

# CS412 Homework 3 – Spring 2024/25

## Question 1: Gradient Descent (50 points)

We wish to minimize the function

$$F(x, y) = x^2 + 4x + y^2 - 4y,$$

with respect to both  $x$  and  $y$ . Starting from the initial point

$$(x_0, y_0) = (23, 27),$$

apply the steepest descent algorithm for **two steps** using a step size  $\eta = 0.1$ :

$$(x, y) \leftarrow (x, y) - \eta \nabla F(x, y).$$

(Subscripts denote iterations, e.g.,  $(x_1, y_1)$ ,  $(x_2, y_2)$ .)

### Worksheet

#### 1. Initial Function Value (5 pts):

$$F(23, 27) = \underline{23^2 + 4 \cdot 23 + 27^2 - 4 \cdot 27 = 529 + 92 + 729 - 108 = 1242}$$

#### 2. Gradient Computation (10 pts):

$$\nabla F(x, y) = \begin{bmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{bmatrix} \rightarrow \begin{bmatrix} 2x+4 \\ 2y-4 \end{bmatrix} \rightarrow (2x+4, 2y-4)$$

#### 3. Gradient at (23, 27) (10 pts):

$$\nabla F(23, 27) = \underline{\begin{bmatrix} 2 \cdot 23 + 4 \\ 2 \cdot 27 - 4 \end{bmatrix} = \begin{bmatrix} 50 \\ 50 \end{bmatrix} \rightarrow (50, 50)}$$

4. Update to Find  $(x_1, y_1)$  (5 pts):

$$(x_1, y_1) = (23, 27) - 0.1 \nabla F(23, 27) = \underline{(23, 27)} - 0.1 \begin{bmatrix} 50 \\ 50 \end{bmatrix} = (18, 22)$$

5. Function Value at  $(x_1, y_1)$  (5 pts):

$$F(x_1, y_1) = \frac{18^2 + 4 \cdot 18 + 22^2 - 4 \cdot 22 = 324 + 72 + 484 - 88}{= 792}$$

6. Second Step (15 pts):

$$(x_2, y_2) = (x_1, y_1) - 0.1 \nabla F(x_1, y_1) = \underline{(18, 22)} - 0.1 [40, 40] = (14, 18)$$

$$\text{ANSWER: } (x_2, y_2) = \underline{(14, 18)}, \quad F(x_2, y_2) = \frac{14^2 + 4 \cdot 14 + 18^2 - 4 \cdot 18}{= 196 + 56 + 324 - 72} = \underline{504}$$

$$\nabla F(18, 22) = \begin{bmatrix} 2 \cdot 18 + 4 \\ 2 \cdot 22 - 4 \end{bmatrix} = \begin{bmatrix} 40 \\ 40 \end{bmatrix}$$

## Question 2: Naïve Bayes Classification (50 points)

Based on the PlayTennis dataset provided on the next page, solve the classification task for the instance:

$$\mathbf{x} = [\text{Overcast}, \text{Mild}, \text{Normal}, \text{Strong}]$$

using the Naïve Bayes algorithm. Show all your probabilities and calculations explicitly and answer under the specified conditions:

a) (20 pts) Without Smoothing:

$$1) P(\text{Yes}) = \frac{9}{14} \quad P(\text{No}) = \frac{5}{14}$$

$$2) \text{Outlook} = \text{Overcast} \text{ and } \text{PlayTennis} = \text{Yes} : \frac{4}{9}$$

$$\text{Temperature} = \text{Mild} \text{ and } \text{PlayTennis} = \text{Yes} : P(\text{Mild} | \text{Yes}) = \frac{4}{9}$$

$$\text{Humidity} = \text{Normal} \text{ and } \text{PlayTennis} = \text{Yes} : P(\text{Normal} | \text{Yes}) = \frac{6}{9}$$

$$\text{Wind} = \text{Strong} \text{ and } \text{PlayTennis} = \text{Yes} : P(\text{Strong} | \text{Yes}) = \frac{3}{9}$$

$$P(x|\text{Yes}) = \frac{4}{9} \cdot \frac{4}{9} \cdot \frac{6}{9} \cdot \frac{3}{9} = \frac{288}{6561}$$

b) (20 pts) With Laplace (Add-1) Smoothing:

$$1) \text{Prior Probabilities: } P(\text{Yes}) = \frac{9}{14}, \quad P(\text{No}) = \frac{5}{14}$$

2) Number of unique values ( $k$ ) for each feature:

Outlook: Sunny, Overcast, Rain  $\rightarrow k=3$

Temperature: Hot, Mild, Cool  $\rightarrow k=3$

Humidity: High, Normal  $\rightarrow k=2$

Wind: Weak, Strong  $\rightarrow k=2$

$$3) P(\text{Overcast} | \text{Yes}) = \frac{4+1}{9+3} = \frac{5}{12}, \quad P(\text{Normal} | \text{Yes}) = \frac{6+1}{9+2} = \frac{7}{11}$$

$$P(\text{Mild} | \text{Yes}) = \frac{4+1}{9+3} = \frac{5}{12}, \quad P(\text{Strong} | \text{Yes}) = \frac{3+1}{9+2} = \frac{4}{11}$$

c) (10 pts) Normalized Probabilities:

Based on your calculations in part (b) find the normalized probabilities  $P(\text{Yes} | \mathbf{x})$  and  $P(\text{No} | \mathbf{x})$ .

1) Denominator for normalization

$$\text{Total} = P(\text{Yes} | \mathbf{x}) + P(\text{No} | \mathbf{x}) = 0.02583 + 0.00273 \\ = 0.02856$$

3) Based on the normalized probabilities:

PlayTennis = Yes with probability 90.45%

2) Normalized Probabilities

$$P(\text{Yes} | \mathbf{x}) = \frac{0.02583}{0.02856} \approx 0.9045$$

$$P(\text{No} | \mathbf{x}) = \frac{0.00273}{0.02856} \approx 0.0955$$

## PlayTennis Dataset

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

## Submission Instructions

- Submission:** Submit your solutions as a single PDF file by 03.04.2025 23:55. Please work within this document by filling in the provided empty lines and using the allocated space.
- File Naming:** Name your submission as CS412-HW3-NameSurname.pdf
- Late Submissions:** Late submissions will be accepted with a penalty of 10 points per day.