Simplified Transfer Function Approach of Rubber Bushing Damping Characteristics for Accurate Prediction of Durability Loads in Multi-Body Dynamics Models

ABSTRACT
Physical systems that consist of parts and vibration isolators such as bushings or mounts are usually modelled in multibody simulations as rigid bodies with mass and inertia properties of parts and Voigt models for isolators to represent dynamic characteristic of the physical system as accurately as possible. Employment of Voigt models in multi-degree-of-freedom (MDOF) systems, however, may result in lower accuracy due to limitations to reflect frequency-dependent dynamic characteristics and modelling complexity. In this study, a ~~method~~ so called simplified frequency-dependent transfer function model, which can determine the bushing forces more accurately by reflecting frequency dependent complex stiffness and damping characteristics of the rubber bushings from vehicle level measurements, will be featured. The proposed methodology is demonstrated on a multi-DOF heavy commercial vehicle cabin system to simplify the model ~~to a simple system model~~ and represent its dynamic characteristic with transfer function model instead of Voigt model. Simulation results are well correlated with the measurements obtained from prototype vehicle measurements over the frequency range of interest. Integration of the proposed method in to multi-body simulation software is also demonstrated with co-simulation between MSC.ADAMS and MATLAB software.

Keywords: multibody dynamics, rubber bushings, frequency-dependent stiffness, damping, cabin suspension design, simplified transfer function model

1. INTRODUCTION

Development of a vehicle has become more challenging ~~than ever~~ due to ever increasing competition ~~between brands~~ and more efficient product demands from customers ~~in automotive industry and legal entities~~. Therefore, analytical design processes have become an essential part of the vehicle product development cycle. Efficient ~~employment of the~~ analytical tools ~~in the design cycle requires that the models~~ must replicate the real life accurately. This ~~would~~ enables the design of the product with less physical testing in shorter time.

Analytical calculation of vehicle performance in early phases of vehicle development enables assessment of new ~~to assess new~~ designs in terms of durability, driving dynamics and noise & vibration points of view without requiring physical prototypes in the product development cycle. Therefore, it is important to have a competitive, state of the art ~~state-of-art~~ simulation methodology for problem specific system hierarchy for the successful prediction of analytical vehicle responses of dynamical system under real life conditions.

Customers want to have durable vehicles as well as vehicles with good vehicle performance and improved fuel economy. These attributes are usually in conflict ~~trade-off with each other~~. Durability of a product in general requires heavier body components to be used however fuel economy and emissions require lighter vehicles. I~~n order~~ To design the vehicles as light as possible and optimize the trade-offs ~~of various attributes~~, the engineers should be able to calculate the loads acting on the vehicle under customer usage. A key challenge for calculating the loads depends on the quality of the simulation model where the quality means simulation models should be accurate, with minimum complexity in terms of model inputs in order to replicate the real life conditions.

~~During the generation of the mathematical models, the complexity of the model may vary significantly especially based on the experience of the engineer and the familiarity with the system to be modeled~~. It is not uncommon to build and use a very complex model of the system during the development cycle, ~~which is believed to be accurate~~. This should reduce costs ~~has important implication on the cost~~ and development time ~~for the product~~. However, the determination of ~~the~~ model parameters ~~usually requires more~~ can require more hardware ~~for testing, more facilities~~ and measurement time, which may add significantly to the cost ~~of the product~~. Therefore, it is desirable to create computer simulation models with minimum complexity. Another disadvantage of the simulation models with increased complexity may be that those models can be very time consuming, especially when the models are to be evaluated multiple times ~~as~~ in design optimization studies (Papalambros and Wilde, 2000). For example, automotive models have more than 250 rigid bodies, and more than 1100 degrees of freedom, which result in long simulation times (Tuncel et al., 2010).

The next challenge besides the decision on the complexity of the simulation model is to obtain the simulation parameters. The inputs to a generic multibody dynamics based simulation model include the mass, inertia, center of mass of the rigid bodies as well as the viscoelastic properties of the joints between the rigid bodies. The parameters for mass, inertia and center of mass of the rigid bodies are usually straightforward to obtain. However, viscoelastic properties of the joints are usually difficult to obtain and to be implemented in the simulation software. Viscoelastic vibration isolators are widely used as attachment components such as in vehicle bushings or mounts. Their viscoelastic material properties, which play an important role in vehicle dynamics' performance and durability loads, are highly nonlinear. Their stiffness and damping properties are dependent on both frequency and amplitude of excitation (Sedlaczek et. al, 2011). Voigt model (Zhang et. al, 2011 and Cao et.al, 2005), which is a semi-analytical model, is most commonly used to represent the viscoelastic properties of rubber bushings for the dynamic simulation of vehicles. However, quasi-static force-deflection tests suggest that bushings can be characterized as a non-linear stiffness with energy dissipation due to viscous damping and material friction, which cannot be modelled only with Voigt parameters. The dynamic sweep tests of the bushings show that stiffness and phase angle both slightly increase over frequency while their damping exponentially decreases due to high viscous damping at lower frequencies (Scheiblegger, et. al, 2014). Despite of the frequent usage of Voigt models in multi-body simulations, Voigt models are limited to predict the dynamic behavior of the vehicle response accurately for the whole frequency range of interest but only capable of predicting the system dynamics when the number of the frequency of interest is just one, which is around the resonant frequency of a Single Degree-of-Freedom (SDOF) system (Scheiblegger, et. al, 2014).

Not clear. I assume the accuracy is limited to a narrow frequency range.

Many different viscoelastic vibration isolator models have been proposed (Li et. al., 2015, Scheiblegger et. al., 2014, Lee and Kim, 2002) to improve Voigt model for its known deficiencies. For example, Li et. al., (2015) and Scheiblegger et. al. (2014) proposed a bushing model, which includes hysteresis characteristic, frequency dependency, non-linear stiffness and non-linear damping characteristics of bushings. However, their methods require component specific test data available and parametrization is required to be performed using external data fitting tools. Lee and Kim, (2002) also has proposed a transfer function model method to treat Voigt model deficiencies that introduces polynomial fraction transfer function model to predict the slightly increasing complex stiffness characteristics over frequency. However, that study does not address the modelling of both material friction and complex damping characteristics of the bushings, which is critical to predict the durability loads especially for the heavy commercial vehicles. Therefore, it is required to derive the frequency dependent characteristics of the rubber bushings more easily such as from the vehicle level tests, which may be already available, instead of single component level testing for each component.

