Social Network

Homework 3

Full Name:	Erfan Shahabi
Student ID:	810103166
Course:	Social Networks
Submission Date:	03-Jan-2026
Github Link:	https://github.com/erfanshahabi/Social-Networks-Fall-2025/tree/main/SN_HW3_810103166

Table of Contents

List of Figures

List of Tables

1. Theoretical Framework

1.1. Optimizing Structural Balance in a Competitive Network

Part a:

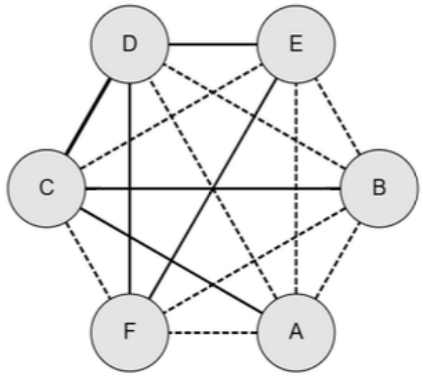

Figure 1

Table 1

Triangle	*Situation*	*Sins*
CDF	*Imbalanced*	*++-*
CDB	*Imbalanced*	*++-*
CDA	*Imbalanced*	*++-*
CBA	*Imbalanced*	*++-*
CDE	*Imbalanced*	*++-*
DBA	*Imbalanced*	*---*
EBA	*Imbalanced*	*---*
EFA	*Imbalanced*	*---*
ABF	*Imbalanced*	*---*

Part b:

Optimal two faction partition:

Group 1: {A, B, C}

Group 2: {D, E, F}

Minimum flips needed: 2 edges

i. Edges that must change sign:

   Edge 1: A-B

   Current: Negative

   Required: Positive

   Reason: A and B must cooperate with faction 1

   Edge 2: C-D

   Current: Positive

   Required: Negative

   Reason: C and D are in opposing factions and must be adversaries.

ii. Yes, The partition is preserved
   Bipartition Analysis
   Analysis of Original Structure:
   Positive edge distribution:
   - Within Group 1: CB, CA
   - Within Group 2: DF, DE, EF
   - Between Groups: CD

   Result: 5 out of 6 positive edges lie within fractions.

   Original Bipartition is preserved. The network naturally exhibits the {A, B, C} vs {D, E, F} faction structure.

   Why this partition is structurally preferable:

   1. Minimal changes required

      Only 2 edge flips (A-B and C-D)

      Alternative partitions need 5+ flips

2. Eliminates all instability

   Removes all 8 imbalanced triangles

   Creates perfect balance structure

3. Natural alignment

   Group 2 {D,E,F} already forms complete positive triangle

   Group 1 {A,B,C} has 2/3 positive edges

   Network naturally exhibits this division

2.1. Graph Reduction in Large and Irregular Signed Networks

Part a

Using the standard graph reduction algorithm, we identify connected components based on positive edges only. Each component becomes a supernode in the reduced graph G'.

Supernodes in G':

S1: {1, 2, 3, 4, 5}

- Positive connections: 1-2, 1-3, 2-3, 2-4, 4-5
- Form densely connected cluster

S2: {6, 7}

- Positive connections: 6-7
- Small coalition

S3: {8}

- Isolated node

S4: {9}

- Isolated node

S5: {10, 11, 12, 13}

- Positive connections: 10-11, 11-12, 12-13
- Forms chain structure

S6: {14, 15, 16, 17, 18}

- Positive connections: 14-15, 14-16, 15-16, 16-17, 17-18
- Forms highly connected cluster

Reduced graph G' contains 6 supernodes

Part b:

Reduced Graph G' Structure

Edges in G' (derived from negative edges in original G):

- S1-S2 (from 2-6 negative edge)
- S1-S3 (from 5-8 negative edge)
- S1-S5 (from 4-11 negative edge)
- S1-S6 (from 1-14, 3-16 negative edges)
- S2-S4 (from 7-9 negative edge)
- S4-S5 (from 9-10 negative edge)
- S5-S6 (from 13-15, 12-18 negative edges)

Bipartiteness Test

- Result: G' is NOT BIPARTITE

Cycle Analysis

- Odd-length cycle detected: S1 → S5 → S6 → S1
- This cycle has length 3 (odd), which prevents bipartite structure.
- Path:
  - S1 connects to S5 (via negative edge 4-11)
  - S5 connects to S6 (via negative edges 13-15, 12-18)
  - S6 connects back to S1 (via negative edges 1-14, 3-16)

Conclusion

The original network G does NOT satisfy strong structural balance

Part c:

Transform G' into bipartite structure by minimizing edge modifications (converting negative edges to positive, or equivalently, removing edges from G').

