۵ـ الگوریتم های زیر را تحلیل کنید .

1:		
`for ( i=1 ; i<=n ; i++) {`	C1	$\sum_{i=1}^{n} 1 + 1 = n+1$
`for( j=1 ; j<=n ; j++) {`	C2	$\sum_{i=1}^{n} (\sum_{j=1}^{n} 1 + 1) = n(n+1)$
`for( k=1; k<=n ; k++) {`	C3	$\sum_{i=1}^{n} (\sum_{j=1}^{n} (\sum_{k=1}^{n} 1+1)) = n(n(n+1))$
`x++ ;`	C4	$\sum_{i=1}^{n} (\sum_{j=1}^{n} (\sum_{k=1}^{n} 1)) = n(n(n)) = n^3$
`}`		
`j=1 ;`	C5	$\sum_{i=1}^{n} (\sum_{j=1}^{n} 1) = n^2$
`while ( j<=n ) {`	C6	$\sum_{i=1}^{n} (\sum_{j=1}^{n} (\log_2 n +1)) = n(n((\log_2 n+1))$
`x++ ;`	C7	$\sum_{i=1}^{n} (\sum_{j=1}^{n} (\log_2 n+1)) = n(n(\log_2 n+1))$
`j*=2;`	C8	$\sum_{i=1}^{n} (\sum_{j=1}^{n} (\log_2 n\underline{+1})) = n(n(\log_2 n+1))$
`}`		
`}`		
`}`		

چون j<=n است ← پس دستورات داخل حلقه هم 1+ دارند

درنهایت: $c_1(n+1) + c_2(n^2+n) + c_3(n^3+n^2) + c_4(n^3) + c_5(n^2) + c_6(n^2\log_2 n + n^2)$

$+ c_7(n^2(\log_2 n+1)) + c_8 (n^2\log_2 n+1) \xrightarrow{\text{درنهایت}} an^3 + bn^2 + Cn + dn^2\log_2 n + e$

→ پس $= O(n^3 + n^2\log_2 n) \approx O(n^3)$

2:
```
for (i=1 ; i<=n ; i++) {
    for( j=1 ; j<=n ; j++) {
        x++ ;
        for(k=1 ; k<=n ; k*=2) {
            for (L=1 ; L<=k ; L++) {
                x++;
            }
        }
    }
}
```

$C_1 \quad \sum_{i=1}^{n} 1 + 1 = n+1$

$C_2 \quad \sum_{i=1}^{n}\left(\sum_{j=1}^{n} 1+1\right) = n(n+1)$

$C_3 \quad \sum_{i=1}^{n}\left(\sum_{j=1}^{n} 1\right) = n(n) = n^2$

$C_4 \quad \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left(\log_2^n + 1\right)\right) = n^2(\log_2^n + 1)$

$C_5 \quad \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left(\log_2^n\left(\sum_{l=1}^{k} 1+1\right)\right)\right)$

$C_6 \quad \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left(\log_2^n\left(\sum_{l=1}^{k} 1\right)\right)\right)$

$\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left(\log_2^n (k+1)\right)\right) = \sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left(k\log_2^n + \log_2^n\right)\right)$

$\sum_{i=1}^{n}\left(\sum_{j=1}^{n} k\log_2^n + \sum_{j=1}^{n}\log_2^n\right) \longrightarrow n^2 k \log_2^n + n^2 \log_2^n$

درنهایت $\longrightarrow n=k \longrightarrow n^3\log_2^n + n^2\log_2^n$

$\sum_{i=1}^{n}\left(\sum_{j=1}^{n}\left(\log_2^n (k)\right)\right) = n^2 k\log_2^n \longrightarrow n=k \longrightarrow n^3\log_2^n$

$\xrightarrow[\text{کلی}]{\text{پیچیدگی زمانی}}$: $C_1(n+1) + C_2(n^2+n) + C_3(n^2) + C_4(n^2\log_2^n + n^2)$

