CFRM 421/521

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Homework 4

- **Due: Tuesday, May 27, 2025, 11:59 PM**
- Total marks: 43
- Late submissions are allowed, but a 20% penalty per day applies. Your last submission is considered for calculating the penalty.
- Use this Jupyter notebook as a template for your solutions. **Your solution must be submitted as both one Jupyter notebook and one PDF file on Gradescope.** There will be two modules on Gradescope, one for each file type. The notebook must be already run, that is, make sure that you have run all the code, save the notebook, and then when you reopen the notebook, checked that all output appears as expected. You are allowed to use code from the textbook, textbook website, or lecture notes.

1. A regression MLP [12 marks]

Consider the original source of the California housing data (used in Homework 2) in Scikit-Learn. The data is obtained and split using the code below, where we split off 20% as the test set, and then split off 20% of the training set as a validation set, and keep the remaining 80% of the training set as the actual training set. The following code creates the training set `X_train`, `y_train`, the validation set `X_valid`, `y_valid` and the test set `X_test`, `y_test`.

In [46]:
```
import numpy as np
import pandas as pd
import tensorflow as tf
import keras_tuner as kt
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_california_housing
from sklearn.model_selection import train_test_split

housing = fetch_california_housing()
X = housing.data
y = housing.target

X_train_tmp, X_test, y_train_tmp, y_test = train_test_split(X, y, test_size=0.2, random_state=42)
X_train, X_valid, y_train, y_valid = train_test_split(X_train_tmp, y_train_tmp, test_size=0.2, random_state=42)
```

(a) [4 marks]

Use `tensorflow.keras` to train a regression MLP with a normalization layer as the first layer (`tf.keras.layers.Normalization(input_shape=X_train.shape[1:])`), and one hidden layer of 50 ReLU neurons. For the output layer, try both a ReLU activation function and no activation function (which is equivalent to the identity function). Explain which choice is better. Use the appropriate weight initialization. Use the Nadam optimizer. Train for 30 epochs, and report the mean squared error on the validation set. In the `.compile()` method, use `loss="mse"`.

In [47]:
```
model = tf.keras.models.Sequential([
    tf.keras.layers.InputLayer(shape=X_train.shape[1:]),
    tf.keras.layers.Normalization(),
    tf.keras.layers.Dense(50, activation="relu", kernel_initializer="he_normal"),
    tf.keras.layers.Dense(1, kernel_initializer="glorot_normal")
])
model.compile(loss='mse', optimizer='nadam')
history = model.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid))
```

```
Epoch 1/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 510us/step - loss: 20406.3281 - val_loss: 56.4079
Epoch 2/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 344us/step - loss: 13.7450 - val_loss: 26.3230
Epoch 3/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 346us/step - loss: 4.2473 - val_loss: 13.1006
Epoch 4/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 365us/step - loss: 2.5797 - val_loss: 6.1220
Epoch 5/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 345us/step - loss: 1.8588 - val_loss: 3.2989
Epoch 6/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 343us/step - loss: 1.4930 - val_loss: 2.5257
Epoch 7/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 351us/step - loss: 1.3062 - val_loss: 2.4276
Epoch 8/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 342us/step - loss: 1.2024 - val_loss: 2.6477
Epoch 9/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 341us/step - loss: 1.1383 - val_loss: 3.1632
Epoch 10/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 338us/step - loss: 1.1060 - val_loss: 3.6771
Epoch 11/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 347us/step - loss: 1.0989 - val_loss: 3.4939
Epoch 12/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 345us/step - loss: 1.1278 - val_loss: 2.6006
Epoch 13/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 353us/step - loss: 1.2875 - val_loss: 2.2067
Epoch 14/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 336us/step - loss: 1.6373 - val_loss: 2.2537
Epoch 15/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 343us/step - loss: 2.1648 - val_loss: 2.3177
Epoch 16/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 342us/step - loss: 2.6374 - val_loss: 2.1554
Epoch 17/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 340us/step - loss: 2.4089 - val_loss: 2.0476
Epoch 18/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 334us/step - loss: 2.3160 - val_loss: 1.9540
Epoch 19/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 331us/step - loss: 2.3658 - val_loss: 1.7539
Epoch 20/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 326us/step - loss: 2.3632 - val_loss: 1.4239
Epoch 21/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 361us/step - loss: 2.3623 - val_loss: 1.9364
Epoch 22/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 378us/step - loss: 2.3975 - val_loss: 4.0520
Epoch 23/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 329us/step - loss: 2.2219 - val_loss: 6.0693
Epoch 24/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 329us/step - loss: 2.4592 - val_loss: 3.5110
Epoch 25/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 330us/step - loss: 2.2295 - val_loss: 2.0622
Epoch 26/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 330us/step - loss: 2.3418 - val_loss: 1.9142
Epoch 27/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 330us/step - loss: 2.2790 - val_loss: 1.2953
Epoch 28/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 330us/step - loss: 2.3250 - val_loss: 1.6342
Epoch 29/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 329us/step - loss: 2.1695 - val_loss: 1.9472
Epoch 30/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 340us/step - loss: 2.2584 - val_loss: 2.5151
```

In [48]:

```
model_relu = tf.keras.models.Sequential([
    tf.keras.layers.InputLayer(shape=X_train.shape[1:]),
    tf.keras.layers.Normalization(),
    tf.keras.layers.Dense(50, activation="relu", kernel_initializer="he_normal"),
    tf.keras.layers.Dense(1, activation="relu", kernel_initializer="he_normal")
])
model_relu.compile(loss='mse', optimizer='nadam')
history_relu = model_relu.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid))
```

```
Epoch 1/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 2/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 346us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 3/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 340us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 4/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 339us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 5/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 339us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 6/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 351us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 7/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 333us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 8/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 343us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 9/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 350us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 10/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 346us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 11/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 342us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 12/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 343us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 13/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 346us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 14/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 340us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 15/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 341us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 16/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 345us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 17/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 354us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 18/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 338us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 19/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 340us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 20/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 341us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 21/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 343us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 22/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 334us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 23/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 342us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 24/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 341us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 25/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 342us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 26/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 351us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 27/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 346us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 28/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 341us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 29/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 357us/step - loss: 5.5688 - val_loss: 5.7567
Epoch 30/30
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 356us/step - loss: 5.5688 - val_loss: 5.7567
```

In [49]:
```python
pd.DataFrame(history.history).iloc[-1, :]
```

Out[49]:
```
loss        6.662468
val_loss    2.515069
Name: 29, dtype: float64
```

In [50]:
```python
pd.DataFrame(history_relu.history).iloc[-1, :]
```

Out[50]:
```
loss        5.597989
val_loss    5.756706
Name: 29, dtype: float64
```

The validation loss for ReLU is 5.75 while for the identity function is 2.51. The identity function activation produces a 2x smaller validation loss. However, using the ReLU activation function results in much closer validation loss to the training loss. The identity activation is better sine the validation loss is much lower.

(b) [6 marks]

Read the section "Fine-Tuning Neural Network Hyperparameters" in the textbook and the corresponding section in the Jupyter notebook on the textbook website using Keras Tuner. You will need to install the package `keras_tuner` if you don't already have it.

Then use Keras Tuner to do a randomized search to search for the best hyperparameters. Do the randomized search over the first 5000 observations of the training set. Use 20 iterations, 20 epochs per iteration. Use the same network architecture as (a) except where otherwise specified below. Use no activation function for the output layer. Use a seed of 42, and the objective is clearly to minimize validation loss. The hyperparameters to search over are:

- Hidden layers: 1 to 5.
- Number of neurons per layer: 1 to 100.
- Learning rate: 1e-4 to 1e-2 using log sampling.
- ℓ_2 regularizers with `l2` value: 1e-4 to 100 using log sampling.
- Optimizer: `tf.keras.optimizers.SGD(learning_rate=learning_rate,clipnorm=1.0)` and `tf.keras.optimizers.Nadam(learning_rate=learning_rate)`.

Print the best hyperparameter. (You can ignore any warning message you may get).