Optimal Bipolar Partition

Coalition 1: {S1, S4} Coalition 2: {S2, S3, S5, S6}

Minimum Changes Required

Number of edge changes: 1

Edge to modify:

- S5-S6: This edge must be removed (or converted to positive)

Verification

After removing S5-S6:

Edges within Coalition 1: None ✓

Edges within Coalition 2: None ✓

Edges between coalitions:

- S1-S2 ✓
- S1-S3 ✓
- S1-S5 ✓
- S1-S6 ✓
- S4-S2 ✓ (S2-S4)
- S4-S5 ✓

Result: Perfect bipartite structure achieved

3.1. Triadic Closure and Tie Stability in Social Networks

Part a

STC: A node A satisfies the STC property if:

- Whenever A has strong ties to two other nodes B and C
- Then B and C must connected by at least some tie (strong or weak)

In this problem node A's situation:

- A has strong tie to B
- A has strong tie to C
- B and C have no connection

So node A violates the strong triadic closure principle.

Changes to Achieve STC Consistency

Option 1: Add Edge B-C

Create a connection between B and C (either strong or weak tie).

Result:

- Satisfies STC: A has strong ties to B and C, and B-C are now connected
- Completes the triad A-B-C
- Most natural solution reflecting social reality

Option 2: Weaken Tie A-B

Change A-B from strong to weak tie.

Result:

- No longer violates STC (STC only applies when both ties are strong)
- A-B becomes weak, A-C remains strong
- Only one strong tie from A, so STC doesn't apply

Option 3: Weaken Tie A-C

Change A-C from strong to weak tie.

Result:

- No longer violates STC
- A-C becomes weak, A-B remains strong
- Only one strong tie from A, so STC doesn't apply

Option 4: Weaken Both A-B and A-C

Change both ties to weak.

Result:

- Definitely satisfies STC (no strong ties from A to multiple nodes)
- Eliminates the precondition for STC

Part b

Given:

- Network satisfies STC
- Edge A, D is a local bridge

Definitions:

- STC: If node A has strong ties to both B and D, then B and D must be connected.
- Local Bridge: Edge A, D is a local bridge if A and D have no common neighbors.

Proof

Claim: A, D cannot be a strong tie

1. Assume A, D is strong
2. We know A-B is Strong
3. By STC: Since A has strong ties to both B and D, edge B-D must exist
4. Then B is a common neighbor of A and D

5. This contradicts the local bridge definition (no common neighbors)
6. Therefore, (A, D) must be weak

2. Practical Implementation

1.2. Question 1

Part (1): Sign Prediction

Dataset: 50 nodes, 221 edges, 88 unknown

Method: Iterative triangle based prediction using structural balance property. For each triangle with 2 known edges, predict third edge using product rule (all balanced triangles have sign product 1)

Result: Predict 911 edge signs from 88 unknowns by iteratively inferring from balanced triangles.

Part (2): Balanced Test

Method: Super node generation:

1. Merge nodes connected by positive edges into supernodes.
2. Build reduced graph with negative edges between super nodes.
3. Test bipartiteness of reduced graph.

Table 2

Network	Nodes	Supernodes	Result
network_a	35	3	Balanced
network_b	38	3	Balanced
network_c	35	2	Balanced
network_d	33	3	Balanced
network_e	26	2	Balanced
network_f	31	5	Balanced
network_g	25	2	Balanced
network_h	30	2	Balanced

All 8 neworks tested were balanced.

Part (3): Clusterability

Method: Detect clusters using connected components on positive edges. Weakly balanced networks have positive edges within clusters and negative edges between clusters.

