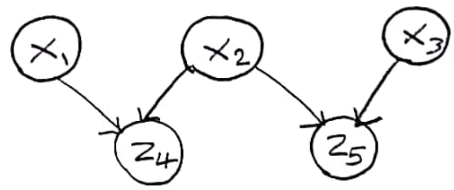
DGM - HW1 810198361 عرفان باقری سولا

① 1.

$$P(x_2, x_3 \mid z_5 = 0) = \begin{cases} \dfrac{q^2}{(1-q)^2 + q^2} & x_2 = 1, x_3 = 1 \\ \dfrac{(1-q)^2}{(1-q)^2 + q^2} & x_2 = 0, x_3 = 0 \\ 0 & \text{o.w} \end{cases}$$

$$P(x_2, x_3 \mid z_5 = 1) = \begin{cases} \dfrac{1}{2} & x_2 = 1, x_3 = 0 \\ \dfrac{1}{2} & x_2 = 0, x_3 = 1 \\ 0 & \text{o.w} \end{cases}$$

2.



$x_1 \perp x_2$

$x_1 \perp x_3$

$x_2 \perp x_3$

$x_1 \perp x_2 \mid z_5$ $\quad$ $x_1 \perp x_3 \mid x_2$

$x_1 \perp x_3 \mid z_5$ $\quad$ $x_1 \perp x_3 \mid x_2, z_4$

$x_2 \perp x_3 \mid z_4$ $\quad$ $x_1 \perp x_3 \mid x_2, z_5$

$x_1 \perp x_3 \mid z_4$ $\quad$ $x_1 \perp x_3 \mid x_2, z_4, z_5$

$x_1 \perp z_5$ $\quad$ $x_1 \perp z_5 \mid x_2, (x_3, z_4)$

$x_1 \perp x_2 \mid x_3$ $\quad$ $x_2 \perp x_3 \mid x_1$

$x_1 \perp x_2 \mid x_3, z_5$ $\quad$ $x_2 \perp x_3 \mid x_1, z_4$

$x_3 \perp z_4$ $\quad$ $x_3 \perp z_4 \mid x_2, (x_1, z_5)$

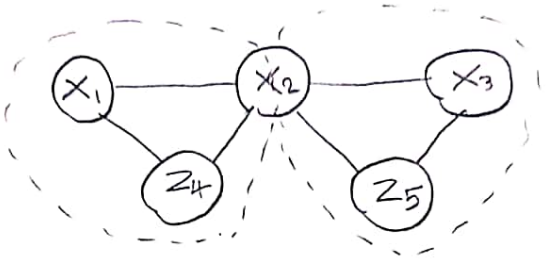
$z_4 \perp z_5 \mid x_2$ $\quad$ $z_4 \perp z_5 \mid x_2, x_3$ $\quad$ $z_4 \perp z_5 \mid x_2, x_1$

conditional p-table	$P(z_4 = 1 \mid x_1, x_2)$	$P(z_4 = 0 \mid x_1, x_2)$
$x_1 = 1, x_2 = 1$	0	1
$x_1 = 1, x_2 = 0$	1	0
$x_1 = 0, x_2 = 1$	1	0
$x_1 = 0, x_2 = 0$	0	1

① proba table

	$P(Z_5=1 \mid X_2, X_3)$	$P(Z_5=0 \mid X_1, X_2)$
$X_3=1, X_2=1$	0	1
$X_3=1, X_2=0$	1	0
$X_3=0, X_2=1$	1	0
$X_3=0, X_2=0$	0	1

① 3.



$X_1 \perp X_3 \mid X_2$

$X_1 \perp X_3 \mid X_2, Z_4$

$X_1 \perp X_3 \mid X_2, Z_5$

$X_1 \perp X_3 \mid X_2, Z_4, Z_5$

$X_1 \perp Z_5 \mid X_2, (Z_4, X_3)$

$Z_4 \perp Z_5 \mid X_2, (X_1, X_3)$

$X_3 \perp Z_4 \mid X_2, (Z_5, X_1)$

$$P(X_1, X_2, X_3, Z_4, Z_5) = g(X_1, X_2, Z_4)\, f(X_2, X_3, Z_5)$$
$$= P(X_1) P(X_2) P(X_3) P(Z_4 \mid X_1, X_2) P(Z_5 \mid X_2, X_3)$$

$$P(Z_5 \mid X_3=0) = \begin{cases} q & 1 \\ 1-q & 0 \end{cases} \qquad P(Z_5 \mid X_3=1) = \begin{cases} 1-q & 1 \\ q & 0 \end{cases}$$

$$P(Z_5) = \begin{cases} 2q(1-q) & 1 \\ (1-q)^2 + q^2 & 0 \end{cases} \quad \Rightarrow \quad q = \frac{1}{2}$$