The objective of the paper is to present a methodology to represent a detailed multi-body dynamics simulation model in the simple equivalent form and improve the deficiencies of the viscoelastic properties of the current modelling strategies using the measurements from vehicle level testing as mentioned above. This paper is organized as follows: Section 2 gives some background information characteristics of rubber bushings and current models of bushing in the literature such as Voigt models. Transformation of a full vehicle model to simple mathematical model and stiffness-damping characterization of the vibration isolators in the new transfer function model representation is explained in Section 3. The proposed methodology is demonstrated on a heavy commercial truck to correlate the methodology to the test data from various durability road tests in Section 4.

2. QUASI-STATIC AND DYNAMIC CHARACTERISTICS OF RUBBER BUSHINGS

Rubber bushings as vibration isolation components are used in ~~automotive such as in~~ vehicle suspension ~~or~~ and engine mounting systems. In this section, the characteristics of the rubber bushings and the deficiency of the current modelling techniques are explained ~~in detail~~.

The measurement ~~procedures~~ of vibration isolators ~~usually~~ ~~consist of~~ involves ~~a~~ quasi-static force-deflection measurements and dynamic sweep excitations at different amplitude of vibration in order to characterize their complex energy dissipation, frequency and amplitude dependent characteristics.

~~A~~ Representative energy dissipation characteristics of a rubber bushing used in the cabin suspension of a heavy commercial truck from quasi-static measurements are ~~is~~ shown in Figure 1.a. ~~As can be seen from the figure,~~ The force-displacement characteristic are ~~is~~ highly nonlinear ~~over the displacement as well as behaving differently under compression and tension loading~~. ~~According to~~ The results of dynamic sweep tests of vibration isolators showing the stiffness and phase angle of vibration isolators ~~both change over~~ with frequency generally ~~increasing with an increase in the frequency as can be seen from~~ are shown in Figure 1.b and Figure 1.c, respectively. ~~It is well known in literature that~~ Phase angle is related to damping of a system and therefore it could participate to energy dissipation (Scheiblegger et. al., 2014). It is also observed that while the stiffness of the component generally is reduced with increasing amplitude of excitation, the phase angle is increased. ~~Dynamic sweep test results also show that phase angle~~

The damping changes significantly over the frequency range from 0 to 20 Hz

with respect to frequency

~~has non-zero initial value at static condition. This could be cross-validated from total energy dissipation at static test and initial phase angle as well.~~ Figure 1.d shows the ~~frequency dependency of~~ damping of a rubber bushing measured from dynamic sweep tests ~~related to material friction of the component. Despite of well-known linear relation between damping and phase angle, "viscous friction" effect is seen from the plot approximately up to 20 Hz.~~ This means that there is a high amount of damping as part of the equivalent damping of the system, which could change the response of the dynamic system significantly. This is especially true for the simulation of ground vehicles for vehicle dynamics and durability assessment since the frequency of the excitations coming from the road surface covers the frequencies up to 20 Hz as well. Therefore, viscous damping effect of the rubber bushings should be also reflected to simulation models in order to predict the durability loads at the engine mounts, suspension attachment points as well as the vehicle ride and impact harshness metrics accurately.

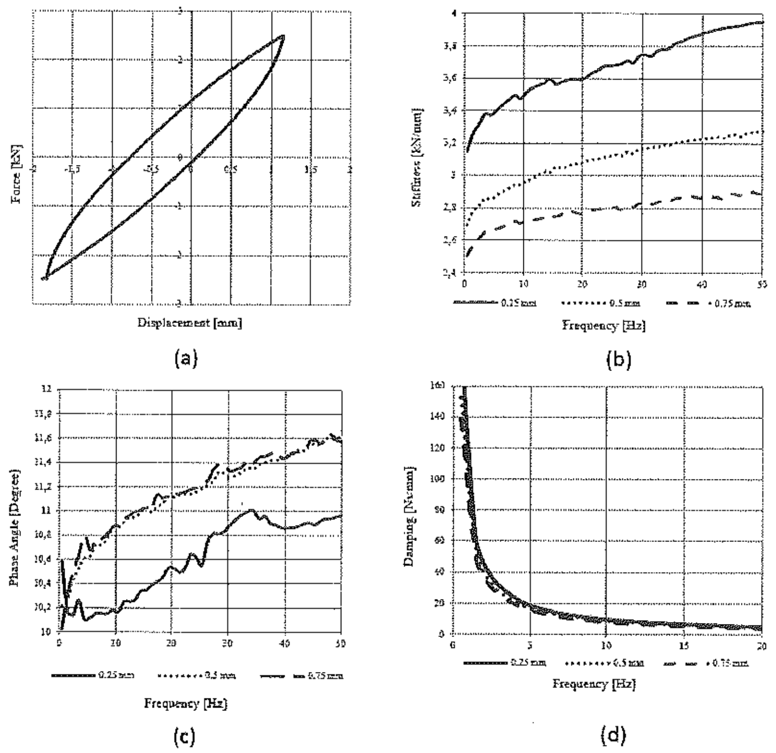
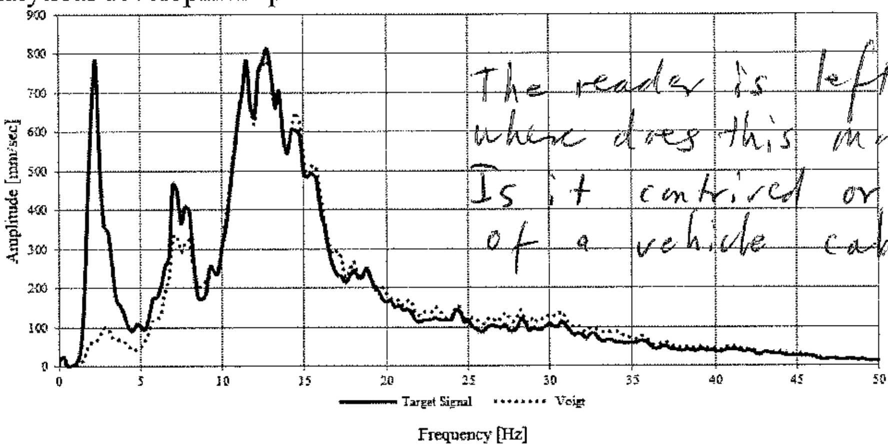


Figure 1: Quasi-static and dynamic characteristics of rubber bushings (a. quasi-static measurement, b. Stiffness versus frequency from dynamic sweep test, c. Phase angle versus frequency from dynamic sweep tests, d. Damping versus frequency from dynamic sweep tests)

The Voigt model is widely used to represent the vibration isolator elements in multi body dynamics models and can be considered as a linear spring element and a linear damper element connected in parallel. The stiffness coefficient of the Voigt model can be obtained from the mean value of the real part of the complex stiffness over the frequency range of interest. The damping characteristic of the Voigt model is derived by measuring energy dissipation per cycle around the resonant frequency.

