



مسئل 1:

Cov(X,Y|Z) = E[(X-E[X|Z])(Y-E[Y|Z])|Z] = E[(XY-XE[Y|Z]-YE[X|Z]+E[X|Z]E[Y|Z])|Z] = E[XY|Z] - E[XE[Y|Z]|Z] - E[YE[X|Z]|Z] + E[E[X|Z]E[Y|Z])|Z] = E[XY|Z] - E[XE[Y|Z]|Z] - E[YE[X|Z]|Z] + E[E[X|Z]E[Y|Z])|Z] = E[XY|Z] - E[X|Z]E[Y|Z] - E[X|Z]E[Y|Z] = E[X|Z]E[X|Z] - E[X|Z]E[X|Z] = E[X|Z

= E(XYIZ)_E(XIZ)E(YIZ)_E(YIZ)E(XIZ)_E(XIZ)E(YIZ)

Cov(X,Y|Z) = E[XYIZ] _ E[XIZ] E[YIZ]

ب) = E[XY]_E[X]E[Y] بدد از قانون طی اسیسامی بره ی وسم:

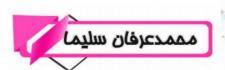
Cov(X,Y)=E[E[XYIZ]]-E[E[XIZ]]E[E[YIZ]]

ر ارسم عنس الف السياده مي لنم:











Cov(X,Y) = E[cov(X,Y|Z) + E[x|z]E[Y|Z]]-E[E[X|Z]]E[E[Y|Z]]

= E (COV (X,Y | Z)] + E [E[X | Z] E [Y | Z]] - E [E[X | Z]] E [E[Y | Z]]

ما تعمر متعمر مرای شهدد بهتردری ماسم له:

E[E[XIZ]E[YIZ]]_E[E[XIZ]]E[E[YIZ]] = CON(E[XIZ], E[YIZ])

. بس سجه می شودله: نام

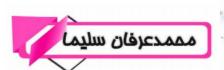
 $C_{ov}(X,Y) = E[c_{ov}(X,Y|Z)]_{+}c_{ov}(E[X|Z],E[Y|Z])$

 $var(X|Y) = E[(X - E[X|Y])^{1}|Y]$ $= E[X^{2}|Y] - 2(E[X|Y])^{2} + (E[X|Y])^{2}$ $= E[X^{2}|Y] - (E[X|Y])^{2}$ $= E[var(X|Y)] = E[X^{2}] - E[(E[X|Y])^{2}]$











var(E[XIY]) = E[E[XIY])] = E[E[XIY]] 2 (E[X]) أبطرا وأراجع ي نيم:

var(E[XIY]) + E[var(XIY)] = E[X]-E[X]=var(X)

وهميس از طيق زيرانيات مي تسود:

در آساوی بخسش ب به طای X و ۲ ، X و به جای X ، ۲ قرار می دهم:

ومى دارنم لر Cov(X,XIY) = Var(X) و Cov(X,X) = Var(X)

ر پین شجری شودلر: .

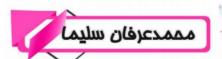
var(X)= E[var(X|Y)] + Cov(E[X|Y], E[X|Y])

var(X)=E[var(X|Y)] + var(E[X|Y])











: 2 Lima

$$Y = \sum_{i=1}^{5} X_i \approx Normal, \frac{\sum_{i=1}^{5} X_i - n}{\sum_{i=1}^{5} X_i} \approx N(0,1) \Rightarrow Z = \frac{Y - 100}{5\sqrt{2}} \approx N(0,1)$$

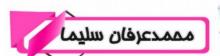
$$P(9.611.0) = P(\frac{9.10.0}{5.52}) < Z(\frac{11.10.0.0}{5.52}) = P(-\sqrt{2}(Z(\sqrt{2}))$$

$$= \phi(\sqrt{2}) - \phi(-\sqrt{2}) = 2 \phi(\sqrt{2}) - 1 \approx 0/8427$$











: 3 Lima

$$E[X] = \sum_{n=1}^{\infty} x P(X=n) \ge \sum_{n=1}^{k} x P(X=n)$$

(لن)

رر/ از انجاله مرد ۲ و نرولی است ، بس ۲ (X = x) ۲ (X = x) مال داریم لیر:

$$E[X] = \sum_{n=1}^{\infty} {_{n}P(X=n)} \ge \sum_{n=1}^{k} {_{n}P(X=n)} \ge \sum_{n=1}^{k} {_{n}P(X=k)}$$

$$\sum_{k=1}^{k} x P(X=k) = P(X=k) \sum_{i=1}^{k} x = \frac{k(k+1)}{2} P(X=k)$$

$$\longrightarrow E[X] \geq \frac{k(k+1)}{2} P(X=k) \longrightarrow P(X=k) \leq \frac{2}{k(k+1)} E[X] \leq \frac{2}{k^2} E[X]$$

$$\longrightarrow \mathcal{P}(X=k) \leq \frac{2E[X]}{k^2}$$

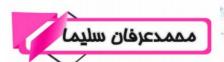
ب) ما مد محسن قبل ، ازانجا له علا و لخ نزولی است ، بس (۴(م) له علی الله علی الله علی الله الله الله علی الله ا

