_{\text{first two}} \times \underbrace{\binom{5}{1}}_{\text{last}} \times \underbrace{\binom{7}{2} \times 2!}_{3^{rd} \text{ and } 4^{th} \text{ digits}} =$$

$$= 4200$$

$$\frac{\binom{4200}{5}\binom{95\,800}{3}}{\binom{100\,000}{8}}$$

3)

A

At least 2 dice 4 or above.

1) 2 dice 4 or above
2) 3 dice 4 or above

1) $\frac{1}{2} \times \left(\frac{1}{2}\right)^2 \times \binom{3}{2} = \frac{3}{8}$

2) $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Adding two mutually ex cases = $\frac{4}{8}$

$P(A) = 1/2$

B

$\left(\frac{1}{6}\right)^3 \times 6 = \frac{1}{6^2} = \frac{1}{36}$

$P(A \cap B)$ = P(All of them 4) + P(All of them 5) + P(All of them 6) =

$= \frac{1}{6^3} \times 3 = \frac{3}{216} = \frac{1}{72} =$

$= P(A) \times P(B) = \frac{1}{2} \times \frac{1}{36} = \frac{1}{72}$

So, they are independent.

4)

Probability of a flush =

$$= \frac{\binom{13}{5} \times 4}{\binom{52}{5}} = p$$

x = no of hands until there is a flush

$$E[x] = \frac{1}{p} = 504.848 \approx 505$$

5)

W = win when there is superstar
H = superstar is healthy
K = win when there is no superstar
G = 4 out of 5 games won

$$P(H|G) = \frac{P(G|H)P(H)}{P(G|H)P(H) + P(G|H')P(H')}$$

$$P(G|H) = \binom{5}{4} \times P(w)^4 \times (1 - P(w)) =$$

$$= 5 \times (0.7)^4 \times (0.3) = 0.36015$$

$P(H) = 0.75$

$P(H') = 0.25$

$$P(G|H') = \binom{5}{4} P(k)^4 \times (1 - P(k)) =$$

$$= 5 \times (0.5)^4 \times 0.5 = 5 \times 0.5^5 = 0.15625$$

$$P(H|G) = \frac{0.36015 \times 0.75}{0.36015 \times 0.75 + 0.15625 \times 0.25} =$$

$$= 0.87365$$