It is not clear! The Voigt model can only model the correct damping at one frequency if the damping varies with frequency. A SDOF has one resonant frequency.

The use of Voigt models is limited to Single Degree-of-Freedom (SDOF) systems since the Voigt models are only capable of predicting the system dynamics when the number of the frequency of interest is one. Therefore, the Voigt models are not accurate for the Multi-Degree-of-Freedom (MDOF) systems where the numbers of resonance frequencies are more such as automotive ride models. The deficiency of the Voigt model is demonstrated in Figure 2 for the ~~multi~~MDOF system with excitation characteristics shown as target signal, which has three significant resonance peaks up to 50 Hz. ~~Multi-DOF~~ The MDOF system has been modelled with Voigt model with system rigid body mode tuned to 12 Hz. Therefore, the model can predict the dynamics around 12 Hz frequency range while the other two resonance peaks (around 2.5 Hz and 7 Hz) are ~~ignored~~ inaccurately modelled due deficiencies of Voigt models explained in the section. If the responses of the system at these two frequencies are important, the simulation model is not considered valid and ~~therefore should not be used as a part of analytical development process.~~



The reader is left wandering where does this model come from. Is it contrived or representative of a vehicle cabin?

Figure 2: Voigt model response versus target signal

3. Modelling of multi-DOF systems with simplified transfer function models

In traditional way of modelling mechanical systems such as ground vehicles, every part in the system is modelled as lumped mass and connection between parts are modelled with the proper constraint equation and elastic behavior of the joint (Tebbe et. al.,2006). This modelling strategy so called full-body modelling is disadvantageous especially in terms of increase in modelling effort since it requires the measurement of the rigid body properties of each part and elastic characteristics between the rigid bodies. Besides simulation of the analytical model usually takes longer due to the increased number of equations of motion. Longer simulation times get even more critical during design optimization studies where the model is calculated thousands of times. Therefore, an analytical modelling approach, while accurately capturing the dynamics of system is vital. For example, cabin suspension system of a heavy duty truck is shown in Figure 3. Modelling the kinematics of suspension, and obtaining the mass, inertia, center of mass of rigid bodies in the system model as well as the viscoelastic characteristics of isolation elements and converting them to multi-body dynamics model requirement is a challenging and time-consuming task. In Figure 4, corresponding MSC.ADAMS model of the full cabin suspension system is shown.

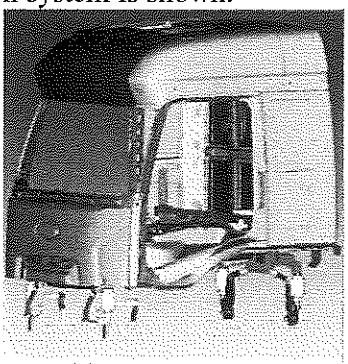


Figure 3: Cabin suspension system of a heavy duty truck

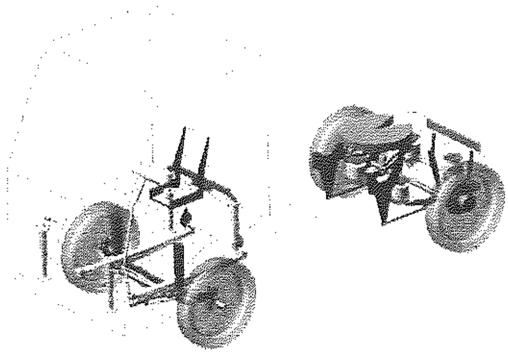


Figure 4: Multibody dynamics model in MSC.ADAMS software

This paper proposes to follow a systematic approach to calculate the durability loads at the cabin attachment points more accurately with a simplified mathematical model compared to the full model. The details of the approach are shown in Figure 5. The first step is to transform the full model of a truck to simple model in order to simplify the modelling assumptions and eliminate the need for obtaining many input parameters. Once the system model is simplified, the next step is to calculate the inertial force and moment at the center of gravity of the system. Then, the equivalent stiffness of the viscoelastic elements (rubber bushings) is determined from the rigid body modes of the system. Damping characterization including the friction characteristics of the viscoelastic elements is performed next before the calculation of the overall response of the simple model to road inputs. The details of each step are explained in the following sections in this order.

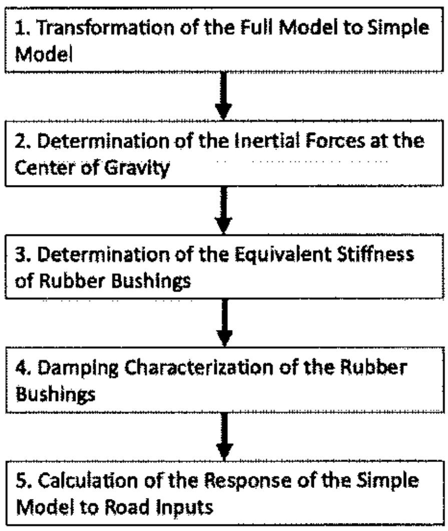


Figure 5: Systematic approach to calculate the durability loads

3.1. Simplified System Model

To be able to simulate complex systems that consist of many degree of freedom with vibration isolators, it is essential to reduce number of parameters needed to be tuned and frequency dependency should be investigated despite of time domain analysis is performed. Therefore, the simplified model that is equivalent to the original full model in terms of kinematic and dynamics response is desired. The simplified model of the truck cabin suspension system, shown in Figure 6, only consists of the equivalent mass and inertia of the cabin at the center of gravity of the cabin and connected to 4 bushings representing the overall stiffness and damping characteristics at the cabin attachment points.