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② Bayesian Network

1. False : a head to tail connection
2. True : they are disconnected
3. False : $S \to D \to H$ connection
4. True : they are disconnected
5. False : $S \to F \to H \to Z \to N$ connection
6. True : they are disconnected
7. False : head to head connection
8. False : given H, the v-structure gets connected
9. True : the v-structure is disconnected by default

③ 10. False : N is a child of v-structure and given that the v-structure gets connected

11. False : head to head connection

12. False : they are still connected

Markov Network

1. $P(S, F, D, C, H, N, Z) =$
$\delta_1(S, F)\, \delta_2(S, D)\, \delta_3(F, C)\, \delta_4(F, H)\, \delta_5(D, H)\, \delta_6(D, N)$
$\delta_7(H, Z)\, \delta_8(N, Z)$

2. $P(S, F, D, C, H, N, Z) =$
$\delta_1'(S)\, \delta_2'(F)\, \delta_3'(D) \cdots \underbrace{\delta_1(S, F)\, \delta_2(S, D) \cdots \delta_8(N, Z)}_{\text{like the previous part}}$

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② 1. $P(F = \text{true}) = P(F = \text{true} \mid S = \text{winter})\, P(S = \text{winter})$
$+ P(F = \text{true} \mid S = \text{summer})\, P(S = \text{summer}) =$
$0.4 \times 0.5 + 0.1 \times 0.5 = 0.25$

2. $P(F = \text{true} \mid S = \text{winter}) = 0.4$

3. $P(F = \text{true} \mid S = \text{winter}, H = \text{true})$
$= \dfrac{P(H = \text{true} \mid F = \text{true}, S = \text{winter})\, P(F = \text{true}, S = \text{winter})}{P(H = \text{true}, S = \text{winter})}$
↳ Bayes Rule

$P(H = \text{true} \mid F = \text{true}, S = \text{winter}) =$
$P(H = \text{true} \mid F = \text{true}, S = \text{winter}, D = \text{true})\, P(D = \text{true} \mid F = \text{true}, S = \text{winter})$
$+ P(H = \text{true} \mid F = \text{true}, S = \text{winter}, D = \text{false})\, P(D = \text{false} \mid F = \text{true}, S = \text{winter})$
$= 0.9 \times 0.1 + 0.8 \times 0.9 = 0.81$

$P(F = \text{true}, S = \text{winter}) = P(F = \text{true} \mid S = \text{winter})\, P(S = \text{winter})$
$= 0.4 \times 0.5 = 0.2$

$P(H=\text{true}, S=\text{winter}) = P(H=\text{true} \mid S=\text{winter})\, P(S=\text{winter})$

$P(H=\text{true} \mid S=\text{winter}) = 0.4 \times 0.1 \times 0.9 + 0.4 \times 0.9 \times 0.8$
$+ 0.6 \times 0.1 \times 0.8 + 0.6 \times 0.9 \times 0.3 = 0.534$

$\Rightarrow P(H=\text{true}, S=\text{winter}) = 0.534 \times 0.5 = 0.267$

$\Rightarrow P(F=\text{true} \mid S=\text{winter}, H=\text{true}) = \dfrac{0.81 \times 0.2}{0.267} = 0.607$

4. $P(F=\text{true} \mid S=\text{winter}, H=\text{true}, D=\text{true})$

$$= \frac{P(H=\text{true} \mid S=\text{winter}, F=\text{true}, D=\text{true})\, P(S=\text{winter}, F=\text{true}, D=\text{true})}{P(S=\text{winter}, H=\text{true}, D=\text{true})}$$

$P(H=\text{true} \mid S=\text{winter}, F=\text{true}, D=\text{true}) = 0.9$

$P(S=\text{winter}, F=\text{true}, D=\text{true}) =$
$P(F=\text{true} \mid S=\text{winter})\, P(D=\text{true} \mid S=\text{winter})\, P(S=\text{winter}) =$
$0.4 \times 0.1 \times 0.5 = 0.02$

$P(S=\text{winter}, H=\text{true}, D=\text{true}) =$
$P(H=\text{true} \mid D=\text{true}, S=\text{winter})\, \underbrace{P(D=\text{true}, S=\text{winter})}_{0.1 \times 0.5 = 0.05}$

$P(H=\text{true} \mid D=\text{true}, S=\text{winter}) =$
$P(H=\text{true} \mid F=\text{true}, D=\text{true}, S=\text{winter})\, P(F=\text{true} \mid S=\text{winter})$
$+ P(H=\text{true} \mid F=\text{false}, D=\text{true}, S=\text{winter})\, P(F=\text{false} \mid S=\text{winter})$
$= 0.9 \times 0.4 + 0.8 \times 0.6 = 0.84$

$\Rightarrow P(F=\text{true} \mid S=\text{winter}, H=\text{true}, D=\text{true}) = \dfrac{0.9 \times 0.02}{0.84 \times 0.05} = 0.429$

5. knowing you're dehydrated decreases your likelihood of having the flue. this makes sense because without knowing this, there is a good probability that your headache is caused by the flue; but if you know that you're dehydrated, there is a good probability that your headache is only caused by that.