In [6]:
```
def build_model(hp: kt.HyperParameters):
    n_hidden = hp.Int("n_hidden", min_value=1, max_value=5)
    n_neurons = hp.Int("n_neurons", min_value=1, max_value=100)
    learning_rate = hp.Float("learning_rate", min_value=1e-4, max_value=1e-2,
        sampling="log")
    l2 = hp.Float("l2", min_value=1e-4, max_value=100, sampling="log")
    optimizer = hp.Choice("optimizer", values=["sgd", "nadam"])
    if optimizer == "sgd":
        optimizer = tf.keras.optimizers.SGD(
            learning_rate=learning_rate,
            clipnorm=1.0
        )
    else:
        optimizer = tf.keras.optimizers.Nadam(
            learning_rate=learning_rate
        )
    model = tf.keras.Sequential()
    model.add(tf.keras.layers.Flatten())
    for _ in range(n_hidden):
        model.add(tf.keras.layers.Dense(
            n_neurons,
            activation="relu",
            kernel_regularizer=tf.keras.regularizers.l2(l2)))
    model.add(tf.keras.layers.Dense(1, kernel_initializer="glorot_normal"))
    model.compile(loss="mse", optimizer=optimizer,)
    return model
```

In [7]:
```
random_search_tuner = kt.RandomSearch(
    build_model, objective="val_loss", max_trials=20, overwrite=True,
    directory="my_fashion_mnist", project_name="my_rnd_search", seed=42)
random_search_tuner.search(X_train[:5000], y_train[:5000], epochs=20,
    validation_data=(X_valid, y_valid))
```

```
Trial 20 Complete [00h 00m 02s]
val_loss: 47.08333969116211

Best val_loss So Far: 0.7223991751670837
Total elapsed time: 00h 00m 46s
```

best model and params are below

In [51]:
```
top_model = random_search_tuner.get_best_models(num_models=3)
best_model = top_model[0]

top_params = random_search_tuner.get_best_hyperparameters(num_trials=3)
top_params[0].values
```

```
/Users/erevtsov/dev/cfrm/.venv/lib/python3.12/site-packages/keras/src/saving/saving_lib.py:757: UserWarning: Skipping variable loadin
g for optimizer 'nadam', because it has 2 variables whereas the saved optimizer has 27 variables.
  saveable.load_own_variables(weights_store.get(inner_path))
/Users/erevtsov/dev/cfrm/.venv/lib/python3.12/site-packages/keras/src/saving/saving_lib.py:757: UserWarning: Skipping variable loadin
g for optimizer 'nadam', because it has 2 variables whereas the saved optimizer has 11 variables.
  saveable.load_own_variables(weights_store.get(inner_path))
```

Out[51]:
```
{'n_hidden': 5,
 'n_neurons': 62,
 'learning_rate': 0.006718710759425462,
 'l2': 0.0003483686981793893,
 'optimizer': 'nadam'}
```

(c) [2 marks]

For the best model in (b), train the model on the full training data for 200 epochs. Plot the learning curve. Does it look like the model is overfitting?