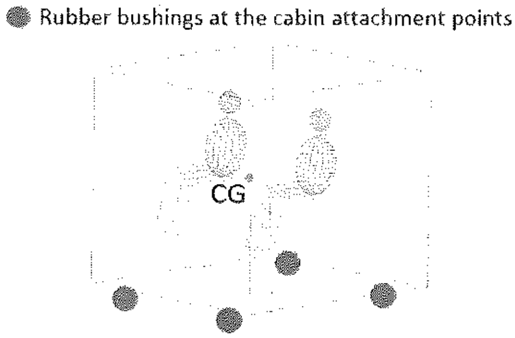
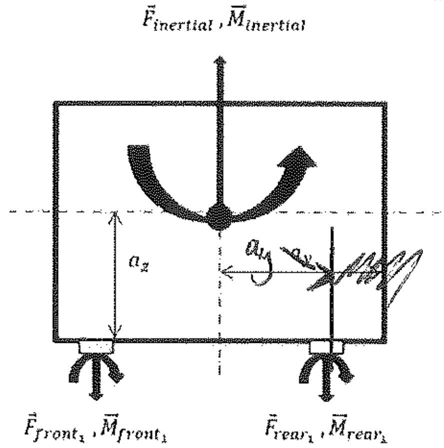


Figure 6: Simplified representation of the cabin model

Simplified model requires the determination of 1) system excitation from road inputs, 2) mass / inertia and center of gravity of the body, 3) the equivalent stiffness properties of the rubber bushings and 4) determination of the damping characteristics of the rubber bushings. The free-body diagram of simplified system model could be seen in Figure 7 and the equilibrium equations are given in Equation (1) and Equation (2).



ay incorrect
Not SAE coordinates as x is fore-aft in SAE.

Figure 7: Free-body diagram of simplified system model

Not clear what i=1 or i=2 represent but it is likely i=1 is left and i=2 is right.

$$\sum_{i=1}^{2} \vec{F}_{inertial} - \vec{F}_{front_i} - \vec{F}_{rear_i} = 0 \tag{1}$$

$$\sum_{i=1}^{2} \vec{M}_{inertial} - \vec{M}_{front_i} - \vec{M}_{rear_i} = 0 \tag{2}$$

3.2. Determination of Inertial Loads

In order to calculate the inertial loads at the center of gravity (CG) of the

cabin, acceleration vector of the CG of the cabin is needed. Cabin CG accelerations are usually obtained by measuring at least six-channels for accelerations located at three locations of the cabin. The equations to calculate the cabin CG accelerations are shown in Equation (3) and Equation (4), where A_{cg} is 6xN (N: number of measured data point) matrix containing CG accelerations with both translational and angular components. A_{mes} is measurement data, which contain the acceleration data of the cabin attachment points where the accelerometers are rigidly attached to. B is the conversion matrix related to the location of accelerometers relative to the cabin CG location. Sensor locations should be selected such that the matrix B is non-singular.
Once the CG accelerations of the cabin are calculated as described above, the inertial loads acting at the CG of the cabin are calculated by multiplying the acceleration matrix with the mass matrix as in Equation (5). $F_{inertial}$, a 6xN matrix, contains the translational forces and rotational moments in all directions acting at the CG of the cabin (Yang et. al., 2014.

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & z_2 & -y_2 \\ 0 & 1 & 0 & -z_2 & 0 & x_2 \\ 0 & 0 & 1 & y_2 & -x_2 & 0 \\ 0 & 0 & 1 & y_3 & -x_3 & 0 \\ 0 & 0 & 1 & y_1 & -x_1 & 0 \\ 0 & 1 & 0 & -z_3 & 0 & x_3 \end{bmatrix} \tag{3}$$

$$A_{cg} = B^{-1} \cdot A_{mes} \tag{4}$$

$$F_{inertial} = m \cdot A_{cg} \tag{5}$$

One underlying assumption of the formulation is that the body in consideration behaves like a rigid body and there is no relative motion between any two points on the same body for accurate calculation of the inertial force and moments at the center of gravity (Yang et. al., 2014).

3.3. Determination of Equivalent Stiffness

Correlation between the physical system and simulation model depends on the determination of the equivalent stiffness of the viscoelastic elements such as rubber bushings accurately in the model. The equivalent stiffness is related to the rigid body modes of the physical system. Once the rigid body modes of the system are known, equivalent stiffness of the elastic elements are calculated from Equation (6) to Equation (11). In these equations f_{fx}, f_{fy}, f_{fz} are translational rigid body modes, f_{tx}, f_{ty}, f_{tz} are rotational rigid body modes, k_x, k_y, k_z are equivalent stiffness of the system, m is mass of the body and ρ_x, ρ_y, ρ_z are radius of gyration in given directions (Harris and Piersol, 2002).

$$f_{fx} = \frac{1}{2\pi}\sqrt{\frac{\sum k_x}{m}} \tag{6}$$

$$f_{fy} = \frac{1}{2\pi}\sqrt{\frac{\sum k_y}{m}} \tag{7}$$

$$\cancel{f_{fz} = \frac{1}{2\pi}\sqrt{\frac{\sum k_z}{m}}} \tag{8}$$

$$f_{tx} = \frac{1}{2\pi}\sqrt{\frac{\sum(k_y a_z^2 + k_z a_y^2)}{m\rho_x^2}} \tag{9}$$

model only has 3 DOF see Figure 7.

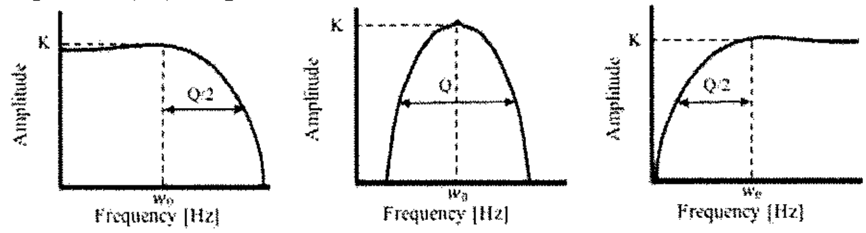
$$\cancel{f_{ty} = \frac{1}{2\pi}\sqrt{\frac{\sum(k_x a_z^2 + k_z a_x^2)}{m\rho_y^2}}} \tag{10}$$

$$\cancel{f_{tz} = \frac{1}{2\pi}\sqrt{\frac{\sum(k_x a_y^2 + k_y a_x^2)}{m\rho_z^2}}} \tag{11}$$

3.4. Damping Characterization and Transfer Function Model

According to vibration isolator characteristics explained in Section 2, the mathematical models should have the details in terms of static stiffness, dynamic stiffness, phase angle and frequency dependency in terms of damping characteristics. The paper proposes to derive the equivalent damping characteristic of the system from the frequency response of the measurement data from vehicle level tests (will be referred as the target signal) and that of the simplified model. The transfer function between the measurement of A_{mes} and simplified model response of similar locations of mounted sensor in the body with the stiffness are used to find the order of the transfer function for modelling the damping characteristics.

