

materials lecture notes

erentar

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Contents

2	Stiffness, stress, and strain	3
2.1	Stress and strain	3
3	Atomic structure	6
3.1	Polymers	6
4	Thermal Expansion	7

The lectures follow a question-answer style, which is less suited for the textbook-style notes i normally take. This will be less organised than the other courses.

Chapter 2

Stiffness, stress, and strain

By convention, tension forces are written as positive, and compression forces as negative.

Definition 1. Elastic deformation: body changes *temporarily* shape under load, but returns to its original shape after load is removed.

Definition 2. Plastic deformation: body *permanently* changes shape under load, does not return to its original shape after load is removed.

Definition 3. Fatigue: as the material is deformed and relaxed; cracks form, move, and finally accumulate near each other, which eventually causes failure.

Definition 4. Strength: maximum load endured before failure.

Definition 5. Stiffness: resistance to deformation. Stiffer materials deform less.

Definition 6. Resilience: maximum stress and strain endured before plastic deformation occurs. More resilient materials stay elastic under higher loads.

Definition 7. Poisson coefficient $\left(\nu = \frac{-\epsilon_y}{\epsilon_z}\right)$: ratio of $\underbrace{\text{lateral strain}}_{\text{change in diameter}}$ to $\underbrace{\text{axial strain}}_{\text{change in length}}$ for a cylindrical bar.

Definition 8. Isotropic material: material whose properties are the same regardless of where and which direction you take the slice. Composites are anisotropic: composites are much stronger when the force is applied parallel to the fibres but not strong at all if force is perpendicular to the fibres.

2.1 Stress and strain

Stress $\left(\sigma = \frac{F}{A}\right)$ is defined as force per unit area.

In *true stress*, the area variable A can vary as one goes through the length of the material.

For simplifying calculations (eg. eliminating a second variable from an integral), *engineering stress* σ_{eng} is defined, which neglects changes in area.

$$\sigma_{\text{eng}} = \frac{F}{A_0}$$

where A_0 is the initial area.

Engineering stress is used to simplify calculations when dealing with elastic-deformed metals, because they do not change much in cross-section. However, if the metal is plastically deforming, or we are dealing with a material which does change cross-section under elastic deformation (such as elastomer rubber), we use *real stress*.

Strain $\varepsilon_{\text{eng}} = \left(\frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0} = \frac{L_1}{L_0} - 1 \right)$ is the elongation in relation to the original length of a material. This is the *engineering strain*.

With large deformations, it makes less relevant to talk of strain in relation to the original length (TODO why?). In this case, we calculate the *true strain* as $\varepsilon_{\text{true}} = \frac{dL}{L}$ where L is the instantaneous length at that time.

The true strain from one point to another is the integral of this fraction:

$$\int_{L_0}^{L_1} \left(\frac{dL}{L} \right) = \ln \left(\frac{L_1}{L_0} \right)$$

The full formulation of hooke's law takes into account poisson's law:

$$\varepsilon_x = \frac{1}{E} * (\sigma_x - \nu * (\sigma_y + \sigma_z)) \quad (2.1)$$

$$\varepsilon_y = \frac{1}{E} * (\sigma_y - \nu * (\sigma_z + \sigma_x)) \quad (2.2)$$

$$\varepsilon_z = \frac{1}{E} * (\sigma_z - \nu * (\sigma_x + \sigma_y)) \quad (2.3)$$

However, the following cancellations can be made when we constrain the scenario to a uni-lateral load scenario (i.e when $\sigma_x = \sigma_y = 0$):

$$\varepsilon_x = \frac{1}{E} * (\cancel{\sigma_x} - \nu * (\cancel{\sigma_y} + \sigma_z)) \quad (2.4)$$

$$\varepsilon_y = \frac{1}{E} * (\cancel{\sigma_y} - \nu * (\sigma_z + \cancel{\sigma_x})) \quad (2.5)$$

$$\varepsilon_z = \frac{1}{E} * (\sigma_z - \nu * (\cancel{\sigma_x} + \cancel{\sigma_y})) \quad (2.6)$$

which yields

$$\varepsilon_x = \frac{-\nu \sigma_z}{E} \quad (2.7)$$

$$\varepsilon_y = \frac{-\nu \sigma_z}{E} \quad (2.8)$$

$$\varepsilon_z = \frac{\sigma_z}{E} \quad (2.9)$$

[TODO incomplete]