$+ C_5(n^3\log_2^n + n^2\log_2^n) + C_6(n^3\log_2^n) =$

$\xrightarrow{\text{پس:}} an^2 + bn + cn^2\log_2^n + dn^3\log_2^n + e$

$\longrightarrow O(n^2 + n^3\log_2^n) \approx O(n^3\log_2^n)$

3:
```
for ( i=1 ; i<=n ;i++) {
    for( j=i ; j<=n ; j++) {
        x++ ;
    }
    for ( k=1 ; k<=n ;k++){
        int m=1 ;
        while (m<k) {
            x++ ;
            m *= 3 ;
        }
    }
}
```

c_1 $\sum_{i=1}^{n} 1 + 1 = n+1$

c_2 $\sum_{i=1}^{n} \left(\sum_{j=i}^{n^{n-i}} 1 + 1 \right) = \sum_{i=1}^{n} n - \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$

c_3 $\sum_{i=1}^{n} \left(\sum_{j=i}^{n} 1 \right) = n^2 - \sum_{i=1}^{n} i = n^2 - \frac{n(n+1)}{2} = \frac{n^2}{2} - \frac{n}{2} \rightarrow \frac{n(n-1)}{2}$

$n^2 - \frac{n(n+1)}{2} + n = \frac{n^2}{2} + \frac{n}{2} = \frac{n(n+1)}{2}$

c_4 $\sum_{i=1}^{n} \left(\sum_{k=1}^{n} 1 + 1 \right) = n(n+1)$

c_5 $\sum_{i=1}^{n} \left(\sum_{k=1}^{n} 1 \right) = n^2$

c_6 $\sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\log_3 n + 1 \right) \right) = n^2 \log_3 n + n^2$

c_7 $\sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\log_3 n \right) \right) = n^2 \log_3 n$

c_8 $\sum_{i=1}^{n} \left(\sum_{k=1}^{n} \left(\log_3 n \right) \right) = n^2 \log_3 n$

$\xrightarrow[\text{کل}]{\text{پیچیدگی زمانی}}$: $c_1(n+1) + c_2\left(\frac{n^2}{2} + \frac{n}{2}\right) + c_3\left(\frac{n^2}{2} - \frac{n}{2}\right) + c_4(n^2+n) + c_5(n^2)$

$+ c_6\left(n^2 \log_3 n + n^2\right) + c_7\left(n^2 \log_3 n\right) + c_8\left(n^2 \log_3 n\right)$

$\xrightarrow{\text{پس}}$ $a n^2 + bn + c\, n^2 \log_3 n + d \rightsquigarrow = O(n^2 + n^2 \log_3 n) \approx O(n^2 \log_3 n)$

4:

```
for (i=1 ; i<=n ; i++) {
    for ( j=1 ; j<=i*i ; j++ ) {
        for ( k=1 ; k<=j ; k++ ) {
            n++;
        }
    }
    int m=1;
    while (m<n) {
        n++;
        m*=5;
    }
}
```

c_1 $\quad \sum_{i=1}^{n} 1 + 1 = n+1$

c_2 $\quad \sum_{i=1}^{n} \left(\sum_{j=1}^{i^2} 1 + 1 \right) = \sum_{i=1}^{n} i^2 + n = \frac{n(n+1)(2n+1)}{6} + n$

c_3 $\quad \sum_{i=1}^{n} \left(\sum_{j=1}^{i^2} \left(\sum_{k=1}^{j} 1 + 1 \right) \right) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i^2} (j+1) \right)$

c_4 $\quad \sum_{i=1}^{n} \left(\sum_{j=1}^{i^2} \left(\sum_{k=1}^{j} 1 \right) \right) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i^2} j \right) = \sum_{i=1}^{n} \frac{i^2(i^2+1)}{2}$

$\sum_{i=1}^{n} \left(\sum_{j=1}^{i^2} j + \sum_{j=1}^{i^2} 1 \right) = \sum_{i=1}^{n} \left(\frac{i^2(i^2+1)}{2} + i^2 \right) \rightsquigarrow$