A generic transfer function between the target signal and the simulation output is shown in Figure 8. This plot is used to identify the damping requirements of the simplified model in order to show similar dynamics as in the target signal (vehicle level test). It is a measure of discrepancy between the target signal and simplified model, and used to determine the damping characteristic as a function frequency that needs to be added to the simplified model. If the damping characteristic is as in Figure 8.b, the damping of the viscoelastic element is modelled for the frequency bands shown in the figure. Similarly, for the transfer functions where damping characteristic is increasing with increased frequency, the damping should be modelled as in Figure 8.c. The transfer functions for low damping, band damping and high damping types are formulated from Equation (12) to Equation (14), respectively.



(a) low damping (b) band damping (c) high damping

Figure 8: Frequency dependent damping

$$TF_{low} = K \cdot \frac{{\omega_0}^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \tag{12}$$

$$TF_{band} = K \cdot \frac{\frac{\omega_0}{Q} \cdot s}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \tag{13}$$

$$TF_{high} = K \cdot \frac{s^2}{s^2+\frac{\omega_0}{Q}s+\omega_0^2} \tag{14}$$

where $s = \sigma + j \cdot \omega$, where σ is amplitude or gain and ω_0 is angular frequency.

Q is known to be the "quality factor", which is a unitless parameter for determining the bandwidth of the systems in frequency domain where higher Q indicates lower rate of energy loss; it means low damping rate at the system. So Q determines system behavior in terms of its damping characteristics such as overdamped, underdamped or critically damped. In Equations (12) to (14), ω_0 determines the center frequency of the transfer function, which will be extended with Q for the frequency band. Finally, the K term in the damping transfer functions in Equations (12) to (14), is the gain of the transfer function and scales the amplitude of the damping transfer function. This term is used to minimize the error between responses of the target signal and simple model.

3.5. Determination of the transfer function parameters

Three unknown parameters, Q, ω_0 and K in Equation (12) to Equation (14), are determined by minimizing the response of the simple model and target signal. For this purpose, two objective metrics to measure the accuracy of the simulation with respect to test data are chosen: 1)

Coefficient of Determination 2) Root Mean Squared Error (RMSE). The coefficient of determination is a measure of how similar the shape of the simulation result is with respect to the test data. The coefficient of determination, shown by symbol r, is given by Equation (15). It could range from -1 to 1, i.e. $-1 \leq r \leq 1$, where r=1 is where the simulation results are perfectly correlated to test results. The coefficient of determination is used to tune the parameters Q and ω_0 in Equation (12) to Equation (14).

$$r = \frac{n \cdot \sum(f \cdot y) - (\sum f) \cdot (\sum y)}{\sqrt{n \cdot (\sum f^2) - (\sum f)^2} \cdot \sqrt{n \cdot (\sum y^2) - (\sum y)^2}} \tag{15}$$

where y is the target signal (measurement data), f is the simulation signal of simplified model and n is the number of simulation data points.
The third parameter K, the transfer function gain, is determined by minimizing the Root Mean Squared Error (RMSE) according to Equation (16):

$$min\left[\sqrt{\frac{1}{N}\sum_{i=1}^{n}(y_i - f_i)^2}\right] \tag{16}$$

Similarly, where y is the target signal (measurement data), f is the simulation signal and i is the number of simulation data points. In Equation 16, $\sqrt{\frac{1}{N}\sum_{i=1}^{n}(y_i - f_i)^2}$ represents the Root Mean Squared Error (RMSE) between simplified model and test data.
The deficiency of the Voigt model can therefore be avoided with the damping characterization proposed which maximizes the coefficient of determination (therefore forcing the shape of the frequency response of the simple model to be similar to the test data) and minimizes the RMSE between the simple model and test results.

4. EXPERIMENTAL RESULTS

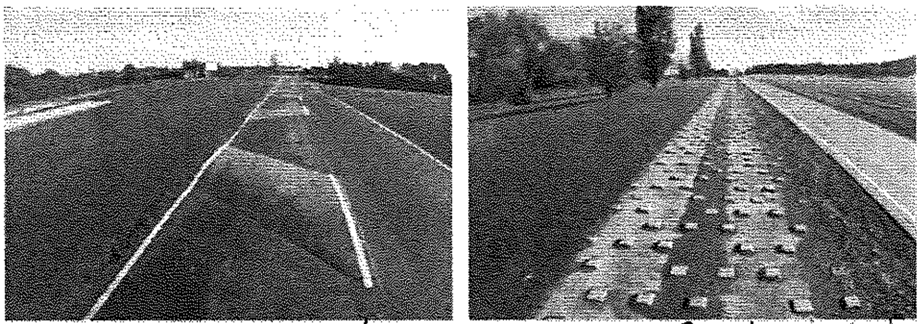
In this section, the proposed methodology will be demonstrated on a heavy commercial truck cabin for calculating the loads at the attachment points in order to demonstrate the effectiveness of the simulation model compared to the test data from the prototype testing.
A 6x4 heavy commercial truck with mechanical suspension system at both chassis and cabin suspension system is selected to demonstrate the methodology. The properties for mass, inertia properties, spring and damping rates of chassis and cabin suspension system are given in Table 1.

