CSCI567 – Machine Learning
Assignment 2

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Question 1-1

$$\frac{\partial l}{\partial u} = -[W^{(2)T}\frac{\partial l}{\partial a}].*H(u)$$

$$\frac{\partial l}{\partial a} = [diag(z) - zz^T][-y.*\frac{1}{z}]$$

$$\frac{\partial l}{\partial W^{(1)}} = \frac{\partial l}{\partial u}x^T$$

$$\frac{\partial l}{\partial b^{(1)}} = \frac{\partial l}{\partial u}$$

$$\frac{\partial l}{\partial W^{(2)}} = \frac{\partial l}{\partial a}h^T$$

Question 1-2

$$W^{(2)} = 0 \Rightarrow \frac{\partial l}{\partial u} = 0 \Rightarrow \frac{\partial l}{\partial W^{(1)}} = \frac{\partial l}{\partial b^{(1)}} = 0$$

$$W^{(1)} = b^{(1)} = 0 \rightarrow u = W^{(1)}x + b^{(1)} = 0 \Rightarrow h = max(0, u) = \mathbf{0} \Rightarrow \frac{\partial l}{\partial W^{(2)}} = \frac{\partial l}{\partial a}h^T = \mathbf{0}$$

We have shown that the initial value for these parameters and the change on them (derivative) is zero, this is why they will not get updated and remain zero forever.

Question 1-3

$$\begin{aligned} a = W^{(2)}u + b^{(2)} = W^{(2)}(W^{(1)}x + b^{(1)}) + b^{(2)} = \\ W^{(2)}W^{(1)}x + W^{(2)}b^{(1)} + b^{(2)} = \mathbf{U}x + \mathbf{v} \Rightarrow \\ \mathbf{U} = W^{(2)}W^{(1)} \\ \mathbf{v} = W^{(2)}b^{(1)} + b^{(2)} \end{aligned}$$

Question 2-1

$$J(w) = \sum l(w^T\phi(x_n), y_n) + \frac{\lambda}{2}||w||^2 \Rightarrow minJ(w) \Rightarrow \frac{\partial J}{\partial w} = 0$$
$$\Rightarrow \sum l'(w^T\phi(x_n), y_n).\phi(x_n) + \lambda\mathbf{w} = 0$$
$$\Rightarrow w^* = \frac{-1}{\lambda}\sum l'(w^T\phi(x_n), y_n).\phi(x_n)$$
$$note : l' = \frac{\partial l(w)}{\partial w}$$

Question 2-2

$$\alpha_n = \frac{-1}{\lambda} l'(w^T\phi(x_n), y_n) \Rightarrow w^* = \sum \alpha_n\phi(x_n) = \Phi^T\alpha$$
$$\Rightarrow J(\alpha) = \sum l(\alpha^T\Phi\phi(x_n), y_n) + \frac{\lambda}{2}||\Phi^T\alpha||^2 = \sum l(\alpha^T\Phi\phi(x_n), y_n) + \frac{\lambda}{2}\alpha^T\Phi\Phi^T\alpha$$
$$\rightarrow J(\alpha) = \sum l(\alpha^T K_n, y_n) + \frac{\lambda}{2}\alpha^T K\alpha,$$

$$K_n = \begin{bmatrix} \phi(x_1)^T.\phi(x_n) \\ \phi(x_2)^T.\phi(x_n) \\ \phi(x_3)^T.\phi(x_n) \\ ... \\ ... \\ \phi(x_N)^T.\phi(x_n) \end{bmatrix}$$