Table 3

Network	Nodes	Clusters	Cluster Sizes
network_a	14	3	5, 5, 4 nodes
network_b	20	4	5, 5, 5, 5 nodes
network_c	30	4	8, 8, 7, 7 nodes
network_d	40	6	7, 7, 7, 7, 6, 6 nodes
network_e	60	8	8, 8, 8, 8, 7, 7, 7, 7 nodes

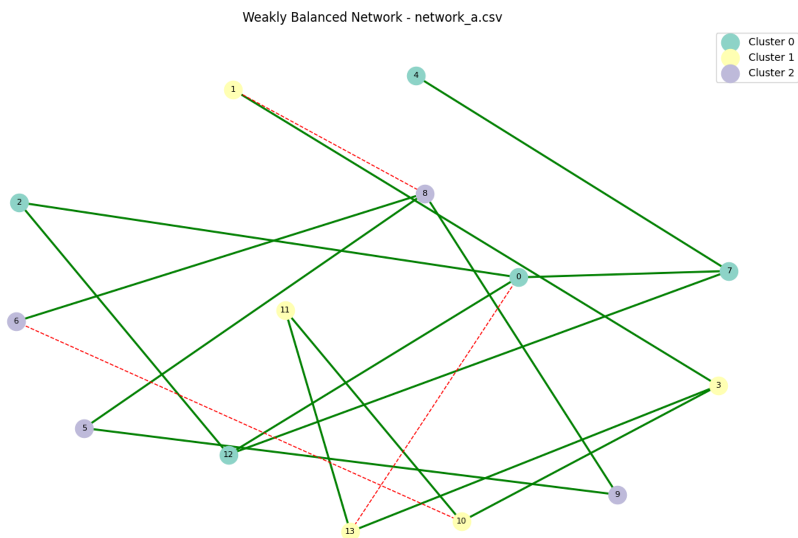

Figure 2

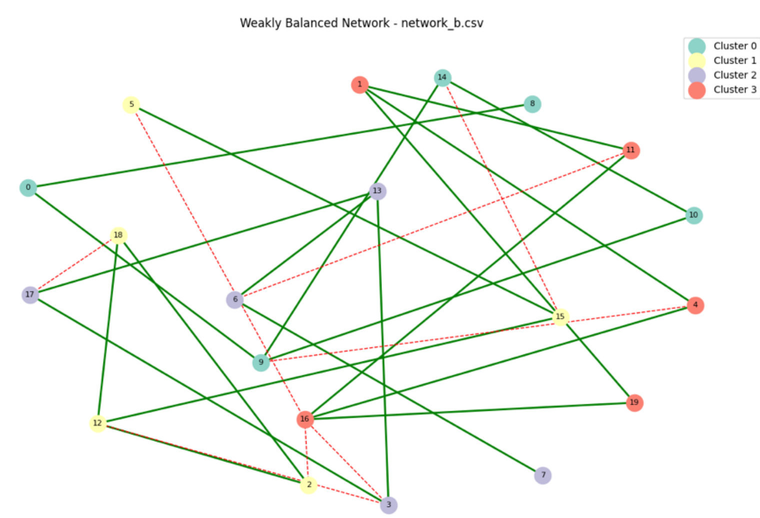

Figure 3

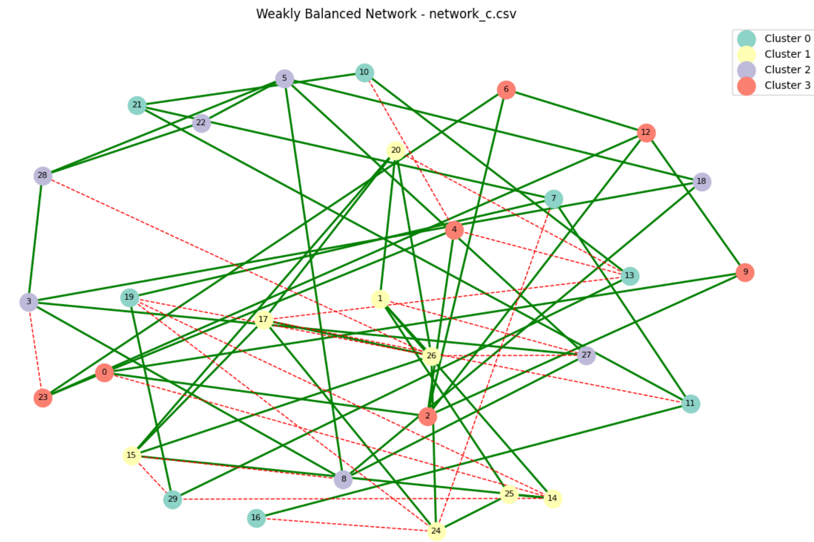

Figure 4

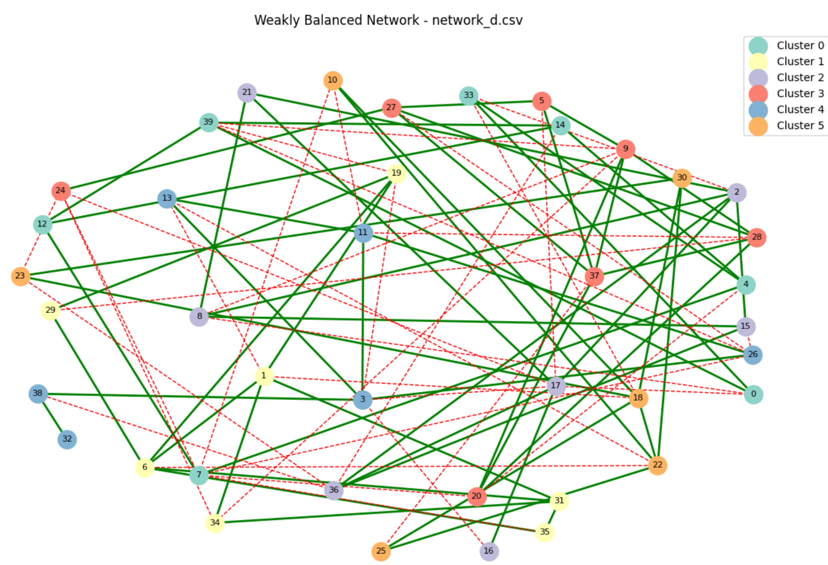

Figure 5

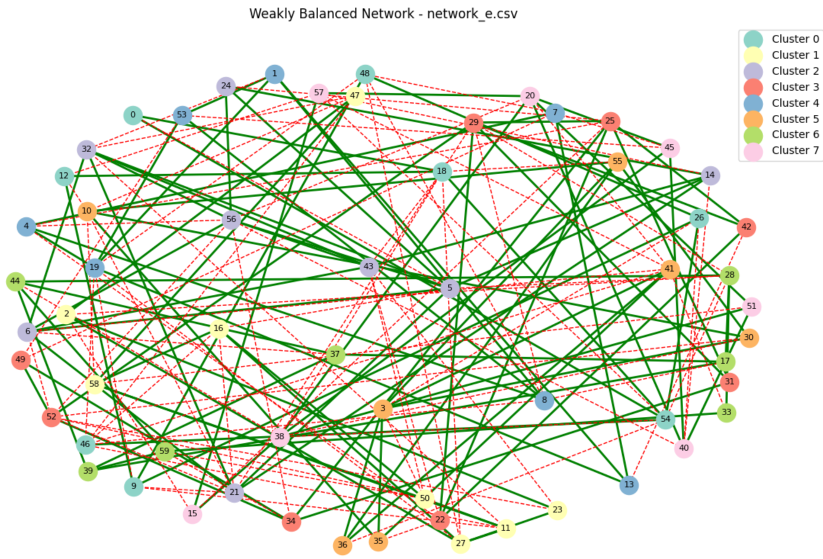

Figure 6

Part (4): Line Index

Formula: LI = 0.5·P + 0.5·N (P=positive between, N=negative within)

Random: LI = 38.50

Heuristic (2000 iterations): LI = 0.00

Improvement: 38.50 (perfect clustering)

Part (5): Transitivity

Dataset: 200 nodes, 820 directed edges

Transitive triples: 70 / 3,277

Transitivity ratio: 2.14%

Missing edges: 3,069

Total needed: 3,889 edges

2.2. Question 2

Network Overview

120 students tracked over 90 days (Day 1, 30, 60, 90)

Network growth: 25 → 312 → 1,022 → 1,845 edges

Smoking spread: 20 → 29 → 42 → 70 smokers (250% increase)

Closure Analysis

Table 4

Period	Triadic	Focal (Smoking)	Membership	New Edges
Day 1→30	14 (4.9%)	171	187	287
Day 30→60	758 (106.8%)	382	321	710
Day 60→90	801 (97.3%)	459	1043	823

Key Findings

Triadic Closure: Accelerated after Day 30 (4.9% → 106.8% → 97.3%). 738 smokers involved in Day 60→90 triadic closures, indicating smoking spreads through mutual friends.

Focal Closure (Smoking): 459 new connections between smokers in Day 60→90. Smokers increasingly connect with each other, creating smoking subgroups.

Membership Closure: Strong class-based clustering (1043 same-class connections in Day 60→90). Students primarily befriend classmates.