Going from engineering strain (ϵ_{eng}) to true strain (ϵ_{true}):

Definition of engineering strain: $\frac{L-L_0}{L_0} = \frac{L}{L_0} - 1$

Definition of true strain: $\ln\left(\frac{L}{L_0}\right)$

$$\epsilon_{\text{eng}} = \frac{L}{L_0} - 1 \quad \wedge \quad \epsilon_{\text{true}} = \ln\left(\frac{L}{L_0}\right) \quad (2.10)$$

$$1 + \epsilon_{\text{eng}} = \frac{L}{L_0} \quad \wedge \quad \epsilon_{\text{true}} = \ln\left(\frac{L}{L_0}\right) \quad (2.11)$$

$$\therefore \epsilon_{\text{true}} = \ln(1 + \epsilon_{\text{eng}}) \quad (2.12)$$

Going from engineering stress (σ_{eng}) to true stress (σ_{true}):

Definition of engineering stress: $\sigma_{\text{eng}} = \frac{F}{A_0}$

Definition of true stress: $\sigma_{\text{true}} = \frac{F}{A}$

Volume does not change under elastic deformation: $V = A * L = A_0 * L_0 = V_0$

$$\Rightarrow \frac{L}{L_0} = \frac{A_0}{A}$$

$$\Rightarrow \sigma_{\text{true}} = \frac{F}{A_0} * \frac{A_0}{A} \therefore \sigma_{\text{true}} = \sigma_{\text{eng}} * \underbrace{(1 + \epsilon_{\text{eng}})}_{\text{can be neglected since } \epsilon_{\text{eng}} \text{ is close to 0.}}$$

Proof that under elastic deformation, true strains ϵ_1 and ϵ_2 can be added to calculate ϵ_{total} :

the material goes from $L_0 \rightarrow L_1 \rightarrow L_2$

$$\epsilon_{\text{true}_1} = \ln\left(\frac{L_1}{L_0}\right) \quad \epsilon_{\text{true}_2} = \ln\left(\frac{L_2}{L_1}\right)$$

$$\epsilon_{\text{true}_{\text{total}}} = \ln\left(\frac{L_2}{L_0}\right) = \ln\left(\frac{L_2}{L_1} * \frac{L_1}{L_0}\right) = \ln\left(\frac{L_2}{L_1}\right) + \ln\left(\frac{L_1}{L_0}\right) \blacksquare$$

Chapter 3

Atomic structure

- Nylon has greater E-modulus than polyethylene because of intermolecular forces between oxygen atoms of one chain and hydrogen atoms of another chain. A permanent dipole force will form, bonding the chains to each other stronger.
- Diamond is denser than graphite because of its bond angles. Graphite is planar, while diamond has a 3-dimensional structure, which fills more space with mass.
- Covalent bonds are less dense than ionic or metallic bonds. Covalent bonds have individual molecules defined, and are strongly directional. On the other hand, metals and ionic bonds have lattice structure where their structure is optimised to be as close as possible.
- PET bottles are easier to recycle than rubber tyres because rubber tyres are vulcanised, meaning there will be disulfide cross-links between the chains.
- Creep: elastic deformation held for a long time will become plastic deformation. Will be covered in chapter 12.

3.1 Polymers

- Thermoplastics (do not have crosslinks, free chains) can be remolten.
- Thermosets (have crosslinks) cannot be remolten, they burn.

Thermoplastics can be used in regular old injection molding, however thermosets require a more elaborate setup called Reaction Injection Molding, where the reaction to thermoset happens in the mold, and the plastic is shaped permanently.

Chapter 4

Thermal Expansion

Most materials which thermally shrink and expand are not stiff. (TODO why?)

Thermal expansion coefficient (α): amount of lengthening in relation to the original length. per temperature change. unit: $\frac{1}{\text{kelvin}}$

$$\alpha * \Delta T * L_0 = \Delta L$$