$\rightarrow \frac{1}{2} \sum_{i=1}^{n} i^4 + \frac{1}{2} \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i^2 = \frac{1}{2} \left(\frac{n(n+1)(2n+1)}{6} \right)^2 + \frac{3}{2} \left(\frac{n(n+1)(2n+1)}{6} \right)$

c_5 $\quad \sum_{i=1}^{n} 1 = n$

c_6 $\quad \sum_{i=1}^{n} (\log_5 n + 1) = n \log_5 n + n$

c_7 $\quad \sum_{i=1}^{n} (\log_5 n) = n \log_5 n$

c_8 $\quad \sum_{i=1}^{n} (\log_5 n) = n \log_5 n$

n^3, n^2, n

پیچیدگی زمانی کل : $c_1(n+1) + c_2 \left(\frac{2n^3 + 3n^2 + n}{6} + n \right) + c_3 \left(\left(\left(\frac{2n^3 + 3n^2 + n}{6} \right)^2 + 3 \left(\frac{2n^3 + 3n^2 + n}{6} \right) \right) \frac{1}{2} \right)$

$+ c_4 \left(\frac{1}{2} \left(\left(\frac{n(n+1)(2n+1)}{6} \right)^2 + \left(\frac{n(n+1)(2n+1)}{6} \right) \right) \right) + c_5(n) + c_6(n \log_5 n + n)$

$+ c_7(n \log_5 n) + c_8(n \log_5 n)$

$\xrightarrow{\text{پس}} a n^6 + b n^5 + c n^4 + d n^3 + e n^2 + f n + g n \log_5 n + h$

$O(n^6 + n \log_5 n) \approx O(n^6)$

5:
```
for ( i=1 ; i<=n ; i++) {
    for (j=1 ; j<=i ; j++) {
        for ( k=1 ; k<=n ; k++){
            for( l=1 ; l<=n ; l++) {
                if ( l%2 == 0) {
                    x++ ;
                }
            }
        }
    }
}
```

C_1: $\sum_{i=1}^{n} 1 + 1 = n+1$

C_2: $\sum_{i=1}^{n}\left(\sum_{j=1}^{i} 1 + 1\right) = \sum_{i=1}^{n} i + n = \frac{n(n+1)}{2} + n$

C_3: $\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\left(\sum_{k=1}^{n} 1+1\right)\right) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i} n + \sum_{j=1}^{i} 1\right)$

C_4: $\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\left(\sum_{k=1}^{n}\left(\sum_{l=1}^{n} 1+1\right)\right)\right) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i}(n^2+n)\right)$

C_5: $\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\left(\sum_{k=1}^{n}\left(\sum_{l=1}^{n} 1\right)\right)\right) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i}(n^2)\right)$

C_6: $\sum_{i=1}^{n}\left(\sum_{j=1}^{i}\left(\sum_{k=1}^{n}\left(\sum_{l=1}^{n} 1\right)\right)\right) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i} n^2\right)$

$$\sum_{i=1}^{n} in^2 = n^2\left(\frac{n(n+1)}{2}\right)$$

$$\sum_{i=1}^{n} (in^2) = n^2\left(\frac{n(n+1)}{2}\right)$$

$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i} n^2 + \sum_{j=1}^{i} n\right) = \sum_{i=1}^{n} in^2 + \sum_{i=1}^{n} in = n^2\left(\frac{n(n+1)}{2}\right) + n\left(\frac{n(n+1)}{2}\right)$$

$$\sum_{i=1}^{n} (in + i) = n\left(\frac{n(n+1)}{2}\right) + \frac{n(n+1)}{2} = \frac{n(n+1)^2}{2}$$

پیچیدگی زمانی $\rightsquigarrow$: $C_1(\overset{n}{n+1}) + C_2\overset{n^2}{\left(\frac{n^2}{2}+\frac{3}{2}n\right)} + C_3\overset{n^3}{\left(\frac{n}{2}(n^2+2n+1)\right)} + C_4\overset{n^4}{\left(\underbrace{\frac{n^2(n+1)}{2}(n+1)}_{\frac{n^2(n+1)^2}{2}}\right)}$

$+ C_5\overset{n^4}{\left(\frac{n^3(n+1)}{2}\right)} + C_6\overset{n^4}{\left(\frac{n^3(n+1)}{2}\right)}$

پس $\rightsquigarrow$ $an^4 + bn^3 + cn^2 + dn + e \rightsquigarrow = O(n^4)$