In [9]:
```
history_best = best_model.fit(X_train, y_train, epochs=200, validation_data=(X_valid, y_valid))
```

```
Epoch 1/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 1s 670us/step - loss: 19.5976 - val_loss: 0.8240
Epoch 2/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 458us/step - loss: 0.7437 - val_loss: 0.7497
Epoch 3/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 461us/step - loss: 0.6942 - val_loss: 0.6400
Epoch 4/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 458us/step - loss: 0.6485 - val_loss: 0.6205
Epoch 5/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 455us/step - loss: 0.6490 - val_loss: 0.6008
Epoch 6/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 0.6496 - val_loss: 0.7716
Epoch 7/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.1948 - val_loss: 0.6451
Epoch 8/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 531us/step - loss: 0.6555 - val_loss: 0.6094
Epoch 9/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 630us/step - loss: 0.7499 - val_loss: 1.4235
Epoch 10/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 630us/step - loss: 1.3470 - val_loss: 1.4212
Epoch 11/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 648us/step - loss: 1.3166 - val_loss: 0.7169
Epoch 12/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 558us/step - loss: 0.7065 - val_loss: 0.6855
Epoch 13/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 484us/step - loss: 0.7374 - val_loss: 1.4259
Epoch 14/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3625 - val_loss: 1.3276
Epoch 15/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 539us/step - loss: 0.8789 - val_loss: 1.8999
Epoch 16/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 532us/step - loss: 1.4314 - val_loss: 1.4234
Epoch 17/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 493us/step - loss: 1.3638 - val_loss: 1.4182
Epoch 18/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 489us/step - loss: 1.3822 - val_loss: 1.4169
Epoch 19/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 631us/step - loss: 1.3533 - val_loss: 1.4211
Epoch 20/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 592us/step - loss: 1.3618 - val_loss: 1.4083
Epoch 21/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 531us/step - loss: 1.3199 - val_loss: 1.4061
Epoch 22/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 535us/step - loss: 1.3686 - val_loss: 1.4150
Epoch 23/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 517us/step - loss: 1.3403 - val_loss: 1.4267
Epoch 24/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 518us/step - loss: 1.3715 - val_loss: 1.4500
Epoch 25/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 500us/step - loss: 1.3660 - val_loss: 1.4006
Epoch 26/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 485us/step - loss: 1.3392 - val_loss: 1.4048
Epoch 27/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3520 - val_loss: 1.3967
Epoch 28/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3429 - val_loss: 1.3908
Epoch 29/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3527 - val_loss: 1.3890
Epoch 30/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 510us/step - loss: 1.3055 - val_loss: 1.3884
Epoch 31/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3331 - val_loss: 1.3856
Epoch 32/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - loss: 1.3522 - val_loss: 1.3843
Epoch 33/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3294 - val_loss: 1.3949
Epoch 34/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - loss: 1.3293 - val_loss: 1.3853
Epoch 35/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3339 - val_loss: 1.3824
Epoch 36/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 504us/step - loss: 1.3509 - val_loss: 1.3829
Epoch 37/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 493us/step - loss: 1.3219 - val_loss: 1.3821
Epoch 38/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3396 - val_loss: 1.3911
Epoch 39/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.3335 - val_loss: 1.3808
Epoch 40/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3614 - val_loss: 1.3818
Epoch 41/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3006 - val_loss: 1.3821
Epoch 42/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - loss: 1.2953 - val_loss: 1.3835
Epoch 43/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.3161 - val_loss: 1.3823
Epoch 44/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.2906 - val_loss: 1.3808
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Epoch 45/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - loss: 1.3380 - val_loss: 1.3854
Epoch 46/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 476us/step - loss: 1.3464 - val_loss: 1.3803
Epoch 47/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3425 - val_loss: 1.3810
Epoch 48/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.3396 - val_loss: 1.3802
Epoch 49/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3264 - val_loss: 1.3812
Epoch 50/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - loss: 1.3203 - val_loss: 1.3801
Epoch 51/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 493us/step - loss: 1.3132 - val_loss: 1.3800
Epoch 52/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 476us/step - loss: 1.3198 - val_loss: 1.3800
Epoch 53/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3188 - val_loss: 1.4018
Epoch 54/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3261 - val_loss: 1.3802
Epoch 55/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3228 - val_loss: 1.3821
Epoch 56/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3444 - val_loss: 1.3830
Epoch 57/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 478us/step - loss: 1.3164 - val_loss: 1.3801
Epoch 58/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - loss: 1.3360 - val_loss: 1.3919
Epoch 59/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 477us/step - loss: 1.3599 - val_loss: 1.3871
Epoch 60/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.3252 - val_loss: 1.3805
Epoch 61/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3405 - val_loss: 1.3800
Epoch 62/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3407 - val_loss: 1.3807
Epoch 63/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.3338 - val_loss: 1.3801
Epoch 64/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3367 - val_loss: 1.3801
Epoch 65/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 483us/step - loss: 1.3232 - val_loss: 1.3855
Epoch 66/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 501us/step - loss: 1.3355 - val_loss: 1.3811
Epoch 67/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 491us/step - loss: 1.3315 - val_loss: 1.3835
Epoch 68/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 463us/step - loss: 1.3570 - val_loss: 1.3835
Epoch 69/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.3253 - val_loss: 1.3857
Epoch 70/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 461us/step - loss: 1.3416 - val_loss: 1.3849
Epoch 71/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 463us/step - loss: 1.3279 - val_loss: 1.3832
Epoch 72/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.3508 - val_loss: 1.3809
Epoch 73/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 481us/step - loss: 1.3689 - val_loss: 1.3800
Epoch 74/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - loss: 1.3593 - val_loss: 1.3820
Epoch 75/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 461us/step - loss: 1.3537 - val_loss: 1.3828
Epoch 76/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - loss: 1.3386 - val_loss: 1.3801
Epoch 77/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 509us/step - loss: 1.3473 - val_loss: 1.3822
Epoch 78/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - loss: 1.3232 - val_loss: 1.3816
Epoch 79/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 481us/step - loss: 1.3472 - val_loss: 1.3806
Epoch 80/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 484us/step - loss: 1.3759 - val_loss: 1.3805
Epoch 81/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 494us/step - loss: 1.3260 - val_loss: 1.3817
Epoch 82/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 476us/step - loss: 1.3330 - val_loss: 1.3820
Epoch 83/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - loss: 1.3177 - val_loss: 1.3851
Epoch 84/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3306 - val_loss: 1.3800
Epoch 85/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 457us/step - loss: 1.3220 - val_loss: 1.3836
Epoch 86/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3718 - val_loss: 1.3816
Epoch 87/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 460us/step - loss: 1.2829 - val_loss: 1.3803
Epoch 88/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 479us/step - loss: 1.3507 - val_loss: 1.3825
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Epoch 89/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3243 - val_loss: 1.3846
Epoch 90/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3008 - val_loss: 1.3802
Epoch 91/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 463us/step - loss: 1.3493 - val_loss: 1.3800
Epoch 92/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.2875 - val_loss: 1.3800
Epoch 93/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.3273 - val_loss: 1.3801
Epoch 94/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - loss: 1.3532 - val_loss: 1.3822
Epoch 95/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 482us/step - loss: 1.3282 - val_loss: 1.3813
Epoch 96/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 498us/step - loss: 1.3278 - val_loss: 1.3800
Epoch 97/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3071 - val_loss: 1.3800
Epoch 98/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.3150 - val_loss: 1.3805
Epoch 99/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 478us/step - loss: 1.3140 - val_loss: 1.3803
Epoch 100/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 487us/step - loss: 1.3475 - val_loss: 1.3835
Epoch 101/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.3313 - val_loss: 1.3833
Epoch 102/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3439 - val_loss: 1.3815
Epoch 103/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 477us/step - loss: 1.3508 - val_loss: 1.3830
Epoch 104/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.3261 - val_loss: 1.3800
Epoch 105/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3310 - val_loss: 1.3801
Epoch 106/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3303 - val_loss: 1.3820
Epoch 107/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 483us/step - loss: 1.3437 - val_loss: 1.3828
Epoch 108/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3302 - val_loss: 1.3808
Epoch 109/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - loss: 1.3197 - val_loss: 1.3807
Epoch 110/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 489us/step - loss: 1.3133 - val_loss: 1.3800
Epoch 111/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 477us/step - loss: 1.3161 - val_loss: 1.3801
Epoch 112/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3349 - val_loss: 1.3800
Epoch 113/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - loss: 1.3392 - val_loss: 1.3815
Epoch 114/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 483us/step - loss: 1.3577 - val_loss: 1.3802
Epoch 115/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 510us/step - loss: 1.3397 - val_loss: 1.3811
Epoch 116/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.3164 - val_loss: 1.3807
Epoch 117/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 482us/step - loss: 1.3297 - val_loss: 1.3802
Epoch 118/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3260 - val_loss: 1.3806
Epoch 119/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 484us/step - loss: 1.3260 - val_loss: 1.3803
Epoch 120/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 485us/step - loss: 1.3098 - val_loss: 1.3824
Epoch 121/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3549 - val_loss: 1.3861
Epoch 122/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3264 - val_loss: 1.3801
Epoch 123/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 460us/step - loss: 1.3260 - val_loss: 1.3807
Epoch 124/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 463us/step - loss: 1.3370 - val_loss: 1.3828
Epoch 125/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 511us/step - loss: 1.2781 - val_loss: 1.3804
Epoch 126/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3386 - val_loss: 1.3823
Epoch 127/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 486us/step - loss: 1.2941 - val_loss: 1.3800
Epoch 128/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.3615 - val_loss: 1.3809
Epoch 129/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3369 - val_loss: 1.3804
Epoch 130/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3577 - val_loss: 1.3815
Epoch 131/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - loss: 1.3406 - val_loss: 1.3810
Epoch 132/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3677 - val_loss: 1.3810
```
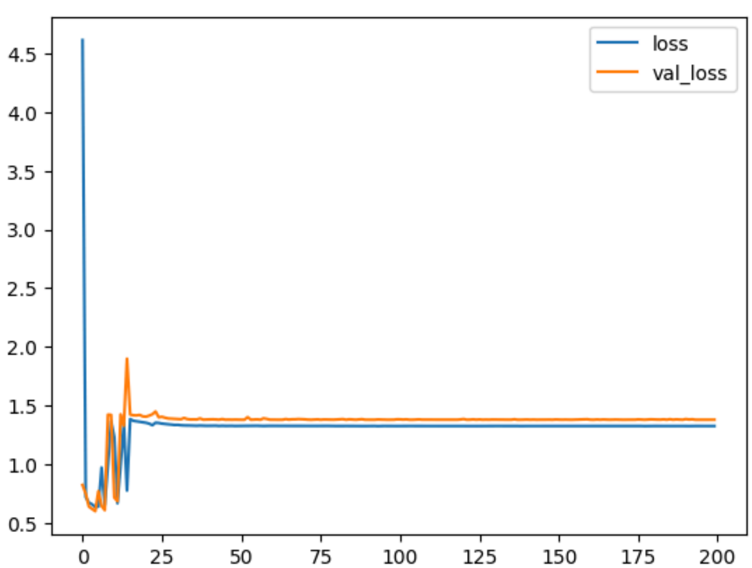

```
Epoch 133/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.2986 - val_loss: 1.3800
Epoch 134/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 483us/step - loss: 1.3262 - val_loss: 1.3802
Epoch 135/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3084 - val_loss: 1.3804
Epoch 136/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - loss: 1.2931 - val_loss: 1.3802
Epoch 137/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - loss: 1.3224 - val_loss: 1.3838
Epoch 138/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3004 - val_loss: 1.3800
Epoch 139/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - loss: 1.3368 - val_loss: 1.3800
Epoch 140/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 490us/step - loss: 1.3110 - val_loss: 1.3815
Epoch 141/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 479us/step - loss: 1.3449 - val_loss: 1.3817
Epoch 142/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3025 - val_loss: 1.3800
Epoch 143/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - loss: 1.3248 - val_loss: 1.3812
Epoch 144/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3395 - val_loss: 1.3801
Epoch 145/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.3140 - val_loss: 1.3804
Epoch 146/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - loss: 1.3548 - val_loss: 1.3802
Epoch 147/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3749 - val_loss: 1.3806
Epoch 148/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 524us/step - loss: 1.3201 - val_loss: 1.3808
Epoch 149/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.2937 - val_loss: 1.3800
Epoch 150/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.2780 - val_loss: 1.3819
Epoch 151/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - loss: 1.3349 - val_loss: 1.3805
Epoch 152/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3119 - val_loss: 1.3802
Epoch 153/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 461us/step - loss: 1.3209 - val_loss: 1.3809
Epoch 154/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3381 - val_loss: 1.3802
Epoch 155/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 514us/step - loss: 1.3038 - val_loss: 1.3800
Epoch 156/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.3464 - val_loss: 1.3806
Epoch 157/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3128 - val_loss: 1.3800
Epoch 158/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3371 - val_loss: 1.3826
Epoch 159/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3162 - val_loss: 1.3824
Epoch 160/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.3421 - val_loss: 1.3841
Epoch 161/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3015 - val_loss: 1.3803
Epoch 162/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3186 - val_loss: 1.3800
Epoch 163/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 481us/step - loss: 1.3484 - val_loss: 1.3822
Epoch 164/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - loss: 1.3266 - val_loss: 1.3803
Epoch 165/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - loss: 1.2940 - val_loss: 1.3821
Epoch 166/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - loss: 1.3414 - val_loss: 1.3808
Epoch 167/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 460us/step - loss: 1.3372 - val_loss: 1.3801
Epoch 168/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - loss: 1.3025 - val_loss: 1.3810
Epoch 169/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 476us/step - loss: 1.3021 - val_loss: 1.3813
Epoch 170/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 491us/step - loss: 1.3052 - val_loss: 1.3801
Epoch 171/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - loss: 1.3034 - val_loss: 1.3800
Epoch 172/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.2973 - val_loss: 1.3800
Epoch 173/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - loss: 1.3455 - val_loss: 1.3800
Epoch 174/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3142 - val_loss: 1.3800
Epoch 175/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 476us/step - loss: 1.3167 - val_loss: 1.3805
Epoch 176/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 476us/step - loss: 1.3239 - val_loss: 1.3828
```

```
Epoch 177/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 481us/step - loss: 1.3395 - val_loss: 1.3815
Epoch 178/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 510us/step - loss: 1.3099 - val_loss: 1.3800
Epoch 179/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3388 - val_loss: 1.3818
Epoch 180/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 479us/step - loss: 1.3520 - val_loss: 1.3833
Epoch 181/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.3177 - val_loss: 1.3818
Epoch 182/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - loss: 1.3245 - val_loss: 1.3814
Epoch 183/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 542us/step - loss: 1.3114 - val_loss: 1.3809
Epoch 184/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 547us/step - loss: 1.3346 - val_loss: 1.3829
Epoch 185/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 511us/step - loss: 1.2907 - val_loss: 1.3803
Epoch 186/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3182 - val_loss: 1.3845
Epoch 187/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3442 - val_loss: 1.3803
Epoch 188/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - loss: 1.3174 - val_loss: 1.3829
Epoch 189/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - loss: 1.3069 - val_loss: 1.3811
Epoch 190/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 515us/step - loss: 1.3243 - val_loss: 1.3800
Epoch 191/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 505us/step - loss: 1.3362 - val_loss: 1.3857
Epoch 192/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - loss: 1.3382 - val_loss: 1.3820
Epoch 193/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 463us/step - loss: 1.3583 - val_loss: 1.3839
Epoch 194/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - loss: 1.3099 - val_loss: 1.3800
Epoch 195/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 476us/step - loss: 1.2900 - val_loss: 1.3801
Epoch 196/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - loss: 1.3090 - val_loss: 1.3800
Epoch 197/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - loss: 1.3269 - val_loss: 1.3811
Epoch 198/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 483us/step - loss: 1.3209 - val_loss: 1.3802
Epoch 199/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 494us/step - loss: 1.3410 - val_loss: 1.3814
Epoch 200/200
413/413 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - loss: 1.3422 - val_loss: 1.3802
```

In [10]:
```
pd.DataFrame(history_best.history).plot()
```

Out[10]:
```
<Axes: >
```



The training and validation loss values are very close together, so the model is not overfitting.

