EX10

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1 Q1 section 1

3 Projects a,b,c. 2 people. 1 dollar budget. Dan support project a and b, Max support project b and c. The project with the majority amount of people is b, giving to b all the budget will cause projects a and c to get zero budget, therefore Dan utility will be zero, and Max utility will be zero. while the best utilization solution is to give $\frac{1}{3}$ of dollar to each project, and that will ensure total utility of $\frac{2}{3}$ which is better.

2 Q1 section 2

We can simply define the problem as Linear programming problem, any linear programming problem with Polynomial number of equations and variables can be solve in Polynomial time, using Vaidya's 89 algorithm.

Definition 2.1 —

- M set of projects
- N set of people
- C is the budget
- d_i is the assignment of budget to specific project
- D is the set of all legal assignments of budgets to projects
- $u_{i,j}$ is the preference of person i to project j
- the target function
$$\arg\max_{d\in D} \sum_{i\in N} \arg\min u_{i,j} \cdot d_j$$

Theorem 2.2 — *The following algorithm is effective and find the best assignment. Solve the following Linear programming issue:*
Find maximum such R, and $D = \{d_1, \dots, d_{|M|}\}$ subject to this constraints,

$$\forall_{(i,j)\in\{(i,j)|i\in N, j\in M, u_{i,j}>0\}} d_j \geq k_i, \quad \sum_{i\in N} k_i \geq R, \quad \sum_{j\in M} d_j = C, \quad \forall_{j\in M} d_j \geq 0$$

Proof. Fact: Any linear programming problem with Polynomial number of equations and variables can be solve in polynomial time using Vaidya's 87 algorithm.
This LP problem have at most $n \cdot m + m + 2$ constraints and at most $m + n + 1$ variables, so it can be solved in polynomial time.

The k_i variable represent the minimum of specific person, this LP problem must find such maximum k_i otherwise it is contraindication to the maximally of R. R is the count of all such minimums, therefore R is the best solution to the maximum sum of minimum and hence the solution D satisfy the "Target function". □

3 Q1 section 3

Yes the algorithm is truthful, First note that the algorithm tend to divide equally budget between specific persons projects. if specific person i will add some project j at the best case the key k_i constraints the LP will divide budget to this extra project so his minimum total utility could only be less, if he remove some project support then in theory this project with less supporters can only get less or even zero budget because of this LP constrains structure so his minimum could only be less.

no more time to complete more solid proof sorry..,