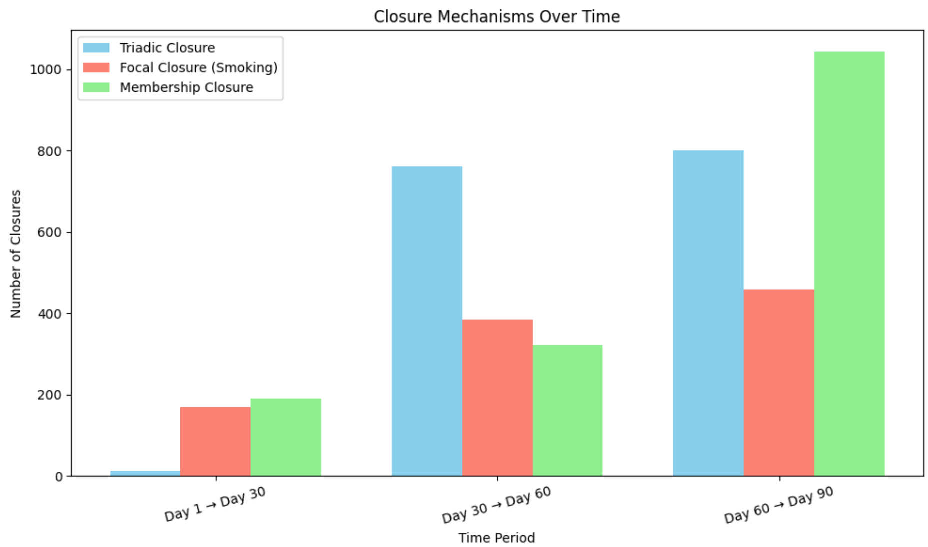

Figure 7

Smoking Behavior Analysis

Smokers vs Non-Smokers

Table 5

Feature	Smokers	Non-Smokers	Observation
Gender (Male)	71.4%	74.0%	Similar distribution
Avg Age	16.11	15.90	Smokers slightly older
Study Hours	1.09	2.06	Smokers study less
Play Football	50/70	27/50	Smokers more athletic

Interpretation: Smokers study significantly less (1.09 vs 2.06 hours) and are more likely to play football. Network analysis shows smoking spreads through social connections, particularly among students with similar behavioral patterns.

