

Coulomb's law

$$F_{1,2} = \frac{1}{4\pi\epsilon_0} * \frac{q_1 * q_2}{r^2}$$

Electric field

Electric field is the amount of force caused by a particle on another particle with arbitrary charge.

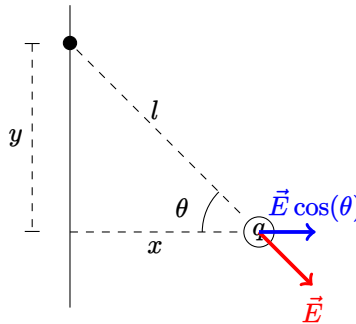
$$\vec{F} = \vec{E} * q_2$$

$$\frac{\vec{F}}{q_2} = \vec{E}$$

$$\frac{F_{q, \text{test charge}}}{q} = \vec{E} = \frac{1}{4\pi\epsilon_0} * \frac{q}{r^2}$$

To measure electric field, one needs a test charge, which is some arbitrary small charge.

Electric field of a charged line



integrate the field force applied by every point on the line, integrate over $y = +\infty$ to $y = -\infty$

notice that since the integral is symmetric in the y -direction, only force in the x -direction remains.

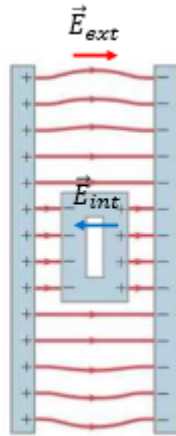
$$\begin{aligned} E &= \int_{y=+\infty}^{-\infty} E_x \\ &= \int_{y=+\infty}^{-\infty} E \cos(\theta) \\ &= \int_{y=+\infty}^{-\infty} \frac{\lambda}{4\pi\epsilon_0 * l^2} dy * \cos(\theta) \end{aligned}$$

λ is the charge per length of the charged wire.

$$l = \sqrt{x^2 + y^2}$$

TODO finish this shit

Induced charges

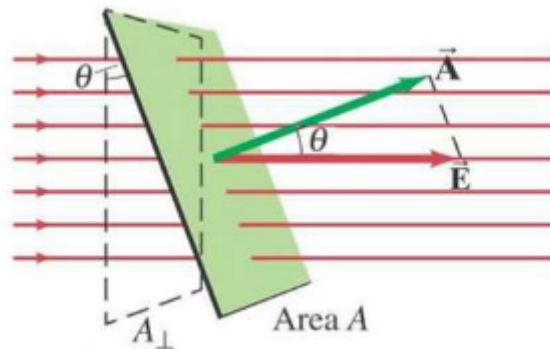


Free charges in a conductor respond to external field such that total field inside the conductor is zero.

$$E_{\text{total}} = E_{\text{external}} + E_{\text{internals}}$$

Faraday cage operates on this principle.

electric flux ϕ is the amount of electric field lines that pass through a surface.



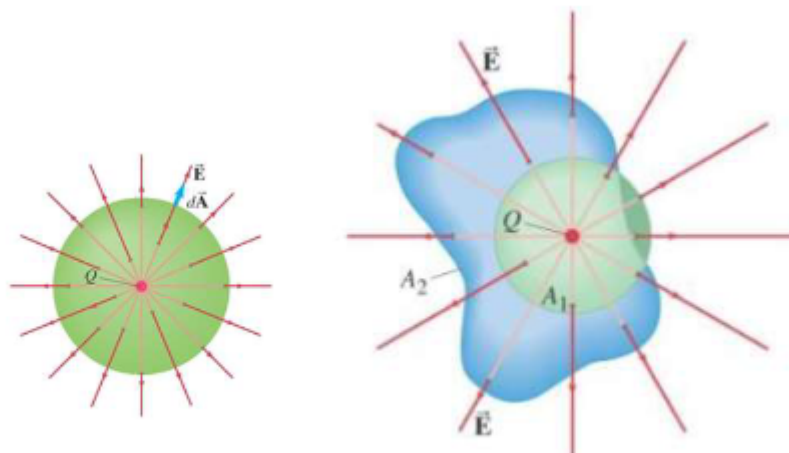
$$\begin{aligned}\phi &= \overbrace{\iint \vec{E} \cdot d\vec{A}}^{\text{surface integral}} \\ &= \iint \vec{E} \cdot d\vec{A} \cdot \cos(\theta)\end{aligned}$$

\vec{A} is the normal vector of the surface, and whose magnitude is the area of the surface.

Gauss's law

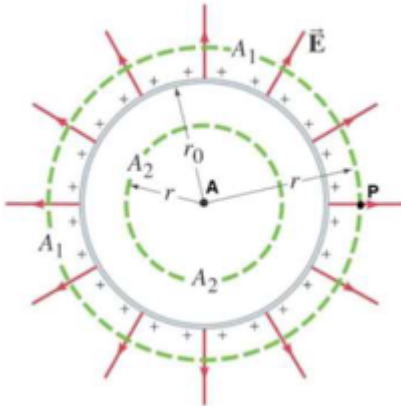
Gauss's law: The electric flux through a closed surface is equal to the net charge enclosed in it divided by ϵ_0

Notice that electric flux is the number of field lines that go through a surface, therefore the following two gauss surfaces have the same flux.



$$\phi_{\text{surface}} = \oiint_{\text{surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Field of a uniformly charged sphere



since all of the field lines are perpendicular to the gauss surface A_1 , and that \vec{E} has the same magnitude everywhere, this is analogous to a point charge. Thusly, uniform homogenous charged spheres can be approximated to point charges.

Field of a non-conducting charge sphere

spoilers, exact same thing as above.

Field of a charged plate