2. Binary classification DNN [17 marks]

Consider the Portuguese Bank Marketing Data Set available at Kaggle. Download the `bank_cleaned.csv` file or from Canvas. Here we want to predict the success or failure of a bank marketing campaign using phone calls to promote a term deposit product. The target variable is `response_binary`.

The following code preprocesses the data. The day and month have been converted into cyclical features(1st day of the month has equal distance to the 2nd and the 31st).

In [118…
```
df = pd.read_csv("datasets/bank_cleaned.csv")

month_dict = {"jan": 1, "feb": 2, "mar": 3, "apr": 4, "may": 5, "jun": 6,
              "jul": 7, "aug": 8, "sep": 9, "oct": 10, "nov": 11, "dec": 12}
day_rad = (df["day"] - 1) * (2 * np.pi / 31)
month_rad = (df["month"].replace(month_dict) - 1) * (2 * np.pi / 12)
df["day_sin"] = np.sin(day_rad)
df["day_cos"] = np.cos(day_rad)
df["month_sin"] = np.sin(month_rad)
df["month_cos"]  = np.cos(month_rad)
df.drop(columns=["Unnamed: 0", "month", "day", "response"], axis=1, inplace=True)
df.head()
```

```
/var/folders/_r/fqfrhk7s0wv6d3lj1cgbhcnc0000gn/T/ipykernel_39975/3076060008.py:6: FutureWarning: Downcasting behavior in `replace` is deprecated and will be removed in a future version. To retain the old behavior, explicitly call `result.infer_objects(copy=False)`. To opt-in to the future behavior, set `pd.set_option('future.no_silent_downcasting', True)`
  month_rad = (df["month"].replace(month_dict) - 1) * (2 * np.pi / 12)
```

Out[118…

	age	job	marital	education	default	balance	housing	loan	duration	campaign	pdays	previous	poutcome	response_binary	day_sin
0	58	management	married	tertiary	no	2143	yes	no	4.35	1	-1	0	unknown	0	0.724793
1	44	technician	single	secondary	no	29	yes	no	2.52	1	-1	0	unknown	0	0.724793
2	33	entrepreneur	married	secondary	no	2	yes	yes	1.27	1	-1	0	unknown	0	0.724793
3	35	management	married	tertiary	no	231	yes	no	2.32	1	-1	0	unknown	0	0.724793
4	28	management	single	tertiary	no	447	yes	yes	3.62	1	-1	0	unknown	0	0.724793

In [119…
```
from tensorflow.keras.layers import BatchNormalization, Dropout
from tensorflow.keras.regularizers import l2
from tensorflow.keras.callbacks import EarlyStopping
```

In [120…
```
from sklearn.preprocessing import StandardScaler, OrdinalEncoder, OneHotEncoder
from sklearn.compose import ColumnTransformer

train_set_tmp, test_set = train_test_split(df, test_size=0.2, random_state=42)
train_set, valid_set = train_test_split(train_set_tmp, test_size=0.2, random_state=42)

X_train_raw = train_set.drop("response_binary", axis=1).copy()
y_train = train_set["response_binary"].copy()
X_valid_raw = valid_set.drop("response_binary", axis=1).copy()
y_valid = valid_set["response_binary"].copy()
X_test_raw = test_set.drop("response_binary", axis=1).copy()
y_test = test_set["response_binary"].copy()

num_attribs = list(X_train_raw._get_numeric_data().columns)
cat_attribs = list(set(X_train_raw.columns) - set(num_attribs))

cat_attribs_ord = ['default', 'housing', 'loan']
cat_attribs_hot = ['job', 'marital', 'education', 'poutcome']

full_pipeline = ColumnTransformer([
        ("num", StandardScaler(), num_attribs),
        ("cat_hot", OneHotEncoder(), cat_attribs_hot),
        ("cat_ord", OrdinalEncoder(categories=[['no','yes'],['no','yes'],['no','yes']]), cat_attribs_ord)
    ])

X_train = full_pipeline.fit_transform(X_train_raw)
X_valid = full_pipeline.transform(X_valid_raw)
X_test = full_pipeline.transform(X_test_raw)
```

(a) [4 marks]

In the next part you will build and fit a DNN with 4 hidden layers of 100 neurons each. Use the following specifications:

(i) He initialization and the Swish activation function.

(ii) The output layer has 1 neuron with sigmoid activation.

(iii) Compile with `loss="binary_crossentropy"` and `metrics=["AUC"]`.

Explain why the choices (i), (ii), and (iii) are justified.

Also, state the proportion of sucesses in the training data.

In [121…
```
dnn_model = tf.keras.Sequential([
    tf.keras.layers.InputLayer(shape=X_train.shape[1:]),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
```

```python
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    tf.keras.layers.Dense(1, activation='sigmoid')
])

# Proportion of successes in the training data:
proportion_successes = np.mean(y_train)
print(proportion_successes)
```

```
0.11168075907717029
```

Justification for (i): He initialization is a good choice because it is a variance-preserving initialization method, which is better than Xavier initialization for neurons with a large number of inputs. Swish activation is used because it is a more recent and better performing alternative to ReLU.

Justification for (ii): The output layer has 1 neuron with sigmoid activation because the task is a binary classification task.

Justification for (iii): The loss is binary cross-entropy because it is the most common and suitable loss function for binary classification tasks. AUC is a suitable metric because it is a measure of the model's ability to distinguish between positive and negative classes.

Proportion of success is ~0.11

(b) [3 marks]

Train the model in (a) for 30 epochs and use exponential scheduling using the function below (`lr0=0.01` , `s=20`) and the NAG optimizer with `momentum=0.9` . Use a learning curve to comment on whether it is overfitting.

At the start of fitting your model, run `reset_session()` given by the following code.

In [122…

```python
def reset_session(seed=42):
    tf.random.set_seed(seed)
    np.random.seed(seed)
    tf.keras.backend.clear_session()

def exponential_decay(lr0, s):
    return lambda epoch: lr0 * 0.1**(epoch / s)
```

In [123…

```python
# I was having issues with the given exponential decay function
# using this instead, from the documentation it's doing the same thing.
exp_decay = tf.keras.optimizers.schedules.ExponentialDecay(
    initial_learning_rate=0.01,
    decay_steps=20,
    decay_rate=0.1,
    staircase=False,
    name='ExponentialDecay'
)

optimizer = tf.keras.optimizers.SGD(
    momentum=0.9,
    nesterov=True,
    learning_rate=exp_decay)

reset_session()
dnn_model.compile(
    loss='binary_crossentropy',
    metrics=['AUC'],
    optimizer=optimizer)

history = dnn_model.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid))
```

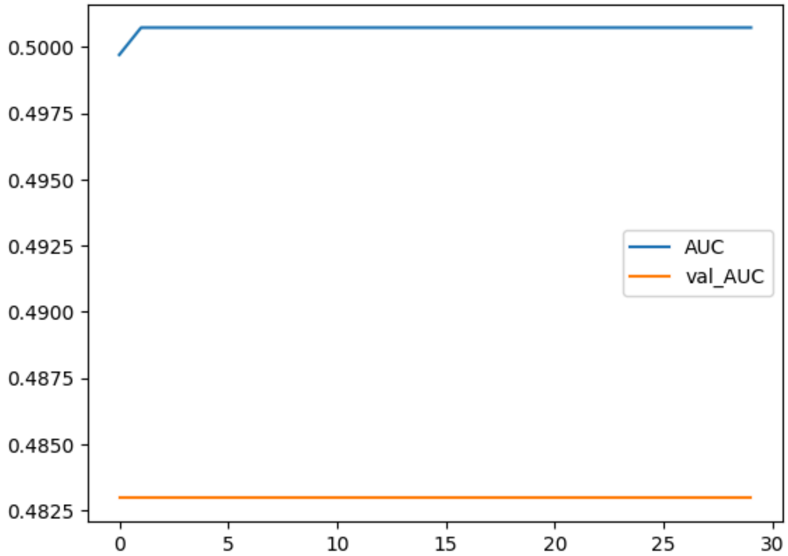
```
Epoch 1/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 616us/step - AUC: 0.5075 - loss: 0.4028 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 2/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 489us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 3/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 486us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 4/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 500us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 5/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 6/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 7/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 479us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 8/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 9/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 10/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 11/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 483us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 12/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 468us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 13/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 14/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 15/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 16/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 17/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 18/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 19/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 20/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 516us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 21/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 482us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 22/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 454us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 23/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 457us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 24/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 457us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 25/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 456us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 26/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 480us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 27/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 28/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 29/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
Epoch 30/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - AUC: 0.5101 - loss: 0.3838 - val_AUC: 0.4830 - val_loss: 0.3919
```

In [124…

```
res = pd.DataFrame(history.history)
res[['AUC', 'val_AUC']].plot()
```

Out[124…

```
<Axes: >
```



The training and vlaue AUC are quite tight, I don't see much overfitting. Re-running the model a few times yields similar results.

