



مسلك ل

ار ۱۲ دا بدس مورت درنظری لیدع:

$$X_{i} = \begin{cases} 1 & \text{where} \\ X_{i} = \begin{cases} 1 & \text{where} \end{cases}$$

ر کر کر کر کوزیع ، X برنونی است ، داریم لد:

$$E[X_i] = P = \frac{1}{6}$$

,
$$Var(X_i) = pq = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

$$Y = \sum_{i=1}^{5..} X_i$$

ع که ۲ دارس مورت فرض می کنیم:

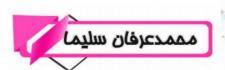
$$Var(Y) = 500 \times \frac{5}{36} = \frac{625}{9} = 8^2 \implies 8 = \frac{25}{3}$$

$$E[Y] = 5... \times \frac{1}{6} = \frac{25.}{3}$$

$$Z = \frac{\gamma - \frac{250}{3}}{\frac{25}{3}} \sim \text{Normal}(0, 1)$$









$$\Rightarrow \frac{250}{3} - 1.96 \times \frac{25}{3} \le Y \le \frac{250}{3} + 1.96 \times \frac{25}{3}$$

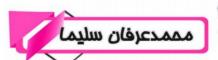
67 Y < 99,66

بازه احسان 95 درصدی برای ۲<mark>، [67,99،66] است.</mark>











$$L(\theta|x_1,x_2,...,x_n) = f_{\chi_1,\chi_2,...,\chi_n}(x_1,x_2,...,x_n|\theta) = \prod_{i=1}^n f_{\chi_i}(x_i|\theta)$$

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$$f_{\chi_i}(x_i,\theta) = \begin{cases} \frac{1}{\theta} & \text{if } i \leq i \leq n, \text{if } i$$

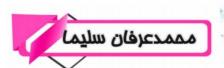
ر لنترس مقدار Θ، ما لسيم ندها است.

$$\theta = Max(x_1,x_2,...,x_n)$$

ب) محس بدی Unbiased است د Θ = Θ علی اقاطی اور اقاطی این استی کنم:









$$F_{Y}(y) = P(Y \le y) = P(X_1 \le y, X_2 \le y, ..., X_n \le y) = P(X_1 \le y)^n$$

$$P(\chi_i \langle y) = F_{\chi_i} \langle y \rangle = \frac{y^{-\circ}}{\theta^{-\circ}} = \frac{y}{\theta}$$

$$\Rightarrow F_{\gamma}(y) = \left(\frac{y}{\theta}\right)^n \Rightarrow f_{\gamma}(y) = \frac{ny^{n-1}}{\theta^n}$$

$$E[\hat{\theta}] = E[Y] = \int_{\theta}^{\theta} \int_{0}^{\pi} \frac{\partial^{n-1}}{\partial x^{n}} dy = \frac{n}{\theta^{n}} \int_{0}^{\theta} \int_{0}^{\pi} dy = \frac{n}{\theta^{n}} \left(\frac{\partial^{n+1}}{\partial x^{n}} \right) \int_{0}^{\theta} dx$$

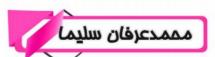
$$\Rightarrow \mathbb{E}\left[\hat{\Theta}\right] = \frac{n}{n+1} \Theta \Rightarrow \mathbb{E}\left[\hat{\Theta}\right] \neq \Theta$$

. - المرابع الم biased الست.











مسئلہ 3.

برای گرو 2 میر لدزیر Unbiased هسسد.

$$X = \frac{\sum_{i=1}^{n} X_i}{n}$$
, $S = \frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1} \Rightarrow E(\overline{S}^2) = \delta^2$

$$var(\overline{X}) = E[\overline{X}^2] - E[\overline{X}]^2$$

$$\Rightarrow E[\bar{X}^2] = var(\bar{X}) + E[\bar{X}]^2$$
, $var(\bar{X}) = \frac{1}{n^2} vn \delta^2 = \frac{\delta^2}{n}$, $E[\bar{X}] = \int_0^{\infty} e^{-\frac{1}{2} xn \delta^2} e^{-\frac{1}{2} xn \delta^2} e^{-\frac{1}{2} xn \delta^2} e^{-\frac{1}{2} xn \delta^2}$

$$\Rightarrow E[\overline{X}] = \frac{3^2}{n} + \int_{0}^{\infty}$$

$$E\left[\frac{5}{n}^{2}\right] = \frac{1}{n} E\left[\frac{5}{5}^{2}\right] = \frac{3}{n}^{2}$$

$$E\left[\overline{X}^{2}, \frac{\overline{5}^{2}}{n}\right] = E\left[\overline{X}^{2}\right] - E\left[\frac{\overline{5}^{2}}{n}\right] = \mu^{2}$$

$$\int_{1}^{2} = \frac{1}{n^{2}} \left(\sum_{i=1}^{n} \chi_{i} \right)^{2} - \frac{1}{n(n-1)} \sum_{i=1}^{n} \left(\chi_{i} - \chi \right)^{2}$$









