EE 150L
Signals and Systems Lab

Lab3 Analysis of Periodic Signals in the Frequency Domain

Date Performed:

Class Id: 1A-105

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1. Get to know the frequency domain:

Find out the amplitude-frequency and phase-frequency of the signal:

$$\mathrm{f(t)} = 1 + 2\sin(\pi t) - \sin(3\pi t) + \sin(4\pi t) + \cos(3\pi t) - \frac{1}{2}\cos(5\pi t - \frac{\pi}{4})$$

The necessary steps need to be given.

提示：

利用三角、和差化积等公式将 f(t)转换为$\mathrm{f(t)} = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_1 t + \varphi_n)$，或利用欧拉公式转换成$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_1 t + \varphi_n}$的形式后，找出角频率与幅度，角频率与相位的对应关系。如：

$\omega = 0$时，$c_0 = 1$，$\varphi_0 = 0$

$$f(t) = 1 + 2\sin(\pi t) - \sqrt{2}\sin(3\pi t - \frac{\pi}{4}) + \sin(4\pi t) - \frac{1}{2}\cos(5\pi t - \frac{\pi}{4})$$

$$= 1 + 2\cos(\pi t - \frac{\pi}{2}) - \sqrt{2}\cos(3\pi t - \frac{3\pi}{4}) + \cos(4\pi t - \frac{\pi}{2}) - \frac{1}{2}\cos(5\pi t - \frac{\pi}{4})$$

$\omega = 0$ 时 $C_0 = 1$. $\varphi_0 = 0$

$W = \pi$ 时 . $C_1 = 2$, $\varphi_1 = -\frac{\pi}{2}$

$\omega = 2\pi$ 时 . $C_2 = 0$. $\varphi_2 = 0$

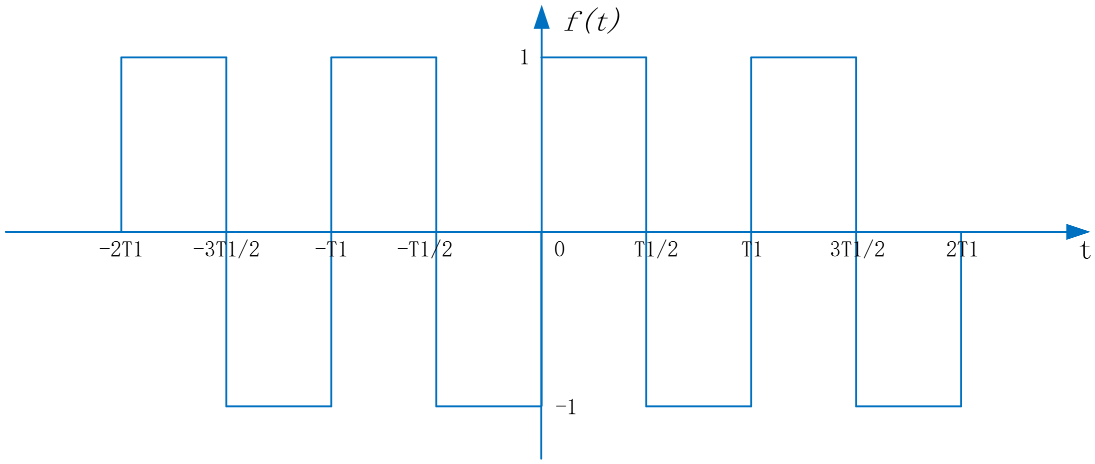
$\omega = 3\pi$ 时. $C_3 = -\sqrt{2}$, $\varphi_3 = -\frac{3\pi}{4}$

$\omega = 4\pi$ 时 $C_4 = 1$, $\varphi_4 = -\frac{\pi}{2}$

$\omega = 5\pi$ 时 $C_5 = -\frac{1}{2}$, $\varphi_5 = -\frac{\pi}{4}$

2. Get to know the Fourier Series:

Find the Fourier series of the following period signal. $T_1 = 2$.



提示:

a) 使用三角或指数形式将上述周期函数展开为傅里叶级数，详细方法请参考Lab 3 Analysis of Periodic Signals in the Frequency Domain 2022-2.pdf。

三角形式：$f(t) = a_{0+} \sum_{n=1}^{\infty}(a_n \cos n\omega_1 t + b_n \sin n\omega_1 t)$

指数形式：$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_1 t}$

b) 请手算（不需要 MATLAB 代码）。

$$a_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt = \frac{1}{2}\int_{-1}^{1} f(t)\,dt = \frac{1}{2}\left[\int_{-1}^{0} -1\,dt + \int_{0}^{1} 1\,dt\right] = 0$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos n\omega t\,dt = \frac{2}{2}\int_{-1}^{1} f(t)\cos n\omega t\,dt$$

$$= \int_{-1}^{0} -1 \times \cos n\pi t\,dt + \int_{0}^{1} 1 \times \cos n\pi t\,dt$$

$$= -\frac{1}{n\pi}\sin n\pi t\Big|_{-1}^{0} + \frac{1}{n\pi}\sin n\pi t\Big|_{0}^{1}$$

$$= -\frac{1}{n\pi}\sin n\pi + \frac{1}{n\pi}\sin n\pi = 0$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\sin n\omega t\,dt = \frac{2}{2}\int_{-1}^{1} f(t)\sin n\omega t\,dt$$

$$= \int_{-1}^{0} -\sin n\pi t\,dt + \int_{0}^{1}\sin n\pi t\,dt$$

$$= \frac{1}{n\pi}\cos n\pi t\Big|_{-1}^{0} - \frac{1}{n\pi}\cos n\pi t\Big|_{0}^{1}$$

$$= \left(\frac{1}{n\pi} - \frac{1}{n\pi}\cos n\pi\right) - \left(\frac{1}{n\pi}\cos n\pi - \frac{1}{n\pi}\right) = \frac{2-2\cos n\pi}{n\pi} = \begin{cases} 0, & n=2,4,6,\cdots \\ \frac{4}{n\pi}, & n=1,3,5,\cdots \end{cases}$$

$$f(t) = \sum_{n=1}^{\infty}\frac{4}{n\pi}\sin n\pi t, \quad n=1,3,5,\cdots$$