(c) [8 marks]

Fit separate models using the same specification as in (b) but with the following regularization techniques:

(i) batch normalization,

(ii) early stopping based on validation AUC with `patience=10` (look at the documentation and note the `mode` argument).

(iii) ℓ_2 regularization with `l2=0.0002`,

(iv) dropout with probability 0.02,

(v) ℓ_2 regularization and early stopping both as above,

(vi) batch normalization and dropout both as above.

At the start of each one of the above models, run `reset_session()`.

The performance measure is validation AUC. State this for the model in (b), and for each of the models here comment on whether it is better than the model in (b).

In [125…

```
# Model with Batch Normalization
reset_session()

optimizer = tf.keras.optimizers.SGD(
    momentum=0.9,
    nesterov=True,
    learning_rate=exp_decay)

dnn_model_bn = tf.keras.Sequential([
    tf.keras.layers.InputLayer(shape=X_train.shape[1:]),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    BatchNormalization(),
    tf.keras.layers.Dense(1, activation='sigmoid')
])


dnn_model_bn.compile(loss='binary_crossentropy', metrics=['AUC'], optimizer=optimizer)
history_bn = dnn_model_bn.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid))
```

```
Epoch 1/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 857us/step - AUC: 0.7234 - loss: 0.5040 - val_AUC: 0.7239 - val_loss: 0.4815
Epoch 2/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 705us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 3/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 602us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 4/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 628us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 5/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 616us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 6/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 586us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 7/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 607us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 8/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 600us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 9/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 591us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 10/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 632us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 11/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 631us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 12/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 596us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 13/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 636us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 14/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 585us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 15/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 587us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 16/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 584us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 17/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 584us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 18/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 592us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 19/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 605us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 20/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 590us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 21/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 595us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 22/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 628us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 23/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 665us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 24/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 692us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 25/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 698us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 26/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 661us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 27/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 617us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 28/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 666us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 29/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 620us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
Epoch 30/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 613us/step - AUC: 0.7475 - loss: 0.4650 - val_AUC: 0.7238 - val_loss: 0.4820
```

In [126…

```
# Model with Early Stopping
reset_session()
early_stopping = EarlyStopping(monitor='val_AUC', patience=10, mode='max', restore_best_weights=True)
dnn_model_es = tf.keras.models.clone_model(dnn_model)
history_es = dnn_model_es.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid), callbacks=[early_stopping])
```

```
Epoch 1/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 571us/step - AUC: 0.5901 - loss: 0.3673 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 2/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 3/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 4/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 5/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 457us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 6/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 7/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 493us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 8/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 9/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 461us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 10/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 458us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
Epoch 11/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 462us/step - AUC: 0.6139 - loss: 0.3492 - val_AUC: 0.6088 - val_loss: 0.3478
```

In [127…

```python
# Model with L2 Regularization
reset_session()

optimizer = tf.keras.optimizers.SGD(
    momentum=0.9,
    nesterov=True,
    learning_rate=exp_decay)

dnn_model_l2 = tf.keras.Sequential([
    tf.keras.layers.InputLayer(shape=X_train.shape[1:]),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal', kernel_regularizer=l2(0.0002)),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal', kernel_regularizer=l2(0.0002)),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal', kernel_regularizer=l2(0.0002)),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal', kernel_regularizer=l2(0.0002)),
    tf.keras.layers.Dense(1, activation='sigmoid')
])
dnn_model_l2.compile(loss='binary_crossentropy', metrics=['AUC'], optimizer=optimizer)
history_l2 = dnn_model_l2.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid))
```

```
Epoch 1/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 601us/step - AUC: 0.4904 - loss: 0.5578 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 2/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 493us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 3/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 4/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 5/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 6/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 473us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 7/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 8/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 9/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 490us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 10/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 475us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 11/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 12/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 13/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 495us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 14/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 15/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 16/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 17/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 491us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 18/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 471us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 19/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 20/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 469us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 21/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 22/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 466us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 23/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 470us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 24/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 474us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 25/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 26/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 464us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 27/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 463us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 28/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 465us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 29/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 472us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
Epoch 30/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 0s 467us/step - AUC: 0.5077 - loss: 0.5386 - val_AUC: 0.4989 - val_loss: 0.5381
```

In [128…

```python
# Model with Dropout
reset_session()

optimizer = tf.keras.optimizers.SGD(
    momentum=0.9,
    nesterov=True,
    learning_rate=exp_decay)
dnn_model_dropout = tf.keras.Sequential([
    tf.keras.layers.InputLayer(shape=X_train.shape[1:]),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    tf.keras.layers.Dense(1, activation='sigmoid')
])
dnn_model_dropout.compile(loss='binary_crossentropy', metrics=['AUC'], optimizer=optimizer)
history_dropout = dnn_model_dropout.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid))
```

```
Epoch 1/30
817/817 1s 645us/step - AUC: 0.5483 - loss: 0.4007 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 2/30
817/817 0s 509us/step - AUC: 0.5628 - loss: 0.3802 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 3/30
817/817 0s 513us/step - AUC: 0.5688 - loss: 0.3793 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 4/30
817/817 0s 516us/step - AUC: 0.5603 - loss: 0.3801 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 5/30
817/817 0s 511us/step - AUC: 0.5638 - loss: 0.3799 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 6/30
817/817 0s 547us/step - AUC: 0.5615 - loss: 0.3809 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 7/30
817/817 0s 564us/step - AUC: 0.5652 - loss: 0.3800 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 8/30
817/817 0s 544us/step - AUC: 0.5603 - loss: 0.3807 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 9/30
817/817 0s 524us/step - AUC: 0.5596 - loss: 0.3806 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 10/30
817/817 0s 546us/step - AUC: 0.5627 - loss: 0.3805 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 11/30
817/817 0s 514us/step - AUC: 0.5630 - loss: 0.3803 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 12/30
817/817 0s 520us/step - AUC: 0.5590 - loss: 0.3807 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 13/30
817/817 0s 517us/step - AUC: 0.5662 - loss: 0.3794 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 14/30
817/817 0s 534us/step - AUC: 0.5625 - loss: 0.3805 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 15/30
817/817 0s 514us/step - AUC: 0.5617 - loss: 0.3804 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 16/30
817/817 0s 515us/step - AUC: 0.5598 - loss: 0.3811 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 17/30
817/817 0s 516us/step - AUC: 0.5656 - loss: 0.3799 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 18/30
817/817 0s 601us/step - AUC: 0.5598 - loss: 0.3811 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 19/30
817/817 0s 532us/step - AUC: 0.5636 - loss: 0.3797 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 20/30
817/817 0s 513us/step - AUC: 0.5620 - loss: 0.3804 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 21/30
817/817 0s 562us/step - AUC: 0.5610 - loss: 0.3800 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 22/30
817/817 0s 534us/step - AUC: 0.5645 - loss: 0.3801 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 23/30
817/817 0s 511us/step - AUC: 0.5655 - loss: 0.3799 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 24/30
817/817 0s 543us/step - AUC: 0.5624 - loss: 0.3804 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 25/30
817/817 0s 560us/step - AUC: 0.5641 - loss: 0.3802 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 26/30
817/817 0s 553us/step - AUC: 0.5624 - loss: 0.3799 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 27/30
817/817 1s 605us/step - AUC: 0.5659 - loss: 0.3797 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 28/30
817/817 0s 530us/step - AUC: 0.5644 - loss: 0.3802 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 29/30
817/817 0s 525us/step - AUC: 0.5646 - loss: 0.3799 - val_AUC: 0.5519 - val_loss: 0.3839
Epoch 30/30
817/817 0s 559us/step - AUC: 0.5620 - loss: 0.3801 - val_AUC: 0.5519 - val_loss: 0.3839
```

In [129…

```python
# Model with L2 Regularization and Early Stopping
reset_session()
dnn_model_l2_es = tf.keras.models.clone_model(dnn_model_l2)
history_l2_es = dnn_model_l2_es.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid), callbacks=[early_stopping])
```

```
Epoch 1/30
817/817 1s 616us/step - AUC: 0.5322 - loss: 0.5418 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 2/30
817/817 0s 512us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 3/30
817/817 0s 529us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 4/30
817/817 0s 484us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 5/30
817/817 0s 482us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 6/30
817/817 0s 482us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 7/30
817/817 0s 525us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 8/30
817/817 0s 543us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 9/30
817/817 0s 557us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 10/30
817/817 0s 596us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
Epoch 11/30
817/817 0s 534us/step - AUC: 0.5499 - loss: 0.5214 - val_AUC: 0.5319 - val_loss: 0.5254
```