مسله ۶۰. الن) تعداد مل بلدرص هار ۱ در نظری لعرم و بیشاندا نله 2 بلدرص کز 6 بلدرص

علامت دار با شد را X مي نامم . حال داريم له:

$$L(N|X) = \frac{\binom{3}{2}\binom{N-3}{4}}{\binom{N}{6}} = \frac{3 \times \frac{(N-3)!}{4! (N-7)!}}{\frac{N!}{6! (N-6)!}} = \frac{3 \times 6!}{4!} \times \frac{(N-6)!}{(N-7)!} \times \frac{(N-3)!}{N!}$$

$$\Rightarrow L(N|X) = \frac{9.(N-6)}{N(N-1)(N-2)}$$

ا توحر به عارت های مالا ۲ × N بی الله .

على الله بعدار Nرا طوري ما بيم لمر عالم السيسة متود: (ازا زيون نسبت بهره مي جريم)

$$\frac{L(N|X)}{L(N-1|X)} = \frac{96(N-6)}{N(N-1)(N-2)} \times \frac{(N-1)(N-2)(N-3)}{96(N-7)} = \frac{N^2-9N+18}{N^2-7N} = 1 + \frac{18-2N}{N^2-7N}$$

ما ترجه ما عمارت بالأ ، بدار أي الا ، بدار أي الله من الله من الدول من المود ، من طامي الست براي بعد الدول









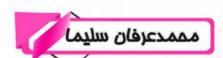
رو المرام : 1. المرام : N = 9 , N = 7 رادر الفر لمرم : المرم : المرم

$$L(N=7|X) = \frac{3}{7}$$
, $L(N=8|X) = L(N=9|X) = \frac{15}{28}$
 $N = \frac{15}{28}$
 $N = \frac{15}{28}$
 $N = \frac{15}{28}$
 $N = \frac{15}{28}$











مسئلہ 5.

(لغ)

$$\hat{\eta} = \sum_{i=1}^{n} \alpha_{i} x_{i} \Rightarrow E[\hat{\eta}] = \sum_{i=1}^{n} E[\alpha_{i} x_{i}] = \sum_{i=1}^{n} \alpha_{i} E[\eta_{+} \varepsilon_{i}]$$

$$\Rightarrow E[\hat{\gamma}] = \gamma \sum_{i=1}^{n} \alpha_i$$

ر - رار م عس لا Unbiased است ، س:

$$E[\hat{\gamma}] = \gamma \Rightarrow \sum_{i=1}^{n} \alpha_i = 1$$

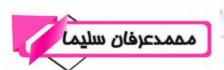
$$\operatorname{Var}(\hat{\gamma}) = \sum_{i=1}^{n} \operatorname{Var}(\alpha_{i} x_{i}) = \sum_{i=1}^{n} \alpha_{i}^{2} \operatorname{Var}(\eta + \varepsilon_{i}) = \sum_{i=1}^{n} \alpha_{i}^{2} \delta_{i}^{2}$$

$$f(\alpha_1, \alpha_2, ..., \alpha_n) = \sum_{i=1}^{n} \alpha_i \beta_i$$
, $g(\alpha_1, \alpha_2, ..., \alpha_n) = \sum_{i=1}^{n} \alpha_i - 1$

$$F(\alpha_1, \alpha_2, ..., \alpha_n) = f(\alpha_1, \alpha_2, ..., \alpha_n) + \lambda g(\alpha_1, \alpha_2, ..., \alpha_n)$$









$$\frac{\partial F}{\partial \alpha_i} = 2\alpha_i \, \delta_{i+1}^2 + \lambda = 0 \implies \alpha_i = \frac{-\lambda}{2 \, \delta_{i+1}^2}$$

$$\frac{\partial F}{\partial \lambda} = \sum_{i=1}^{n} \alpha_{i-1} = 0 \implies \sum_{i=1}^{n} \alpha_{i-1} = 1$$

$$\Rightarrow \sum_{i=1}^{n} \alpha_i = -\frac{\lambda}{2} \sum_{i=1}^{n} \frac{1}{\delta_i^2} \Rightarrow \lambda = \frac{-2}{\sum_{i=1}^{n} \frac{1}{\delta_i^2}}$$

اله دای یام:

$$\alpha'_{i} = \frac{-\lambda}{2 \delta_{i}^{2}} = \frac{2}{\sum_{j=1}^{n} \frac{1}{\delta_{j}^{2}}} \times \frac{1}{2 \delta_{i}^{2}}$$

$$\Rightarrow \alpha_i = \frac{1}{\delta_i^2 \sum_{j=1}^n \frac{1}{\delta_j^2}}$$

- . کرر د محس بریس صورت تولھدبود:

$$\hat{\gamma} = \frac{\sum_{i=1}^{n} \frac{\chi_{i}}{\delta_{i}^{2} \sum_{j=1}^{n} \frac{1}{\delta_{j}^{2}}}$$