Assigment3

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1 Question_1

Definition: Nash-Optimality is when the partition $max(\Sigma_{i=1}^{n} ln(U_i)) \longleftrightarrow max(\prod_{i=1}^{n} U_i)$

1.1

Note: We proved in class that it's envy-free thus Trivial
suppose Ami is the one trying to manipulate.
Resources: A
Players: Ami, Tami :)
EvalFucntion: Ami(A) = V_{Ami}, Tami(A) = V_{Tami}
simple thing is $max((x \cdot V_{Ami}) \cdot ((1-x) \cdot V_{Tami}))$ We know that multiplication is commotative, then Basically we dont need the bracket. Then $max(x \cdot V_{Ami} \cdot (1-x) \cdot V_{Tami})$ $max(x \cdot (1-x)) = -x^2 + x$ $f^{'}(x) = -2x + 1$ We will see that the maximum point would be at $x = 0.5$. Since multiplication is commotative no matter how much we add (or substruct) to Ami we know the highest value would be to multiply by the highest thing therefore we can't manipulate or we can manipulate and it won't do nothing (Technically because it's $\leq$ by definition of the Question).

1.2

Example for intution:
Resources: A,B
Players: Ami, Tami :)
EvalFucntion: $Ami(A, B) = 1, 1$, $Tami(A, B) = 1, 0.1$
Without manipulation it Ami will get 1 and so Tami. With the following manipulation Ami will get 1.5 instead, the new EvalFunction of Ami would be: $Ami(A, B) = 1, 20$. now we will see that $(1 + 10) * 0.5 > 1 * 1, 20 * 0.1$ thus we saw that it's possible in some cases.
Intuition: this example made of think of a good ratio so I could manipulate in each case.
Reduce the problem into 2 Cases:

- Both $Ami(A) = Ami(B) > Tami(A) = Tami(B)$ we will make a status queoe (not change logically), Ami(A,B) = a+5,a+5, if you think about it it can ruin the position. (Technically a manipulation)
- $a > 2b$ And $Ami(A, B) = a, b$ And $Tami(A, B) = b, a$ then the manipulation would be, Ami(B) = a^2/b $(a + \frac{a^2}{b \cdot 2}) \cdot \frac{a}{2} = a^2 \cdot \frac{a}{b} * \frac{1}{4} + \frac{a^2}{2} > a^2$
  $a^2 * a/b * 1/2 > a^2$ Since $a > 2b$ then we got what wanted :) $(1 < 1.5$ "multiplier"), see that I used the first seif (this is why I know it's exactly half).
  Note: I suppose that we could adjust it so the difference between a and b would tend to be the same; $\lim_{x \to b}$ but slight different and then fully Prove.