In [130…

```
# Model with Batch Normalization and Dropout
reset_session()

optimizer = tf.keras.optimizers.SGD(
    momentum=0.9,
    nesterov=True,
    learning_rate=exp_decay)

dnn_model_bn_dropout = tf.keras.Sequential([
    tf.keras.layers.InputLayer(shape=X_train.shape[1:]),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    BatchNormalization(),
    tf.keras.layers.Dense(100, activation='swish', kernel_initializer='he_normal'),
    Dropout(0.02),
    BatchNormalization(),
    tf.keras.layers.Dense(1, activation='sigmoid')
])
dnn_model_bn_dropout.compile(loss='binary_crossentropy', metrics=['AUC'], optimizer=optimizer)
history_bn_dropout = dnn_model_bn_dropout.fit(X_train, y_train, epochs=30, validation_data=(X_valid, y_valid))

# You can now compare validation AUCs for model (b) and the models above
```

```
Epoch 1/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 802us/step - AUC: 0.7218 - loss: 0.5076 - val_AUC: 0.7633 - val_loss: 0.4825
Epoch 2/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 697us/step - AUC: 0.7445 - loss: 0.4763 - val_AUC: 0.7626 - val_loss: 0.4857
Epoch 3/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 646us/step - AUC: 0.7444 - loss: 0.4745 - val_AUC: 0.7631 - val_loss: 0.4830
Epoch 4/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 629us/step - AUC: 0.7474 - loss: 0.4740 - val_AUC: 0.7629 - val_loss: 0.4841
Epoch 5/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 643us/step - AUC: 0.7488 - loss: 0.4744 - val_AUC: 0.7622 - val_loss: 0.4818
Epoch 6/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 778us/step - AUC: 0.7452 - loss: 0.4743 - val_AUC: 0.7623 - val_loss: 0.4824
Epoch 7/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 690us/step - AUC: 0.7482 - loss: 0.4741 - val_AUC: 0.7627 - val_loss: 0.4830
Epoch 8/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 664us/step - AUC: 0.7446 - loss: 0.4756 - val_AUC: 0.7628 - val_loss: 0.4800
Epoch 9/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 661us/step - AUC: 0.7532 - loss: 0.4730 - val_AUC: 0.7624 - val_loss: 0.4809
Epoch 10/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 627us/step - AUC: 0.7497 - loss: 0.4736 - val_AUC: 0.7634 - val_loss: 0.4833
Epoch 11/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 632us/step - AUC: 0.7487 - loss: 0.4745 - val_AUC: 0.7624 - val_loss: 0.4818
Epoch 12/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 648us/step - AUC: 0.7509 - loss: 0.4727 - val_AUC: 0.7635 - val_loss: 0.4859
Epoch 13/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 648us/step - AUC: 0.7475 - loss: 0.4758 - val_AUC: 0.7623 - val_loss: 0.4821
Epoch 14/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 664us/step - AUC: 0.7493 - loss: 0.4749 - val_AUC: 0.7619 - val_loss: 0.4837
Epoch 15/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 638us/step - AUC: 0.7477 - loss: 0.4745 - val_AUC: 0.7624 - val_loss: 0.4830
Epoch 16/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 699us/step - AUC: 0.7431 - loss: 0.4757 - val_AUC: 0.7629 - val_loss: 0.4832
Epoch 17/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 623us/step - AUC: 0.7527 - loss: 0.4723 - val_AUC: 0.7622 - val_loss: 0.4820
Epoch 18/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 626us/step - AUC: 0.7538 - loss: 0.4707 - val_AUC: 0.7631 - val_loss: 0.4836
Epoch 19/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 643us/step - AUC: 0.7463 - loss: 0.4744 - val_AUC: 0.7635 - val_loss: 0.4816
Epoch 20/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 630us/step - AUC: 0.7538 - loss: 0.4723 - val_AUC: 0.7627 - val_loss: 0.4837
Epoch 21/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 625us/step - AUC: 0.7478 - loss: 0.4737 - val_AUC: 0.7628 - val_loss: 0.4819
Epoch 22/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 650us/step - AUC: 0.7531 - loss: 0.4725 - val_AUC: 0.7626 - val_loss: 0.4845
Epoch 23/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 749us/step - AUC: 0.7476 - loss: 0.4736 - val_AUC: 0.7622 - val_loss: 0.4842
Epoch 24/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 723us/step - AUC: 0.7448 - loss: 0.4770 - val_AUC: 0.7627 - val_loss: 0.4829
Epoch 25/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 715us/step - AUC: 0.7527 - loss: 0.4734 - val_AUC: 0.7625 - val_loss: 0.4837
Epoch 26/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 766us/step - AUC: 0.7497 - loss: 0.4733 - val_AUC: 0.7627 - val_loss: 0.4853
Epoch 27/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 698us/step - AUC: 0.7468 - loss: 0.4742 - val_AUC: 0.7623 - val_loss: 0.4804
Epoch 28/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 700us/step - AUC: 0.7537 - loss: 0.4717 - val_AUC: 0.7622 - val_loss: 0.4853
Epoch 29/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 646us/step - AUC: 0.7464 - loss: 0.4741 - val_AUC: 0.7631 - val_loss: 0.4831
Epoch 30/30
817/817 ━━━━━━━━━━━━━━━━━━━━ 1s 707us/step - AUC: 0.7481 - loss: 0.4726 - val_AUC: 0.7628 - val_loss: 0.4855
```

In [131…

```
res_all = pd.Series({
    'SQ': history.history['val_AUC'][-1],
    'Batch': history_bn.history['val_AUC'][-1],
    'Early Stop': history_es.history['val_AUC'][-1],
    'Dropout': history_dropout.history['val_AUC'][-1],
    'L2': history_l2.history['val_AUC'][-1],
    'L2 + Early Stop': history_l2_es.history['val_AUC'][-1],
    'Batch + Dropout': history_bn_dropout.history['val_AUC'][-1],
})
res_all
```

Out[131…

```
SQ                 0.482959
Batch              0.723819
Early Stop         0.608782
Dropout            0.551862
L2                 0.498869
L2 + Early Stop    0.531873
Batch + Dropout    0.762798
dtype: float64
```

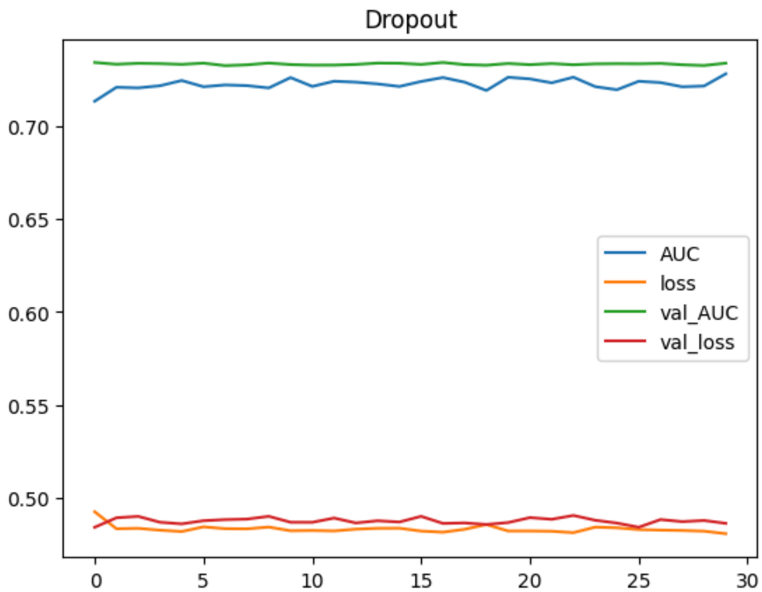
The model in (b) is only bested by L2, and L2 benefits from early stopping, showing that it converges earlier than the 30 epochs.

(d) [1 mark]

For the dropout model in (c)(iv) determine whether or not it is overfitting less than the model in (b).

In [89]:
```
pd.DataFrame(history_bn_dropout.history).plot(title='Dropout')
```

Out[89]: <Axes: title={'center': 'Dropout'}>



The training and validation AUC for for the model with Dropout are pretty tight, just like they were for the original model in (b).

(e) [1 mark]

Of the models in (b) and (c), one would now choose the best model according to the performance metric (validation AUC) to evaluate on the test set. But instead, evaluate the model in (c)(v) on the test set in terms of the AUC and confusion matrix (regardless of whether it is the best model given your results).