Table 1: The properties of the 6x4 truck

Parameters	**Values**
Chassis (front-rear) ~~/ Cabin~~ Suspension (front-rear) Stiffness (kgf/mm) change	37-370 / 50-40 to N/mm
Chassis (front-rear) / Cabin Suspension (front-rear) Damping (Ns/mm)	30-30 / 10-10
Truck Mass / Cabin Mass (kg)	24000 / 1200
Cabin Inertia $I_{xx,yy,zz}$ (kgm^2)	$7.3x10^8$, $5.4x10^8$,$8.1x10^8$

The construction truck has been tested on various road conditions such as the durability ~~inner~~ track shown in Figure 9. The objective of these tracks is to test the truck according to its customer correlated usage specifically in terms of simulating the combined vertical and lateral load conditions. For example, part of the track shown in Figure 9.a provide combined vertical and pitch excitation for the vehicle, while the part of the test track in Figure 9.b provides major excitation only in the vertical direction.

Displacement and acceleration sensors have been instrumented on truck cabin and chassis frame for correlation purposes. For example, four tri-axial acceleration sensors at the cabin attachment points are placed and acceleration data on various durability roads have been collected. Sensor mount locations and sensor specifications are given in Table 2.

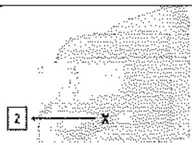
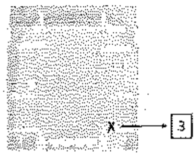
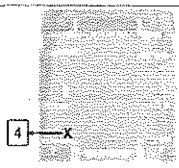


Road with humps
(a) ~~Deterministic road~~

Road with interspersed Belgian pavé
(b) ~~Random road~~

Figure 9: Parts of the durability ~~inner~~ track

Table 2: Sensor specifications and mounting locations

Sensor Location	Sensor Specification	Graphics
Front left on BIW	Silicon Designs DC type tri-axial accelerometer	
Front right on BIW	Silicon Designs DC type tri-axial accelerometer	
Rear left on BIW	Silicon Designs DC type tri-axial accelerometer	
Rear right on BIW	Silicon Designs DC type tri-axial accelerometer	

The inertial loads acting on the center of the gravity of the simplified model are calculated as the first step of the methodology. The acceleration data from the measurements and the locations of the accelerometers are used as part of Equation (1) to Equation (3) in order to calculate the inertial load. The vertical component of the inertial load is shown in Figure 10 as an illustration. Similarly, the forces and moments in all directions are calculated by following a similar approach.

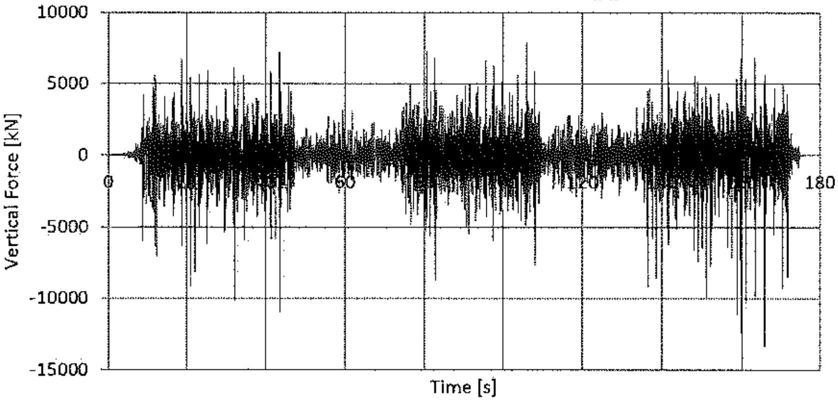


Figure 10: Vertical component of the inertial load acting on the center of the gravity of the cabin

The stiffness of the cabin bushings in each direction is manually tuned in order to match the 6 rigid body modes of the simplified model with the test results from four poster testing (Sendur et. al. 2016). The Equation (6) to Equation (11) are used to calculate required equivalent stiffness of the system. The corresponding parameters for the bushing rates to achieve the rigid body modes from Table 3 (front left, front right, rear left and rear right) are summarized in Table 4.

Table 3: Comparison of rigid body modes: simplified model versus four poster test

Description of the Mode	**Simplified Model [Hz]**	**Four Poster Test [Hz]**
Fore-Aft Mode	30.6	31.5
Lateral Mode	34.7	33.7
Bounce Mode	2.2	2.6
Roll Mode	1.7	2.4
Pitch Mode	1.2	1.5
Yaw Mode	18.7	13.6

Table 4: Equivalent stiffness derived from rigid body modes

	Translational [N/mm]			Rotational [N/mm]		
	Fx	Fy	Fz	tx	ty	tz
Front Left	7976	4600	52.1	572.9	4018.1	11.4
Front Right	7976	4600	52.1	572.9	4018.1	11.4
Rear Left	7976	4600	41.7	572.9	4018.1	11.4
Rear Right	7976	4600	41.7	572.9	4018.1	11.4

Frequency response of the simplified model with the parameters from Table 4 as inputs of the simplified model is compared to that of the frequency of the test results (referred as target signal). Figure 11 shows the frequency spectrum of the vertical acceleration of the cabin front left attachment for the simplified model with no damping and from road test data of random road. The results show that there is a similarity on the peak frequencies between test and simulation results. For example, the peaks at 1-4 Hz band, 5-8 Hz band and 10-15 Hz bands in Figure 11 are well captured by the simplified model with equivalent stiffness of the bushings with respect to test signals that characterized with rigid body modes given in Table 3. The similarity between the results is concluded to be sufficient. Therefore, the damping characterization with the simplified transfer function approach is performed next.

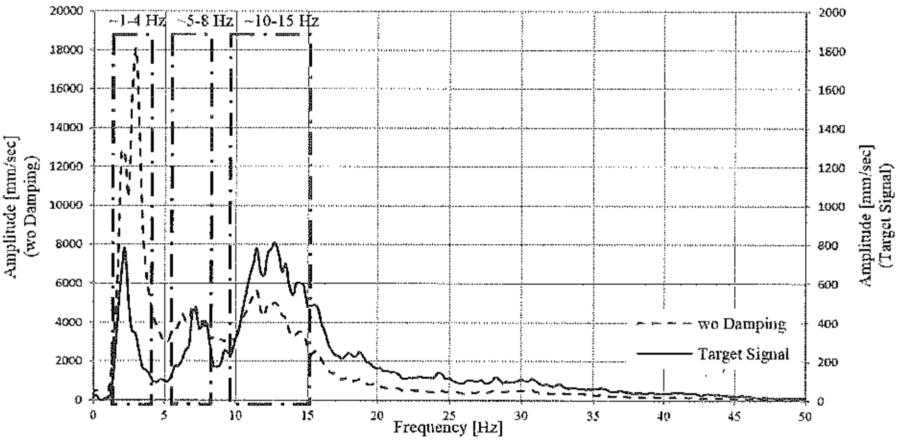


Figure 11: Comparison of frequency response of simple model (with equivalent stiffness and no damping) versus road test data

Damping characterization of the rubber bushings is followed next. The frequency of the target signal is divided by that of simulation signal (with no damping) to identify the damping characteristics of the rubber bushings and plotted in Figure 12. The damping transfer function shows similar characteristics as the band damping as shown in Figure 13.b and given by Equation (13).