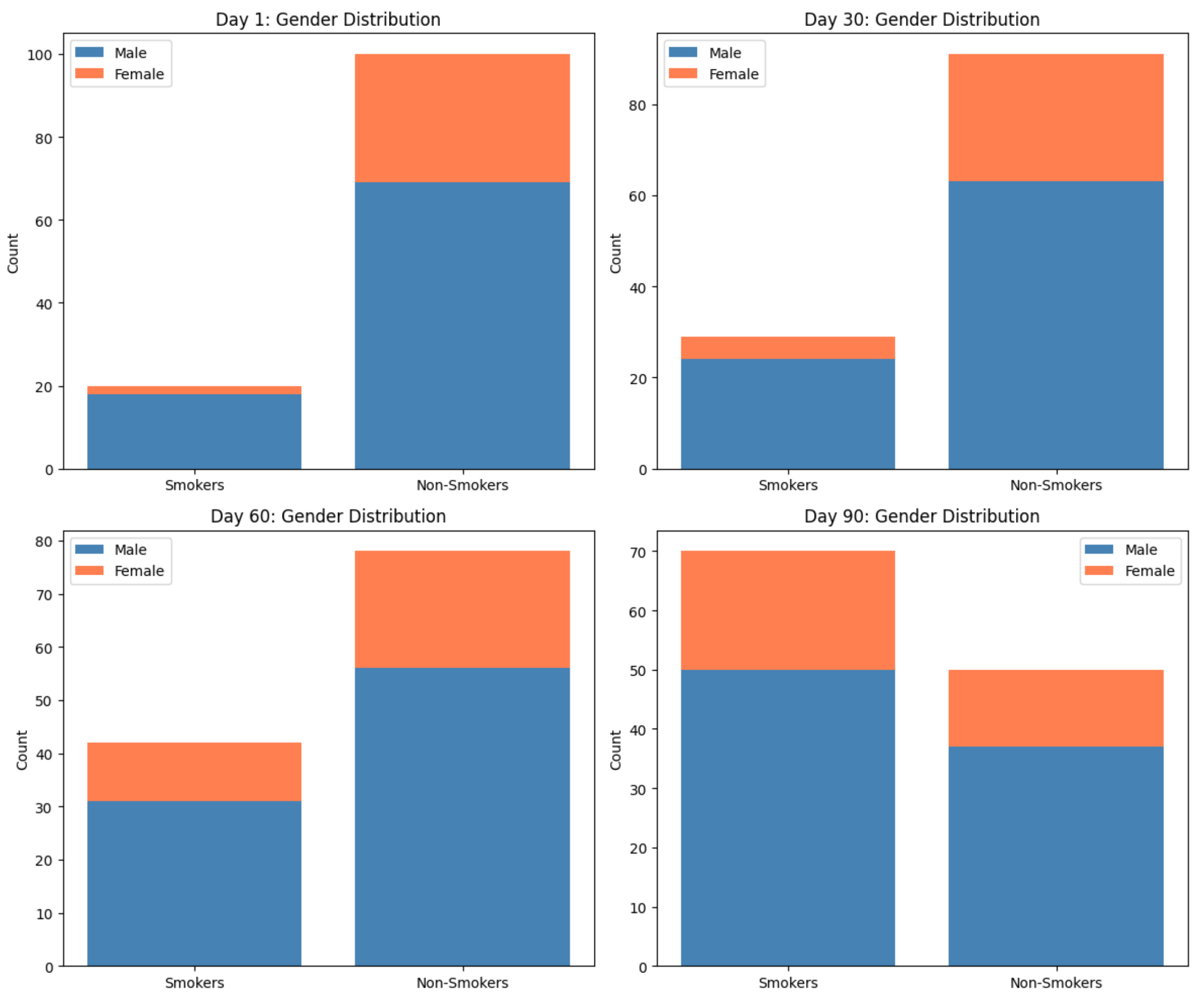

Figure 8

Degree Centrality Analysis

Top 5 most central students at Day 90:

Table 6

Rank	Student ID	Degree	Smoker	Gender	Smoker Contacts
1	52	52	Yes	Boy	37 (71.2%)
2-5	Various	47-50	Mix	Mix	Similar patterns

Role Analysis: Most central student (ID 52) is a smoker with 71.2% smoker connections. Central students act as bridges in the network, potentially accelerating smoking spread through their extensive connections.

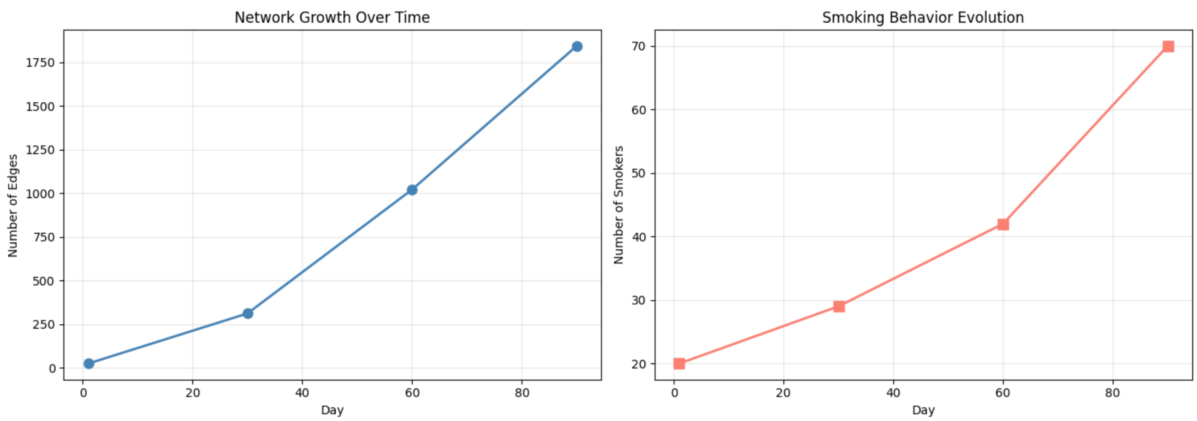

Figure 9

Conclusion

Smoking behavior spreads through social networks via three mechanisms:

1. Triadic closure: Smokers meet through mutual friends (738 cases)

2. Focal closure: Smokers preferentially connect (459 smoking pairs)

3. Membership: Same-class connections dominate (1043 edges)

Central students with high smoker connectivity (71%) play critical roles in behavior diffusion. The 250% increase in smokers (20 to 70) over 90 days demonstrates rapid social contagion effects in school networks.