In [26]:
```
from sklearn import metrics

y_pred_test_l2_es = history_l2_es.model.predict(X_test)
y_pred_test_l2_es_class = np.where(y_pred_test_l2_es > 0.5, 1, 0)
test_auc_l2_es = metrics.roc_auc_score(y_test, y_pred_test_l2_es)
test_confusion_l2_es = metrics.confusion_matrix(y_test, y_pred_test_l2_es_class)

print(f"Test AUC: {test_auc_l2_es:.4f}")
print(f"Test Confusion Matrix:\n{test_confusion_l2_es}")

# interpret results
print(f"Test Accuracy: {(test_confusion_l2_es[0][0] + test_confusion_l2_es[1][1]) / np.sum(test_confusion_l2_es):.4f}")
print(f"Test Precision: {test_confusion_l2_es[1][1] / (test_confusion_l2_es[1][1] + test_confusion_l2_es[0][1]):.4f}")
print(f"Test Recall: {test_confusion_l2_es[1][1] / (test_confusion_l2_es[1][1] + test_confusion_l2_es[1][0]):.4f}")
print(f"Test F1: {metrics.f1_score(y_test, y_pred_test_l2_es_class):.4f}")
```

```
256/256 ━━━━━━━━━━━━━━━━━━━━ 0s 284us/step
Test AUC: 0.5589
Test Confusion Matrix:
[[7166    0]
 [1003    0]]
Test Accuracy: 0.8772
Test Precision: nan
Test Recall: 0.0000
Test F1: 0.0000
/var/folders/_r/fqfrhk7s0wv6d3lj1cgbhcnc0000gn/T/ipykernel_39975/2014987866.py:13: RuntimeWarning: invalid value encountered in scala
r divide
  print(f"Test Precision: {test_confusion_l2_es[1][1] / (test_confusion_l2_es[1][1] + test_confusion_l2_es[0][1]):.4f}")
```

3. Time series using machine learning [14 marks]

Obtain daily values of the Japan/U.S. Foreign Exchange Rate (DEXJPUS) starting from Jan 1, 1990, to Jan 1, 2023, from FRED. This can be obtained using the code below or you can download the data as a csv file from Canvas.

In [255…
```
import pandas as pd
import pandas_datareader as pdr
from datetime import datetime
data = pdr.get_data_fred('DEXJPUS', datetime(1990,1,1),datetime(2023,1,1))
```

(a) [2 marks]

Create a training set (before 2010), a validation set (Jan 2010 to Dec 2015), and a test set (the rest of the data). Turn the time series data into a supervised learning dataset where the features are the value of the exchange rate in the last 10 days inclusive of the current day, and the target is the value of the exchange rate in the next day.

In [256…
```
def create_dataset(data, input_len, target_len):
    data_pivot = pd.DataFrame()
    x_cols = []
    for i in range(input_len):
        data_pivot[f'X_{i}d'] = data['DEXJPUS'].shift(i)
        x_cols.append(f'X_{i}d')

    y_cols = []
    for i in range(1, target_len+1):
        data_pivot[f'y_{i}d'] = data['DEXJPUS'].shift(-i)
        y_cols.append(f'y_{i}d')
    data_pivot = data_pivot.ffill()
    data_pivot = data_pivot.iloc[input_len+1:]
    return data_pivot

def split_data(data, input_len, target_len):
    x_cols = [f'X_{i}d' for i in range(input_len)]
    y_cols = [f'y_{i}d' for i in range(1, target_len+1)]
    X_train = data.loc[:'2009-12-31', x_cols].values
    y_train = data.loc[:'2009-12-31', y_cols].squeeze().values
    X_valid = data.loc['2010-01-01':'2015-12-31', x_cols].values
    y_valid = data.loc['2010-01-01':'2015-12-31', y_cols].squeeze().values
    X_test = data.loc['2016-01-01':, x_cols].values
    y_test = data.loc['2016-01-01':, y_cols].squeeze().values
    return X_train, y_train, X_valid, y_valid, X_test, y_test
```

In [257…
```
# forward-filling the data (missing prices are on holidays so we can assume that the price is the same as T-1)
# converting the prices to returns to create an easier dataset for the model to work with
data_clean = data.ffill().pct_change().dropna().mul(100)

data_pivot = create_dataset(data_clean, input_len=10, target_len=1)
X_train, y_train, X_valid, y_valid, X_test, y_test = split_data(data_pivot, input_len=10, target_len=1)
```

(b) [3 marks]

Fit a random forest regressor to predict the value of the exchange rate in the next day. Using the test set, report the mean squared error and the accuracy for the movement direction.

Hint: You can calculate the accuracy of the movement direction by determining what the actual movement direction is and comparing it to the movement direction corresponding to the predicted value of the exchange rate. For instance, the movement direction of the test set `X_test` and `y_test` where a strictly up movement is `True` can be computed as follows.

NOTE:

- Since I am using return time series and not prices, the formula for direction is slightly different. I just need to determine the sign of the value and compare to test set.
- Trying to predict the price is not a fruitful task. You can just take the mean of the trailing 10-day price and it will appear to be a "good" fit.

In [258…
```
from sklearn.ensemble import RandomForestRegressor
from sklearn.metrics import mean_squared_error, accuracy_score

rf_reg = RandomForestRegressor(random_state=42, oob_score=True, n_jobs=-1)
rf_reg.fit(X_train, y_train)

y_pred_rf = rf_reg.predict(X_test)

mse = mean_squared_error(y_test, y_pred_rf)
print(f"Mean squared error: {mse:.8f}")

accuracy = accuracy_score(
    y_true=(np.sign(y_test) ==  np.sign(X_test[:, 0])), y_pred=(np.sign(y_pred_rf)  == np.sign(X_test[:, 0]))
)
print(f"Accuracy for the movement direction: {accuracy:.4f}")
```

```
Mean squared error: 0.32407133
Accuracy for the movement direction: 0.5411
```

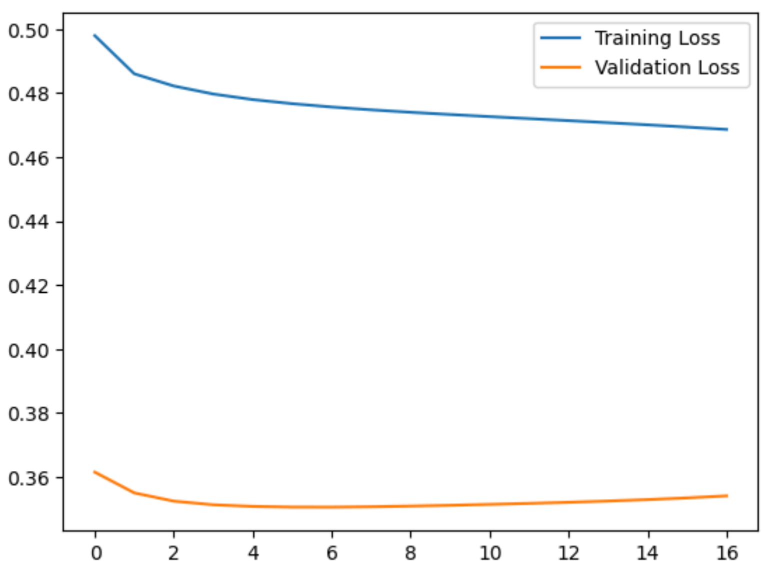
(c) [4 marks]

Repeat (b), but now fit a deep RNN with 2 recurrent layers of 20 and 20 neurons, and an output layer which is 1 dense neuron. Use 100 epochs and the Nadam optimizer. Comment on the result and the learning curve (the validation set is used for the learning curve).

In [260…
```
# (c)
reset_session()

model_ts = tf.keras.Sequential([
    tf.keras.layers.SimpleRNN(20, return_sequences=True, input_shape=[None, 1]),
    tf.keras.layers.SimpleRNN(20),
    tf.keras.layers.Dense(1)
```

```
])

model_ts.compile(loss='mse', optimizer=tf.keras.optimizers.Nadam(learning_rate=0.001))
history_ts = model_ts.fit(
    X_train[..., np.newaxis],
    y_train,
    epochs=100,
    validation_data=(X_valid[..., np.newaxis], y_valid),
    callbacks=[tf.keras.callbacks.EarlyStopping(patience=10)])

y_pred_rnn = model_ts.predict(X_test[..., np.newaxis])

plt.plot(history_ts.history['loss'], label='Training Loss')
plt.plot(history_ts.history['val_loss'], label='Validation Loss')
plt.legend()
plt.show()
```

```
Epoch 1/100
/Users/erevtsov/dev/cfrm/.venv/lib/python3.12/site-packages/keras/src/layers/rnn/rnn.py:200: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the model instead.
  super().__init__(**kwargs)
163/163 ━━━━━━━━━━━━━━━━━━━━ 1s 2ms/step - loss: 0.4952 - val_loss: 0.3615
Epoch 2/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4700 - val_loss: 0.3550
Epoch 3/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4648 - val_loss: 0.3524
Epoch 4/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4615 - val_loss: 0.3513
Epoch 5/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4592 - val_loss: 0.3508
Epoch 6/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4575 - val_loss: 0.3506
Epoch 7/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4561 - val_loss: 0.3505
Epoch 8/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4550 - val_loss: 0.3507
Epoch 9/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4540 - val_loss: 0.3509
Epoch 10/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4532 - val_loss: 0.3511
Epoch 11/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4524 - val_loss: 0.3514
Epoch 12/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4517 - val_loss: 0.3517
Epoch 13/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4510 - val_loss: 0.3521
Epoch 14/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4502 - val_loss: 0.3524
Epoch 15/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4495 - val_loss: 0.3529
Epoch 16/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4487 - val_loss: 0.3534
Epoch 17/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4479 - val_loss: 0.3541
58/58 ━━━━━━━━━━━━━━━━━━━━ 0s 2ms/step
```

