

EX11

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1 Q2

Theorem 1.1 — *The algorithm is not truthful*

Proof.

- 2 projects a and b
- 2 People Max and Dan
- 1 dollar budget
- Max Preference is $(a, b) = (1, 0)$
- Dan Preference is $(a, b) = (0.5, 0.5)$

The median of 2 numbers is the lowest of each preference in this case.
The outcome of the algorithm before normalizing will be

$$(a, b) = (\min(0.5, 1), \min(0, 0.5)) = (0.5, 0)$$

and after normalizing it will be

$$(a, b) = (0.5, 0) + (1 - 0.5) \cdot (0.5, 0.5) = (0.75, 0.25)$$

The utility of Max from this is $-|1 - 0.75| - |0 - 0.25| = -0.5$.

The utility of Dan from this is $-|0.5 - 0.75| - |0.5 - 0.25| = -0.5$.

If Dan will change his preference to $(a, b) = (0, 1)$, the budget before normalization will be

$$(a, b) = (\min(1, 0), \min(0, 1)) = (0, 0)$$

and after normalization it will be

$$(a, b) = (0, 0) + (1 - 0) \cdot (0.5, 0.5) = (0.5, 0.5)$$

The utility of Dan in this case will be $-|0.5 - 0.5| - |0.5 - 0.5| = 0$, which is higher, and that proof that the algorithm is not truthful. \square