 $E[X] = \int_{0}^{\infty} x f(x) dx \ge \int_{0}^{k} x f(x) dx \ge \int_{0}^{k} x f(k) dx$











$$\int_{1}^{k} x f(k) dx = f(k) \int_{1}^{k} x dx = f(k) x \frac{x^{2}}{2} \int_{0}^{k} f(k) x \frac{k^{2}}{2}$$

$$\longrightarrow E[X] \ge \frac{k^2}{2} f(k) \longrightarrow f(k) \le \frac{2 E[X]}{k^2}$$

$$\longrightarrow f(x) \leq \frac{2E[X]}{x^2}$$











مسئله 4: رر المر الم المراد و المراد و

$$Y = \sum_{i=1}^{5} X_i$$
, $E[X_i] = \int_{-2}^{4} = \frac{a+b}{2} = 0$, $var(X_i) = \frac{(b-a)^2}{12} = \frac{1}{12} = \delta^2$

$$Z = \frac{\sum_{i=1}^{5} \chi_{i-1}}{6\sqrt{n}} = \frac{\sum_{i=1}^{5} \chi_{i}}{\frac{1}{\sqrt{12}} \sqrt{5}} = \frac{\sqrt{6}}{5} \sum_{i=1}^{5} \chi_{i} = \frac{\sqrt{6}}{5} Y$$
 Normal(0,1)

$$P(|Y|>3)=1-P(|Y|<3)=1-P(-3$$

=
$$1 - P(-\frac{3}{5} \sqrt{6} < \frac{\sqrt{6}}{5} \sqrt{3} \sqrt{6}) = 1 - P(-\frac{3\sqrt{6}}{5} < Z < \frac{3\sqrt{6}}{5})$$

$$= 1 - (\phi(\frac{3\sqrt{6}}{5}) - \phi(\frac{3\sqrt{6}}{5})) = 2 - 2\phi(\frac{3\sqrt{6}}{5})$$

=
$$2(1-\phi(\frac{3\sqrt{6}}{5})) = 2\phi(\frac{-3\sqrt{6}}{5}) \approx 0.1416$$









ر ما السبعاده لزقصر صومرلزی داریم کر: مسئلہ 5:

$$Y = \frac{\sum_{i=1}^{n} X_i - n^{\frac{1}{p}}}{3 \sqrt{n}}, \quad \beta = 2$$

$$Y = \frac{\sum_{i=1}^{n} X_{i} - n^{\frac{n}{n}}}{2 \sqrt{n}} \sim Normal(0,1)$$

$$P(\left|\frac{\sum_{i=1}^{n} \chi_{i}}{n} - \int_{i=1}^{n} \left| \left\langle \frac{1}{2} \right\rangle \right| = P(\left|\frac{1}{2}\right| \left| \left\langle \frac{\sum_{i=1}^{n} \chi_{i}}{n} - \int_{i=1}^{n} \left\langle \frac{1}{2} \right\rangle \right|$$

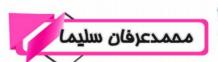
$$\xrightarrow{\times \frac{\sqrt{n}}{2}} P(\frac{-\sqrt{n}}{4} < Y < \frac{\sqrt{n}}{4}) = \Phi(\frac{\sqrt{n}}{4}) - \Phi(\frac{-\sqrt{n}}{4})$$

$$\rightarrow 2 \oplus (\frac{\sqrt{n}}{4}) - 1 \ge 0/95 \Rightarrow \oplus (\frac{\sqrt{n}}{4}) \ge 0/975$$

مر المرابع 62 مار اللوريم را احداكهم.









:6 Lines

$$\longrightarrow P(x \ge a) \le \frac{\beta^2 + b^2}{(a+b)^2}$$

$$\frac{d}{db} \frac{3^{2} + b^{2}}{(a+b)^{2}} = \frac{2b(a+b)^{2} - 2(a+b)(3^{2} + b^{2})}{(a+b)^{4}} = 0$$

$$\Rightarrow 2ba + 2b - 2b^2 - 2b^2 = 0 \Rightarrow b = \frac{b^2}{a}$$









$$\longrightarrow P(X \ge a) \le \frac{Var(X)}{Var(X) + a^2}$$

