1)

one u	two u's	three u's
$5!$	$\frac{5!}{2!} \times \binom{4}{3}$	$\frac{5!}{3!} \times \binom{4}{2}$

Since these are mutually exclusive events we can add the count of these

$$5! + \frac{5!}{2!}\binom{4}{3} + \frac{5!}{3!}\binom{4}{2} =$$

$$= \boxed{480}$$

2)

$$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1} = T$$

We choose 2 possibilities out of 13 possibilities for two pairs.
For each of two pairs there are 2 suits out of for
Out of the 11 remaining cards we choose one of them and there are 4 different possibilites for its suit

$\boxed{T = 123552}$

3)

Fighting couple listens to 0 songs →

→ Dividing 16 songs to 6 couples

16 stars 5 bars (5 bars create 6 sections)

$\binom{21}{5}$

Fighting couple listens to 1 song →

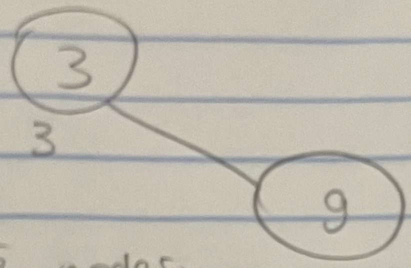
→ Dividing 15 songs to 6 couples

15 stars 5 bars

$\binom{20}{5}$

Total = $\binom{21}{5} + \binom{20}{5} = \boxed{35\,853}$

4)



2 nodes less than 3

5 nodes greater than 3 and less than 9

3 nodes greater than 12

Total # of ways = # of ways with 2 nodes × # of ways with 3 nodes × # of ways with 5 nodes

# of ways with 2 nodes = 2

# of ways with 3 nodes = 5

# of ways with 5 nodes = 42

Total # of possible trees =

$= 2 \times 5 \times 42 = 420$

5)

3 nurses each nurse getting at least one friend

So we can assume each nurse treated one friend and can divide 7 friends to 3 nurses.

Therefore

$$\binom{9}{2}$$

4 nurses (same logic)

$$\binom{9}{3}$$

Total ways = $\binom{9}{2} + \binom{9}{3} = 120$