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روابط بازگشتی

a. $T(n) = \underbrace{3}_{a\geq 1} T(n/\underbrace{3}_{b>1}) + \underbrace{\sqrt{n}}_{f(n)}$

$\exists\, \varepsilon > 0 \quad f(n) \in O(n^{\log_b^a - \varepsilon}) \Rightarrow T(n) = \theta(n^{\log_b^a})$

$\exists\, 0.1 > 0 \quad \sqrt{n} \in O(n^{\cancel{1-0.1}\, 0.9}) \Rightarrow \boxed{T(n) = \theta(n)}$

b. $T(n) = \underbrace{3}_{a\geq 1} T(n/\underbrace{4}_{b>1}) + \underbrace{n\log n}_{f(n)}$

$\exists\, \varepsilon > 0 \quad f(n) \in \Omega(n^{\log_b^a + \varepsilon}) \Rightarrow T(n) = \theta(f(n))$

$\exists\, 0.2 > 0 \quad n\log n \in \Omega(n^{\cancel{0.8+0.2}\, 1}) \Rightarrow \boxed{T(n) = \theta(n\log n)}$

$\exists\, c < 1 \quad af(n/b) < cf(n) \qquad \exists\, 3/4 < 1 \quad 3n/4 \log n/4 < \cancel{3/4}\, n\log n$

c. $T(n) = \underbrace{4}_{a\geq 1} T(n/\underbrace{2}_{b>1}) + \underbrace{n^2}_{f(n)}$

$f(n) \in \theta(n^{\log_b^a}) \Rightarrow T(n) = \theta(n^{\log_b^a} \log n)$

$n^2 \in \theta(n^2) \Rightarrow \boxed{T(n) = \theta(n^2\log n)}$

d. $T(n) = \overbrace{4}^{a\geq 1} T(n/\overbrace{2}^{b>1}) + \overbrace{n^2\log^2 n}^{f(n)} \qquad n > 1$ *

$\exists\, \varepsilon > 0 \quad f(n) \in \Omega(n^{\log_b^a + \varepsilon}) \Rightarrow T(n) = \theta(f(n))$

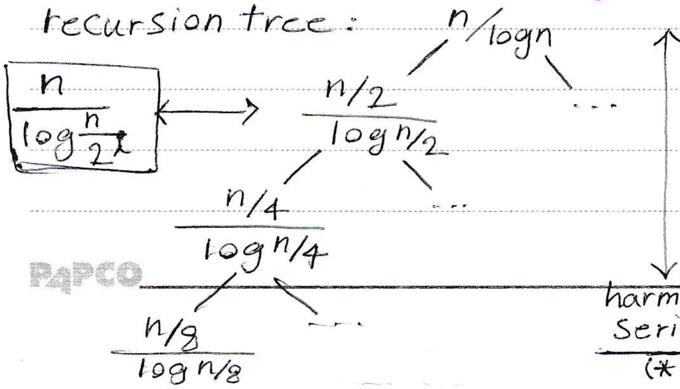
$\exists\, 0.1 > 0 \quad n^2\log^2 n \overset{(*\text{بافرض})}{\in} \Omega(n^{\cancel{2+0.1}\, 2.1}) \Rightarrow \boxed{T(n) = \theta(n^2\log^2 n)}$ (بافرض *)

$\exists\, c < 1 \quad af(n/b) < cf(n) \qquad \exists\, 0.9 < 1 \quad \cancel{4}\, \cancel{n^2/4} \log^2 n/2 < 0.9\, \cancel{n^2} \log^2 n$

نمودارها پیوست می‌شود. (۱ و ۲)

e. $T(n) = 2T(n/2) + n/\log n \qquad \log n \in \theta(\lg n)$ *

recursion tree:



$T(n/2^i) = T(1) = \theta(1)$

$n/2^i = 1 \quad \boxed{i = \lg n}$

$\sum_{i=0}^{\lg n} \frac{n}{\log \frac{n}{2^i}} \overset{(*)}{=} n \sum_{i=0}^{\lg n} \frac{1}{\log n - i}$

$\xrightarrow[(*)]{\text{harmonic series}} n \log\log n \quad \boxed{T(n) = \theta(n\log\log n)}$

PAPCO

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f. $T(n) = T(\sqrt{n}) + 1$

$$T(n) = T(n^{1/2}) + 1$$

$$= T(n^{1/4}) + 2$$

$$= \dots$$

$$= T(n^{1/2^i}) +$$

$$\boxed{T(n) = \theta(\log\log n)}$$

$n^{1/2^i} = 2 \Leftarrow$ نداریم $T(0)$ و $T(1)$

$$\sqrt[2^i]{n} = 2$$

$$n = 2^{2^i}$$

$$2^i = \log_2^n$$

$$i = \log_2 \log_2^n$$