EE150 Signals and Systems
Spring 2020 – Midterm

2 pages, 4 questions, and 100 points in total.

08:15 AM – 10:00 AM, Tuesday, May 19, 2020

1. (10 + 10 points)

   a) For each statement, state (in the following table) if they are true (T) or false (F).

      i) All memoryless systems are causal systems.

      ii) The inverse of a causal LTI system is always causal.

      iii) If an LTI system is causal, then it is stable.

      iv) $y[n] = 3x[n] + 5$ is a linear system.

      v) $y(t) = \int_{-\infty}^{2t} x(\tau)\mathrm{d}\tau$ is time-invariant.

i)	ii)	iii)	iv)	v)

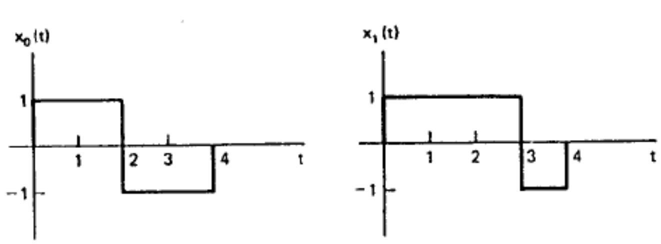
   b) Consider the system $y(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t)$. State (in the following table) if the system is: causal, linear, time-invariant, invertible, stable.

causal	linear	time-invariant	stable

2. (15 + 15 points) Calculate the following two convolutions

   a) Determine $f_1(t) = [u(t) - u(t-1)] * [u(t-1) - u(t-2)]$, where $u(t)$ is the unit step function.

   b) Determine $f_2(t) = x_0(t) * x_1(t)$, when $x_0(t)$ and $x_1(t)$ are given in the following figure.



3. (10 + 10 points) In this problem, we derive two important properties of the continuous-time Fourier series: the multiplication property and Parseval's relation. Let $x(t)$ and $y(t)$ both be continuous-time periodic signals having period T_0 and with Fourier series representations given by ($\omega_0 = 2\pi/T_0$)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k \mathrm{e}^{jk\omega_0 t}, \qquad y(t) = \sum_{k=-\infty}^{\infty} b_k \mathrm{e}^{jk\omega_0 t}.$$

a) Let $z(t) = x(t)y(t)$ and its Fourier series be represented as $z(t) = \sum_{k=-\infty}^{\infty} c_k \mathrm{e}^{jk\omega_0 t}$. Show that the Fourier series coefficients of the signal $z(t)$ are given by the following discrete convolution

$$c_k = \sum_{n=-\infty}^{\infty} a_n b_{k-n}.$$

b) Let $y(t) = x^*(t)$ and $x^*(t)$ denotes the conjugate of $x(t)$. Express b_k in terms of a_k and use the result of (a) to prove the following Parseval's relation for periodic signals:

$$\frac{1}{T_0}\int_0^{T_0} |x(t)|^2 \mathrm{d}t = \sum_{k=-\infty}^{\infty} |a_k|^2.$$

4. (10 + 10 + 10 points)

a) In the lecture, we derived the transform of $x(t) = \mathrm{e}^{-at}u(t)$, where $u(t)$ is the unit step function. Using the linearity and scaling properties, derive the Fourier transform of $\mathrm{e}^{-a|t|} = x(t) + x(-t)$.

b) Using part (a) and the duality property, determine the Fourier transform of $1/(1+t^2)$.

c) If

$$y(t) = \frac{1}{1+(3t)^2}$$

find the Fourier transform of $y(t)$.