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# Homework #1 (70pt), Due. 02-07-2017

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**Q1.** [20pt] We discussed two versions of 3-sum problems: One takes $O(N^3)$ and the other takes $O(N^2 \log N)$. Implement these two algorithms. Your program takes the regular text file as an input. Find sample input data from the attachments and measure the runtime cost of your implementations as function of input data size. Plot a graph and discuss your results.

**Q2.** [20pt] Recall Union-Find algorithms discussed in class. Implement four versions: (i) Quick-Find, (ii) Quick Union, (iii) Weighted Quick Union, (iv) Weighted Quick Union with Path Compression. Using the data provided in the attachments and the client program described in the lecture, determine runtime cost of your implementations as a function of input size. Plot a graph and discuss your results.

**Q3.** [10pt] Recall the definition of Big O notation, where F(N) is O(g(N)) when $F(N) < c\, g(N)$ for $N > N_c$. Estimate the value of $N_c$ for both Q1 and Q2. Explain why with your empirical data.

**Q4.** [10pt] Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

**Q5.** [10pt] Sort the below list using order of growth and discuss why you come with such an ordering.

| | | | | | |
|---|---|---|---|---|---|
| lg(lg*$n$) | $2^{\lg^* n}$ | $(\sqrt{2})^{\lg n}$ | $n^2$ | $n!$ | (lg n)! |
| $(\frac{3}{2})^n$ | $n^3$ | $\lg^2 n$ | lg (n!) | $2^{2^n}$ | $n^{1/lgn}$ |
| ln ln n | lg* n | $n2^n$ | $n^{\lg \lg n}$ | ln n | 1 |
| $2^{\lg n}$ | $(\lg n)^{\lg n}$ | $e^n$ | $4^{\lg n}$ | (n+1)! | $\sqrt{\lg n}$ |
| lg*(lg n) | $2^{\sqrt{2 \lg n}}$ | n | $2^n$ | n lg n | $2^{2^{n+1}}$ |

**Q6.** [10pt] **[Extra Credit]** Recall the percolation problem discussed in the class. Using your own implementation of union-find algorithm acquired in Q2 above, implement a program that enables Monte Carlo simulation. Your program may perform the following sequence of operations.

- Prepare NxN grid and make all sites blocked.
- Choose a blocked site (row i, col j) uniformly at random among all blocked site and open the site.
- Continue until the system percolates.

Run your program as many times as possible, calculate and report the following metrics.

a) For 100x100 grid, report average and standard deviation of percolation thresholds (p*).
b) For varying grid sizes, report runtime as a function of input size.

Note) You can calculate percolation threshold as follows. For 20x20 grid, if the system percolates when 100$^{th}$ site is opened, the percolation threshold is 100/400 = 0.25.