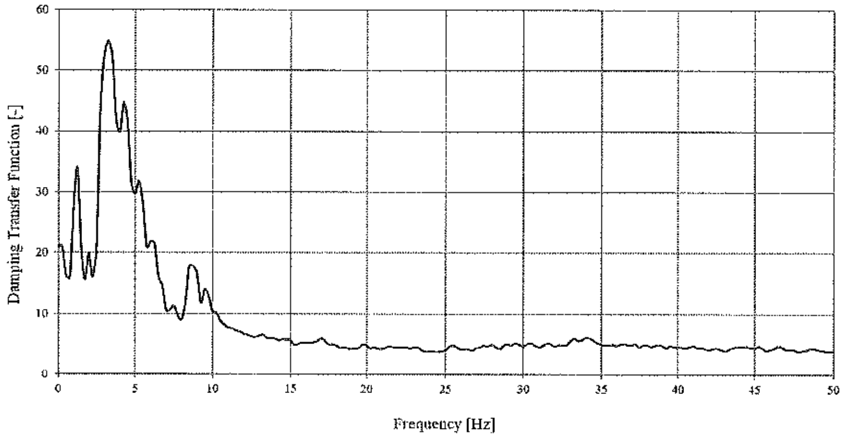


Figure 12. Damping transfer function

The parameters of the damping transfer function are calculated by designing a band-pass character at 2 Hz as a center frequency. Quality factor "Q" is calculated in order to maximize the coefficient of determination between the target signal and simulation signal as given in Equation (15). Similarly, the filter gain "K" of the transfer function is calculated according to Equation (16) by minimizing the RMSE between the target signal and simulation signal. The transfer function parameters are summarized in Table 5.

Table 5: Parameters of the band damping transfer function

Parameter	Value
ω_0 [1/Hz]	12.6
Q [-]	0.5
K [-]	270

The proposed methodology for damping characterization has been implemented in the multi body dynamics modeling environment (MSC.ADAMS) with co-simulation with MATLAB/Simulink software as shown in Figure 13. Instantaneous velocities at the 4 cabin attachment points ($\dot{z}_{fl}$, $\dot{z}_{fr}$, $\dot{z}_{rl}$, $\dot{z}_{rr}$ are the vertical velocities of front-left, front-right, rear-left and rear-right attachment points, respectively) are calculated in MSC.ADAMS for every time step and passed as inputs to MATLAB/Simulink as inputs to the damping transfer function calculations. Then, the damping forces at the cabin attachment points ($TF_{fl}(\dot{z}_{fl})$, $TF_{fr}(\dot{z}_{fr})$, $TF_{rl}(\dot{z}_{rl})$, $TF_{rr}(\dot{z}_{rr})$ are the vertical damping forces at the front-left, front-right, rear-left and rear-right attachment points, respectively) are calculated according to Equation (13) for the band-damping transfer function and sent to MSC.ADAMS for the solution of the equation of motion.

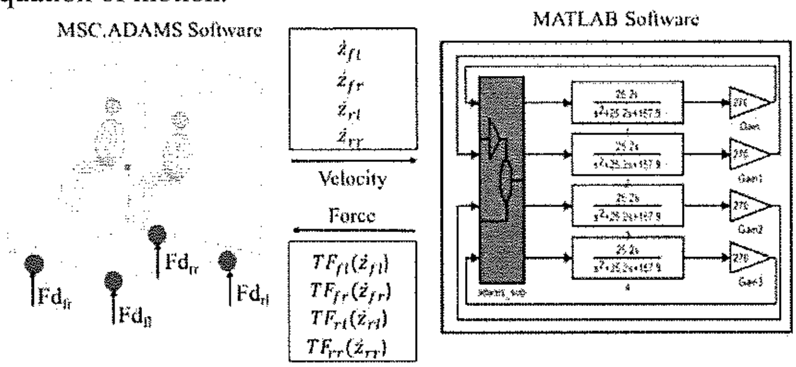


Figure 13: Implementation of the damping characterization in multi-body simulation environment

The results are plotted for the vertical acceleration of front-left cabin attachment on deterministic (road profile from Figure 9.a) and random road profile (road profile from Figure 9.b) in Figure 14 and Figure 15, respectively. The results show good correlation in terms of the resonance frequency and the amplitudes corresponding to those frequencies. The coefficient of determination, r, from Equation (15) between simple model and test data is calculated to be 0.94 for Figure 14 (on deterministic road), and 0.97 for Figure 15 (on random road). If the coefficient of determination is close to 1, it means that two signals are highly similar to each other, while correlation number values close to zero indicate the dissimilarity between two signals (Oppenheim and Schafer, 1975 and Srinath et. al., 1995). Root Mean Squared Error (RMSE) from Equation (16) is calculated to be 134 mm/sec^2 for Figure 14 (on deterministic road), and 52 mm/sec^2 for Figure 15.

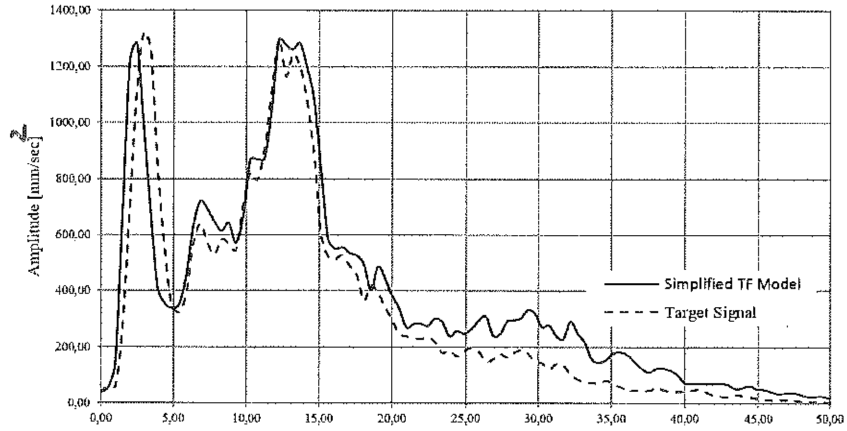


Figure 14: Frequency domain results for the vertical translational acceleration of the front-left cabin attachment point on deterministic road.

