1. **Estimate (using tilde notation) the running time (in seconds) of your two programs as a function of the number of points N. Provide a formal analysis about why you consider the formula provided is correct; you can use a style similar to the proof sketch format used in several sections of the book.**

   Answer:

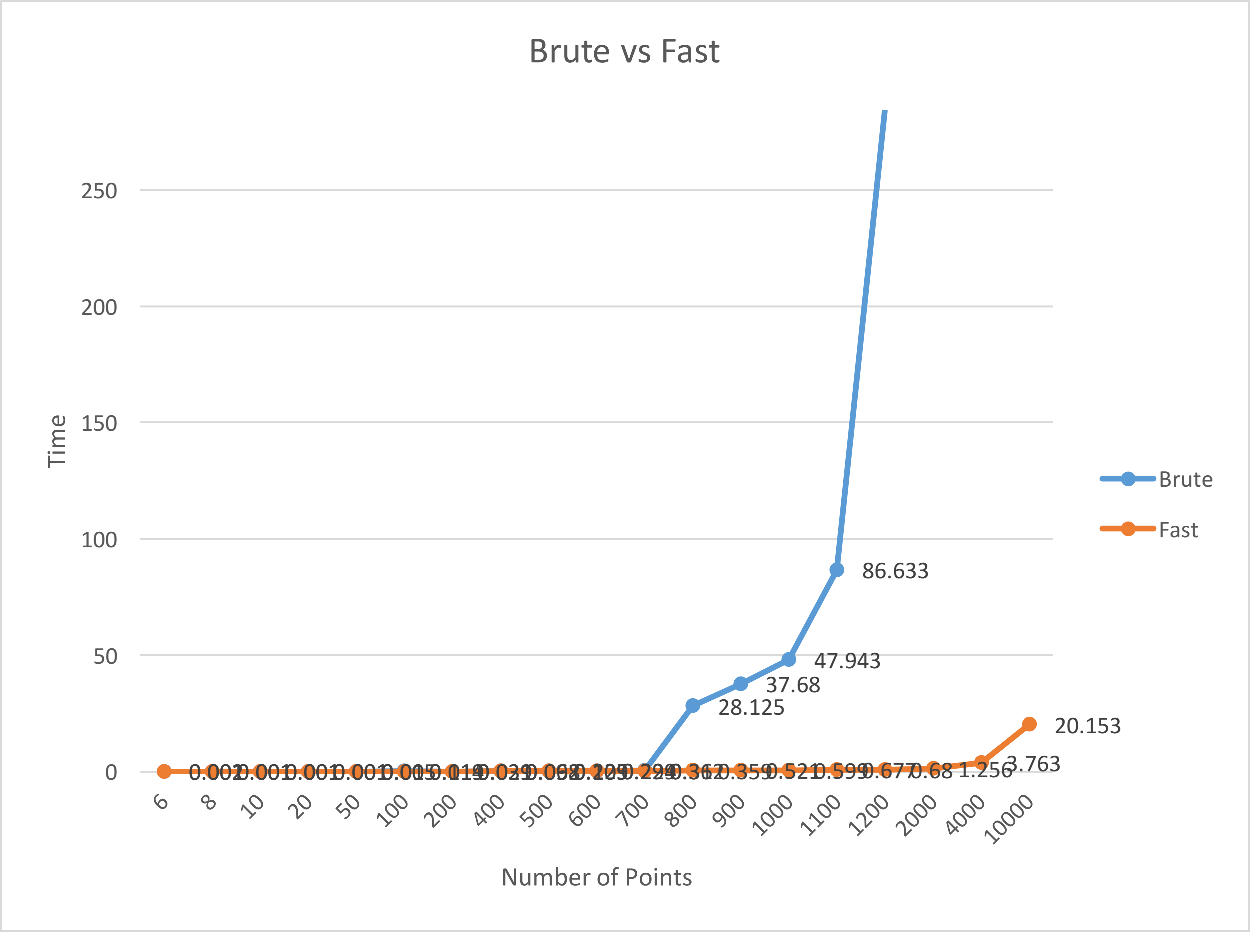
   The running time of Brute(N) is N^4. Analysis: Because my implementation of Brute force using a quadruple nested for loops to exam every possible set of 4-tuples by comparing their slopes, thus the running time is N^4.

   The running time of Fast(N) is Nlog(N) + N*Nlog(N)+N^2Nlog(N) = N^3log(N)(worst case). Analysis: The outer loop which contains all points and sort lexicographically(points[n], and inner loop first resort a second array(copy[n]) by slope order in terms of points[i]. Then it steps through the copy array to search for points that made the same slope with points[i], and if we find 3 or 4 points that fits the condition, it store them into a result array and sort it by x,y(lexicographically). Finally, it checks the result array with points[i] to ensure that this line has not been stored before.

2. **Show, using empirical evidence, the execution time behavior of both versions (brute VS fast). Provide at least a plot and table where you compare the running time observed for different values of N (for example, 10, 20, 50, 100, 200, 400, 1000, 2000, 4000, 10000, etc). You are allowed to test different values of N as long as they don't exceed around 200 seconds of running time. The table should contain three columns: N, brute-force time and fast time. The plot should have the x-axis be the N value and the y-axis will represent running times, where you will show two lines (one for each version).**

   Answer:

N	Brute	Fast
6	0.002	0.001
8	0.001	0.001
10	0.001	0.001
20	0.001	0.001
50	0.01	0.005
100	0.114	0.019
200	0.039	0.021
400	0.092	0.068
500	0.225	0.109
600	0.224	0.299
700	0.362	0.31
800	28.125	0.359
900	37.68	0.521
1000	47.943	0.599
1100	86.633	0.677
1200	284.154	0.68
2000	DNF	1.256
4000	DNF	3.763
10000	DNF	20.153



3. **Estimate how long it would take to solve an instance of size N = 1,000,000 for each of the two algorithms using your computer.**
   Answer: (Using n = 1200 as sample)
   Brute:
   $1.37*10^{-10} * (N^4) = 1.37*10^{-10} * 1000000^4 = 1.37 * 10^{14}$ seconds= 4344241 Years
   Fast:
   $6.66*10^{-8} * (N^2\ln(N) = 920158$ seconds = 10 days