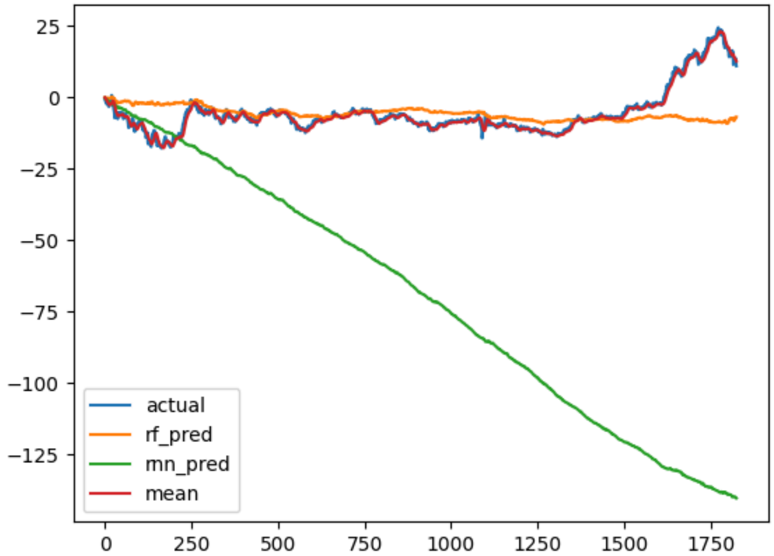


In [269…

```
pd.DataFrame({'actual': y_test, 'rf_pred': y_pred_rf, 'rnn_pred': y_pred_rnn.ravel(), 'mean': X_test.mean(axis=1)}).cumsum().plot()
```

Out[269…

```
<Axes: >
```



The prediction is pretty terrible. the validation loss doesn't seem too bad, but that's because we are predicting small values (even after mulpiplying by 100 to convert to %). You can see on the cumulative plot that you are better off using a simple mean of the trailing returns than using random forest or RNN prediction.

(d) [5 marks]

Create a supervised learning dataset suitable for predicting 3 days ahead instead of 1 day ahead. Adjust the deep RNN in (c) so that it predicts 3 days ahead. Use 100 epochs and the Nadam optimizer. Using the test set, report the mean squared error and the accuracy for the movement direction for each of the 3 days ahead predictions. Comment on the result and the learning curve.

In [270…

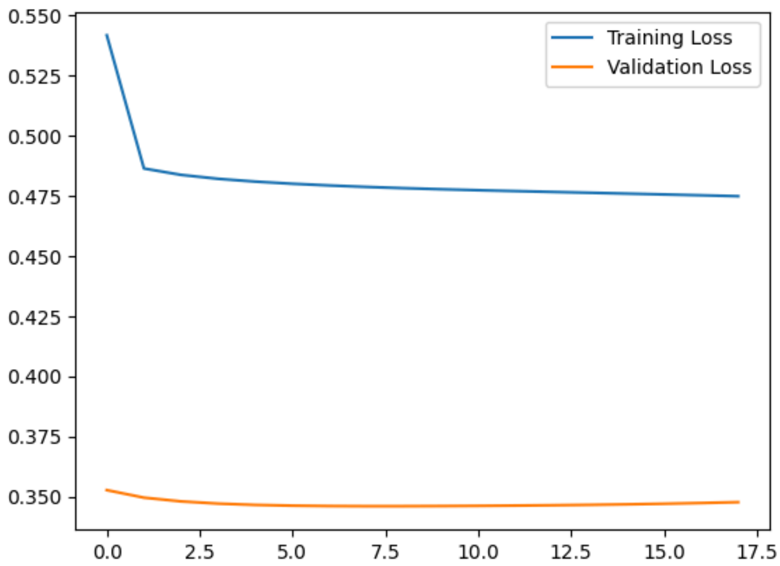
```
data_pivot_3d = create_dataset(data_clean, input_len=10, target_len=3)
X_train_3d, y_train_3d, X_valid_3d, y_valid_3d, X_test_3d, y_test_3d = split_data(data_pivot_3d, input_len=10, target_len=3)
```

In [271…

```
# (c)
reset_session()

model_3d = tf.keras.Sequential([
    tf.keras.layers.SimpleRNN(20, return_sequences=True, input_shape=[None, 1]),
    tf.keras.layers.SimpleRNN(20),
    tf.keras.layers.Dense(3)
])

model_3d.compile(loss='mse', optimizer=tf.keras.optimizers.Nadam(learning_rate=0.001))
history_3d = model_3d.fit(
    X_train_3d[..., np.newaxis],
    y_train_3d[..., np.newaxis],
    epochs=100,
    validation_data=(X_valid_3d[..., np.newaxis], y_valid_3d[..., np.newaxis]),
    callbacks=[tf.keras.callbacks.EarlyStopping(patience=10, restore_best_weights=True)])

plt.plot(history_3d.history['loss'], label='Training Loss')
plt.plot(history_3d.history['val_loss'], label='Validation Loss')
plt.legend()
plt.show()
```

```
Epoch 1/100
/Users/erevtsov/dev/cfrm/.venv/lib/python3.12/site-packages/keras/src/layers/rnn/rnn.py:200: UserWarning: Do not pass an `input_shape`/`input_dim` argument to a layer. When using Sequential models, prefer using an `Input(shape)` object as the first layer in the mode
l instead.
  super().__init__(**kwargs)
```

```
163/163 ━━━━━━━━━━━━━━━━━━━━ 1s 2ms/step - loss: 0.6307 - val_loss: 0.3527
Epoch 2/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4925 - val_loss: 0.3496
Epoch 3/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4893 - val_loss: 0.3480
Epoch 4/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4874 - val_loss: 0.3471
Epoch 5/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4860 - val_loss: 0.3465
Epoch 6/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4850 - val_loss: 0.3462
Epoch 7/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4842 - val_loss: 0.3461
Epoch 8/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4836 - val_loss: 0.3460
Epoch 9/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4831 - val_loss: 0.3460
Epoch 10/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4826 - val_loss: 0.3461
Epoch 11/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4822 - val_loss: 0.3462
Epoch 12/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4819 - val_loss: 0.3463
Epoch 13/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4815 - val_loss: 0.3464
Epoch 14/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4812 - val_loss: 0.3466
Epoch 15/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4809 - val_loss: 0.3468
Epoch 16/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4806 - val_loss: 0.3470
Epoch 17/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4802 - val_loss: 0.3473
Epoch 18/100
163/163 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step - loss: 0.4799 - val_loss: 0.3477
```



In [272…

```
y_pred_3d = model_3d.predict(X_test_3d[..., np.newaxis])
```

```
58/58 ━━━━━━━━━━━━━━━━━━━━ 0s 1ms/step
```

In [273…

```
accuracy = accuracy_score(
    y_true=(np.sign(y_test_3d[:, 0]) ==  np.sign(X_test_3d[:, 0])), y_pred=(np.sign(y_pred_3d[:, 0])  == np.sign(X_test_3d[:, 0]))
)
print(f"Accuracy for the movement direction day 1: {accuracy:.4f}")

accuracy = accuracy_score(
    y_true=(np.sign(y_test_3d[:, 1]) ==  np.sign(X_test_3d[:, 1])), y_pred=(np.sign(y_pred_3d[:, 1])  == np.sign(X_test_3d[:, 1]))
)
print(f"Accuracy for the movement direction day 2: {accuracy:.4f}")

accuracy = accuracy_score(
    y_true=(np.sign(y_test_3d[:, 2]) ==  np.sign(X_test_3d[:, 2])), y_pred=(np.sign(y_pred_3d[:, 2])  == np.sign(X_test_3d[:, 2]))
)
print(f"Accuracy for the movement direction day 3: {accuracy:.4f}")
```

```
Accuracy for the movement direction day 1: 0.5071
Accuracy for the movement direction day 2: 0.5279
Accuracy for the movement direction day 3: 0.5011
```

Similar to the one-observation case, the validation loss plot looks OK. but the accuracy for movement direction is more or less a toss-up.