③ 1. $$P(H|O) = \underbrace{P(H_1|O)}_{q_1(H_1)} \underbrace{P(H_2|O)}_{q_2(H_2)} \cdots \underbrace{P(H_M|O)}_{q_M(H_M)} = q(H)$$

$$q^*(H) = \arg\max_{q_1, q_2, \dots q_M} -KL(q(H) \| P(H,O))$$

$$-KL(q(H) \| P(H,O)) = -\int_H q(H) \log \frac{q(H)}{P(O,H)} dH = E_{q(H)}\left[-\log \frac{q(H)}{P(O,H)}\right]$$

$$= E_{q(H)}[\log P(O,H)] - E_{q(H)}[\log q(H)]$$

$$q(H) = \prod_{m=1}^{M} q_m(H_m) \Rightarrow \quad = E_{q(H)}[\log P(O,H)] - \sum_{m=1}^{M} E_{q(H)}[\log q_m(H_m)]$$

assuming that we only want to find $q_j(H_j)$ and other $q_m(H_m)$ are fixed

$$\sum_{m=1}^{M} E_{q(H)}[\log q_m(H_m)] = \sum_{m=1}^{M} E_{q_m(H_m)}[\log q_m(H_m)]$$

$$= E_{q_j(H_j)}[\log q_j(H_j)] + const$$

$$E_{q(H)}[\log P(O,H)] = \int_{H_j} q_j(H_j) \underbrace{\int_{H_1}\int_{H_2} \cdots \int_{H_M} q_1(H_1) \cdots q_M(H_M) \log P(O,H)\, dH_M \cdots dH_1}_{E_{-j}[\log P(O,H)]} dH_j$$

$$= E_{q_j(H_j)}\left[E_{-j}[\log P(O,H)]\right]$$

$$\Rightarrow -KL(q(H), P(O,H)) = E_{q_j(H_j)}\left[E_{-j}[\log P(O,H)]\right] - E_{q_j(H_j)}[\log q_j(H_j)] + const$$

$$= E_{q_j(H_j)}\left[E_{-j}[\log P(O,H)] - \log q_j(H_j)\right] + const$$

$$= \underbrace{\int_{H_j} q_j(H_j)\left(E_{-j}[\log P(O,H)] - \log q_j(H_j)\right) dH_j}_{G(q_j(H_j))} + const$$

$$q_j^*(H_j) = \arg\max_{q_j} -KL(q(H) \parallel P(O,H))$$

Euler-Lagrange : $\frac{dG}{dq_j} = E_{-j}\left[\log P(O,H)\right] - \log q_j(H_j) - 1 = 0$

$$\Rightarrow \log q_j^*(z_j) = E_{-j}\left[\log P(O,H)\right] + \text{const}$$

$$P(O,H) = P(H_j \mid H_{-j}, O) \cdot P(H_{-j}, O)$$

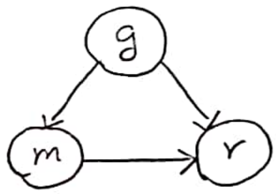
$$\rightarrow \log P(O,H) = \log P(H_j \mid H_{-j}, O) + \underbrace{\log P(H_{-j}, O)}_{\text{constant}}$$

$$\Rightarrow \log q_j^*(z_j) = E_{-j}\left[\log P(H_j \mid H_{-j}, O_{1:N})\right] + \text{const}_2$$

4

	A	B
g1	$\frac{81}{87} = 93\%$	$\frac{234}{270} = 86\%$
g4	$\frac{192}{263} = 73\%$	$\frac{55}{80} = 69\%$
tot	$\frac{273}{350} = 78\%$	$\frac{289}{350} = 82.6\%$

fatty liver grade : g
medicine : m
recovery result : r



When examining the overall success rates for medicine A and B, it initially appears that medicine B outperforms medicine A. However, there is a notable imbalance in the number of grade 1 and grad 4 patients receiving each medicine. A higher proportion of grade 1 patients received medicine B while a larger share of grade 4 patients were treated with medicine A. Since grade 4 fatty liver cases are generally more challenging to treat, this imbalance influences the overall results in favor of medicine B.

when we account for this discrepancy in patient distribution, it becoms evident that medicine A appears to be more effective than B in both grade 1 and grade 4 subgroups.