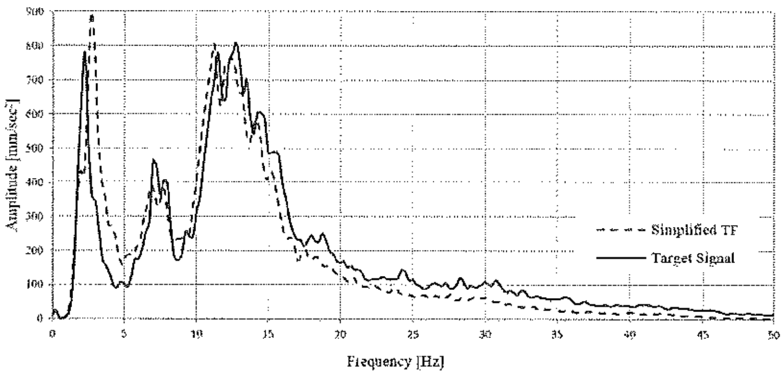


Figure 15: Frequency domain results for the vertical translational acceleration of the front-left cabin attachment point on random road

5. CONCLUSIONS

The employment of simulation models during the design cycle depend on many factors such as the quality of the simulation model. The model quality criteria can be considered as the complexity of the simulation model and accuracy. While the complexity of a simulation model determines the number of input parameters and the computational time, the accuracy of the simulation model determines the validity of the simulation model for the design cycle. In this paper, a simple modeling approach to predict the loads in multi-body systems and the determination of the viscoelastic properties of the model are proposed. This approach enables the simulations with minimum number of inputs. The paper also points out the limitations of widely used Voigt model in viscoelastic models and a methodology to overcome the limitations of the Voigt model is proposed.

First, the paper offers a simplification strategy and determination of inertial loads that replicates motions of multi degree of freedom systems. A simple 6 DOF cabin model is proposed with the equivalent compliance and kinematic properties as the full vehicle model in order to calculate the durability loads at the cabin attachment points. The system is represented by a lumped mass approach where the cabin is represented by mass, inertia and center of gravity and the cabin kinematics is represented by overall equivalent bushings. This representation eliminates the modelling of all the details of the system while requiring a shorter list for the input parameters.

The limitations of the current modelling of rubber bushings with respect to its damping representation were explained, and a new modelling approach is proposed. The new model addresses the damping characterization of the rubber bushings by a band pass filter general transfer function form covering upper and lower bounds, where the parameters can be easily tuned even with simple hand calculations. The implementation of the proposed methodology in the multi body dynamic simulation environment is explained with co-simulation of MSC.ADAMS with MATLAB/Simulink software.

The methodology is shown in the calculation of the durability loads at the cabin attachment points of a heavy duty truck on deterministic road and random road profiles. As illustrated in the case study, the representation of the damping in rubber bushing is of significant importance for accurate model behavior. The accuracy of the methodology with respect to the test data has been compared on two metrics: the coefficient of determination and root mean-squared error (RMSE). The relation between the parameters characterizing the damping transfer function in order to improve the accuracy of the simulation model with respect to physical testing are also presented.

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Originality of the work. The authors have presented their own work in replacing the force transmission between the chassis of a truck and the cabin with a transfer function that is able to account for the frequency dependency of the damping of the rubber bushings. The work is not a meaningful contribution to the literature.

Subject relevance. The challenges faced in modelling rubber bushings is a relevant and worthwhile topic. It is an important research area for mining vehicle manufacturers.

Professional/industrial relevance. While the topic is important, the contribution of the work is not significant as detailed in the paragraph below.

Completeness of the work. The work fails to compare the frequency dependent transfer function method with the Voigt model. The work motivates that a method to determine the frequency dependent damping characteristics of rubber bushings directly from vehicle testing would be beneficial. The work however uses an ADAMS model for the evaluation of the method. If real test measurements are used then this is not clear. Numerous important details such as the vehicle test speed, road profile used, measurement of the road profile, test vehicle details, DAQ sampling rate, measurement accuracy etc. have not been explained. Using field test data to conduct a least-squares regression to determine the damping characteristics of the rubber bushes would highlight numerous challenges and likely invalidate what has been presented. The road profile must be measured, and the vehicle parameters such as tyre characteristics, vehicle suspension characteristics must be measured. The imprecise measurement of all these parameters will influence the accuracy of the least-squares regression.

Acknowledgement of the work of others by references. I am concerned that the pictures in Figure 9 are taken from another manuscript.

Organization of the manuscript. The work motivates that the Voigt model is not accurate for rubber bushings and proposes using a simplified transfer function approach. A comparison of the results obtained using a Voigt model and the simplified transfer function approach is required.

Clarity in writing, tables, graphs and illustrations. The writing is verbose. A scan of some suggested changes to the first two pages is attached to make the writing sharper.

Likelihood of passing the "test of time". See above comments.

QUALITY AND RIGOR:

In your opinion, is the technical treatment plausible and free of technical errors? Yes No

Have you checked the equations and/or statistics? (if applicable) Yes No

Are you aware of prior publication or presentation of this work? Yes No

Is the manuscript free of commercialism? Yes No

Is the paper too long? Yes No

RECOMMENDATION (*)

Honours quality

Acceptable (manuscript can be accepted without changes)

Acceptable with minor revisions (revision do not imply further research, but some clarification or minor corrections)

Reconsider after major revisions (substantial changes should be carried out before acceptance)

Not Acceptable (manuscript contribution is not significant, or there are severe concerns invalidating its conclusions) *

COMMENTS

Suggestions which would improve the quality of the paper but are not essential for publication:

Changes which must be made before publication:

A comparison of the results obtained using a Voigt model and the simplified transfer function approach is required.

The paper must include details of the experimental measurement of accelerations and using field data to evaluate the proposed method.

Some corrections to grammar, diagrams and units are highlighted in the scanned feedback.

Confidential comments only for the editor: