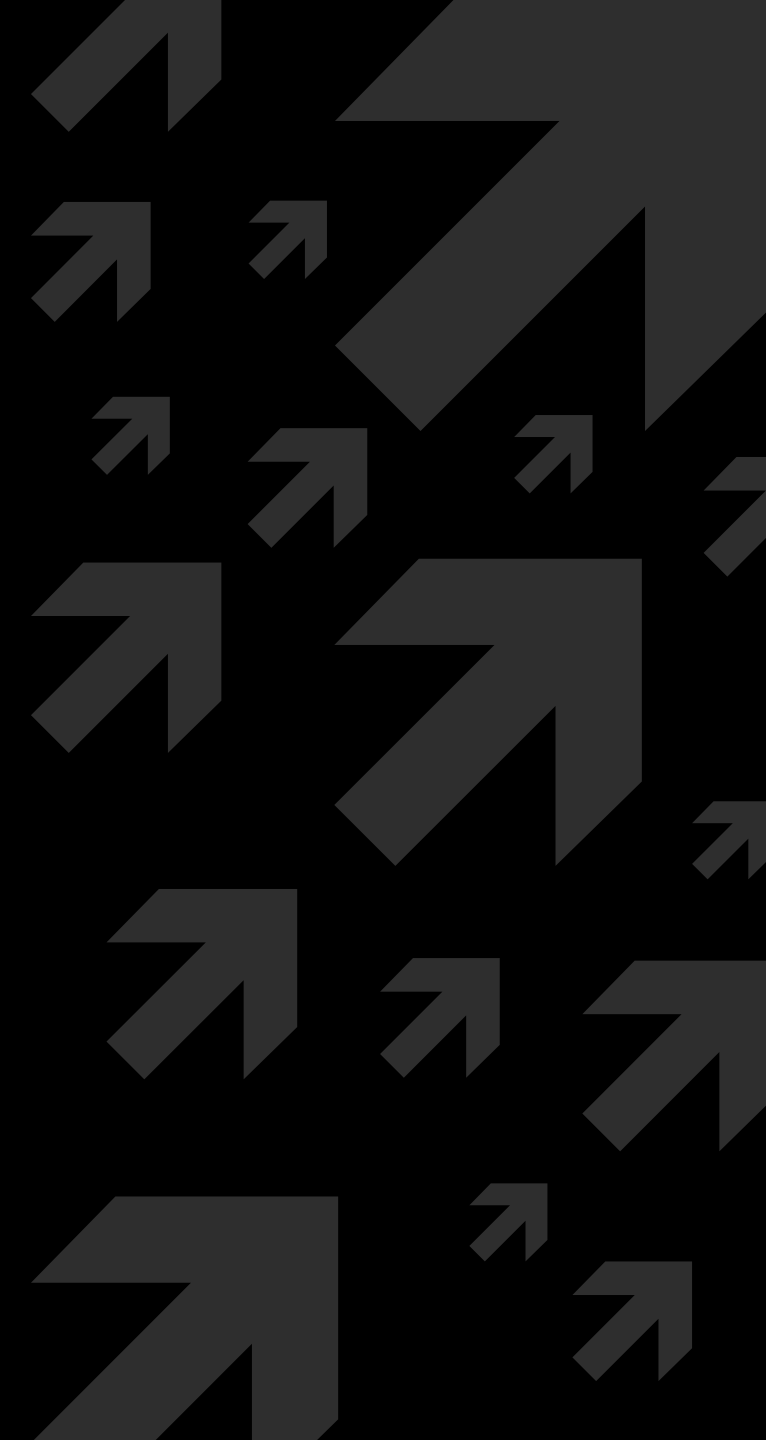
CS101 Midterm Review

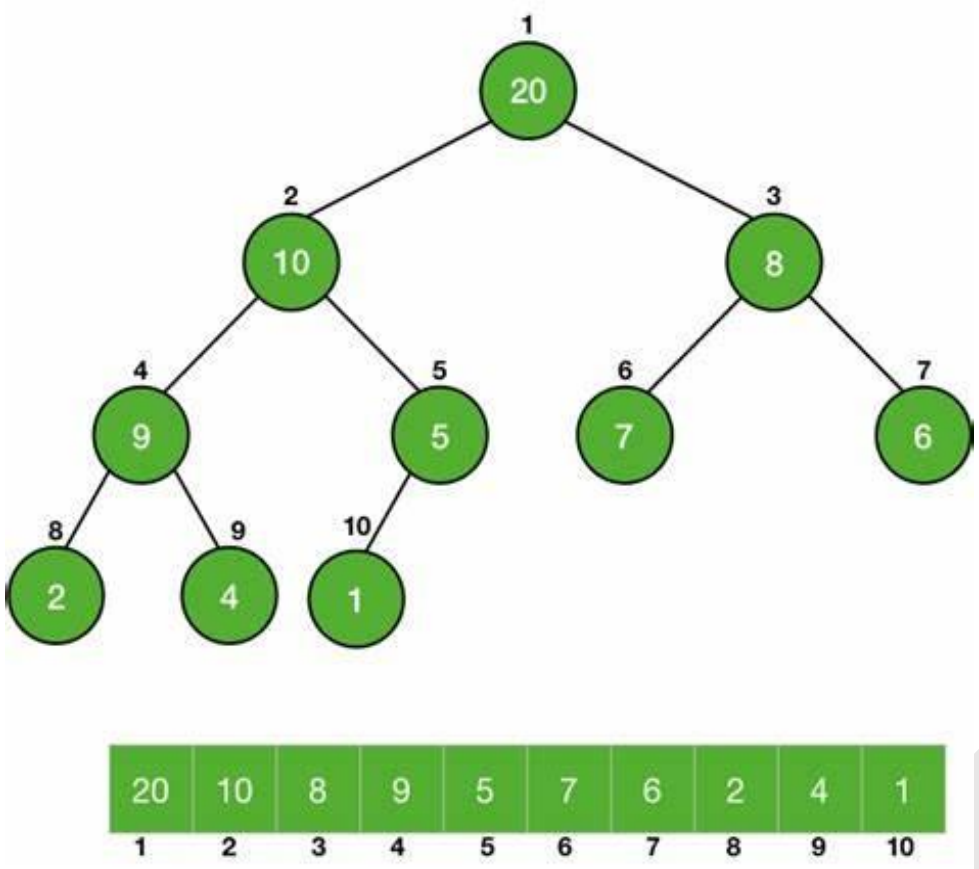
Heap, Huffman, AVL

1.
Heap

Heap concepts/Operations



Heap Concepts



- 1.height:O(logn)
- 2.stored in an array
- 3.complete binary tree

Heap Concepts

- Father > children
- Construction time complexity O(n)

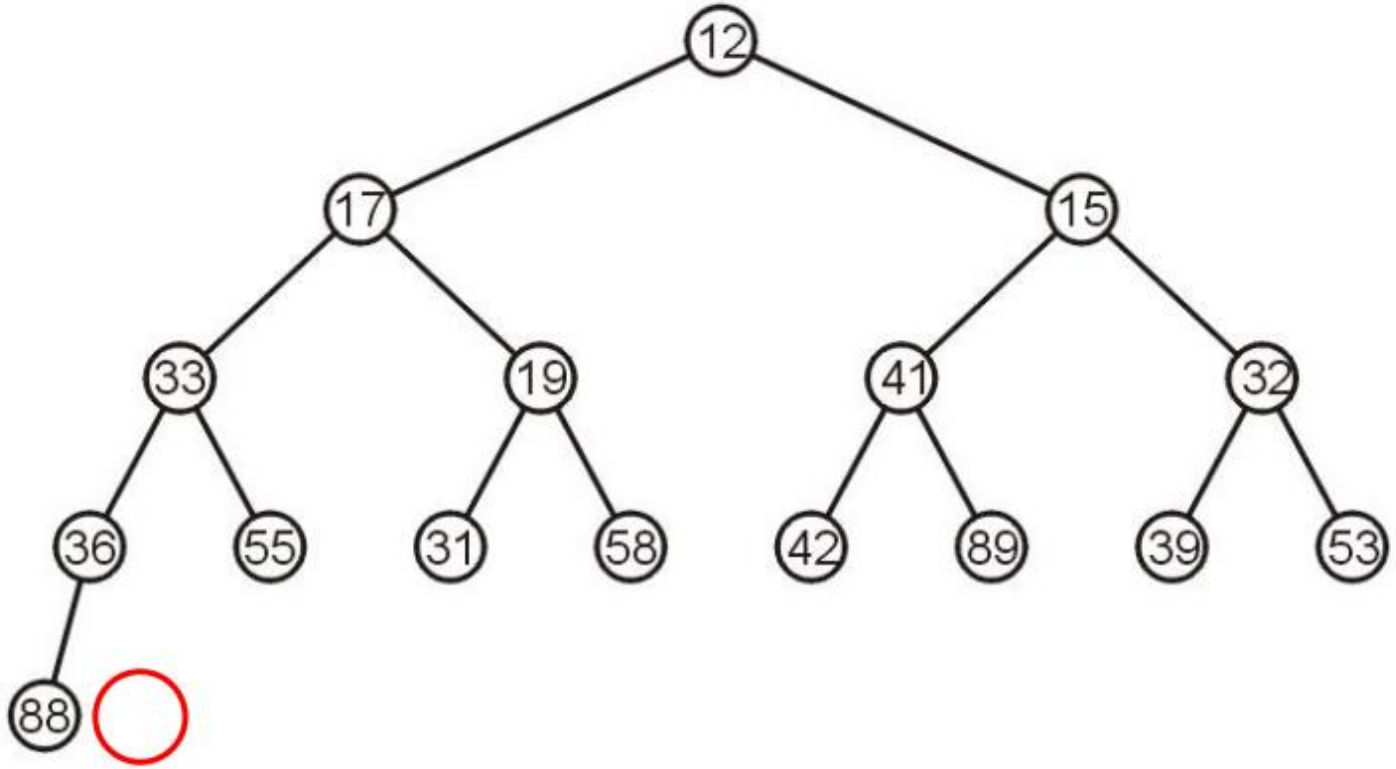
$$\frac{1}{n}\sum_{k=0}^{h}(h-k)2^k = \frac{2^{h+1}-h-2}{n}$$

$$= \frac{n-h-1}{n} = \Theta(1)$$

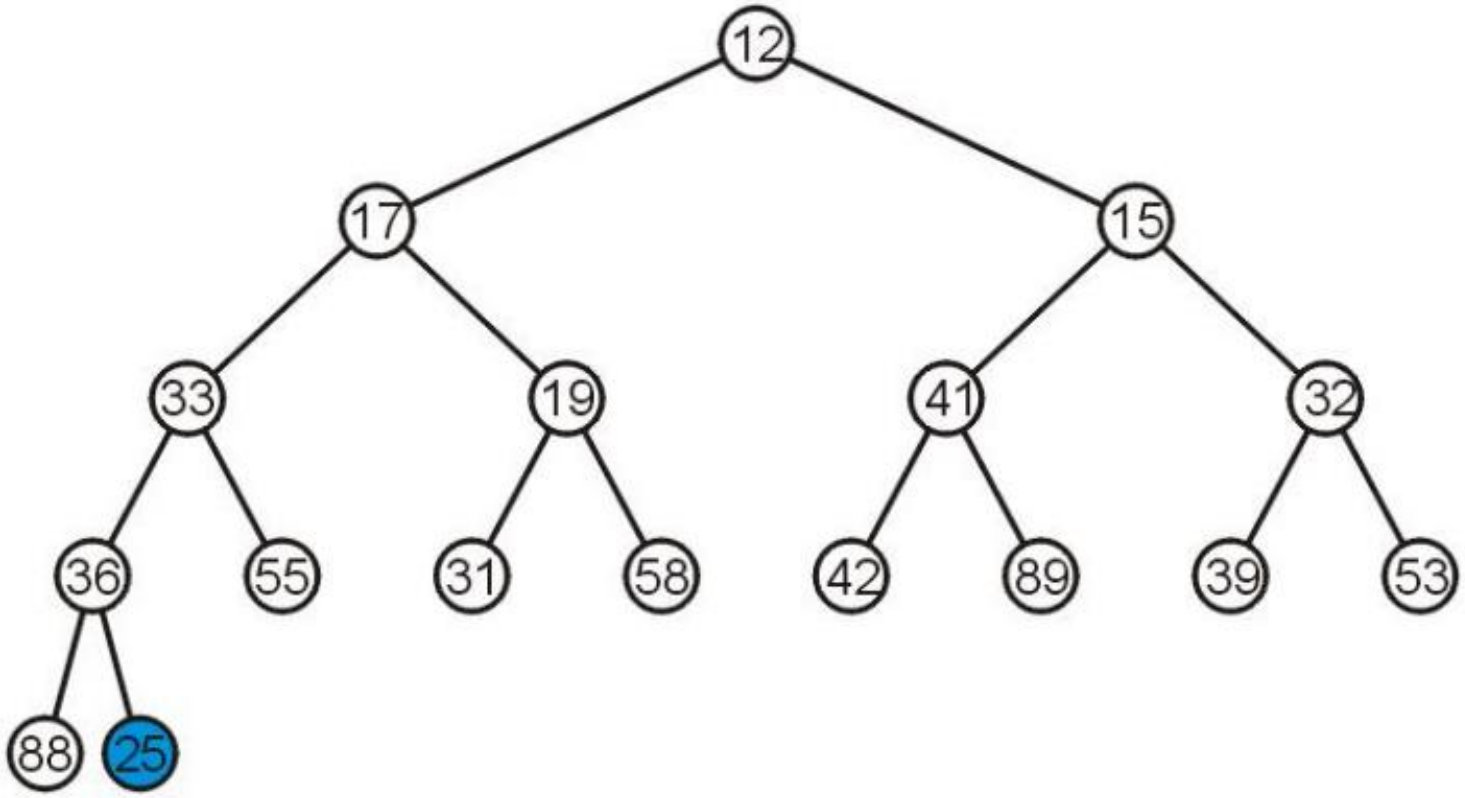
Operations

- 1.precolation
- 2.push
- 3.pop

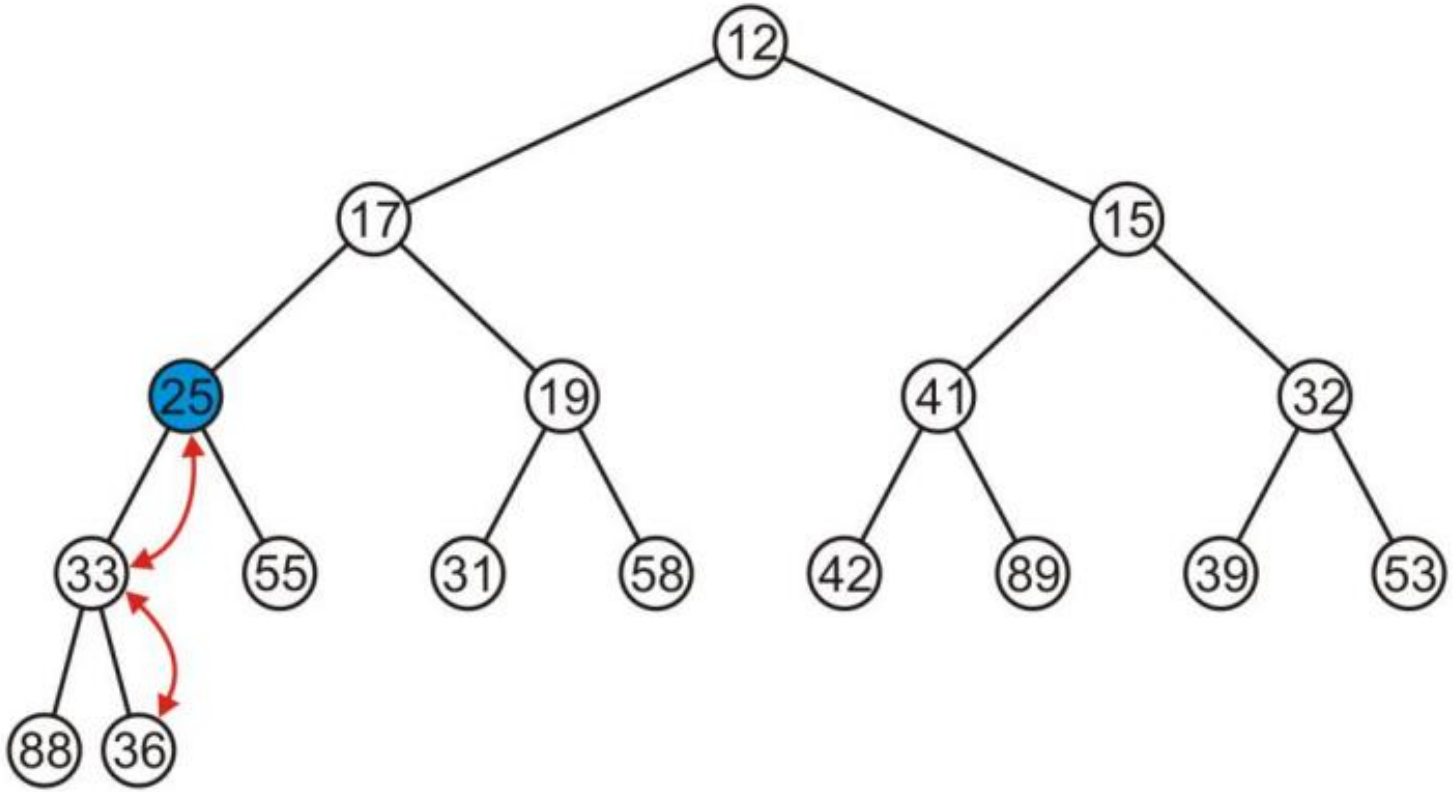
Push



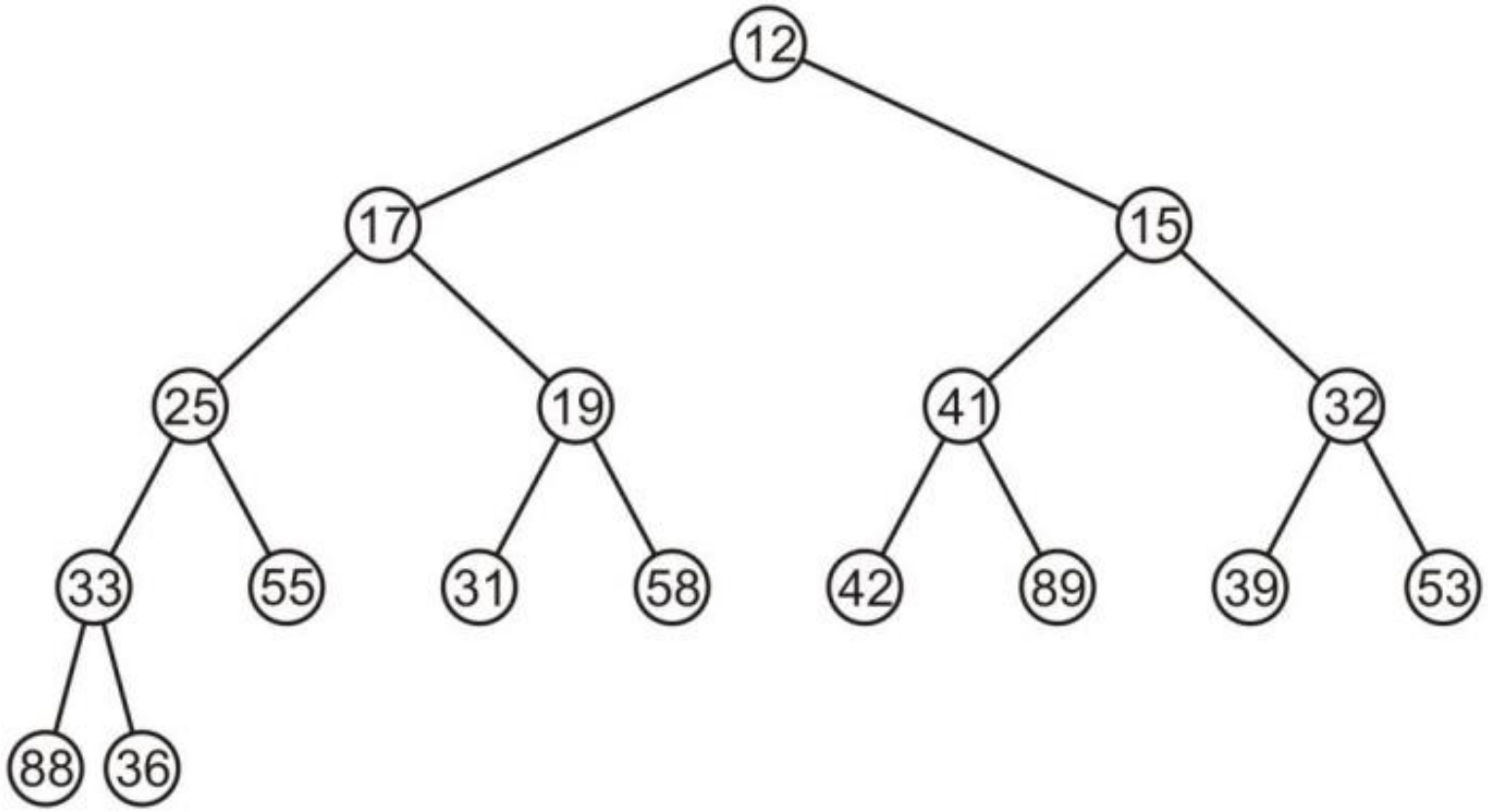
Push



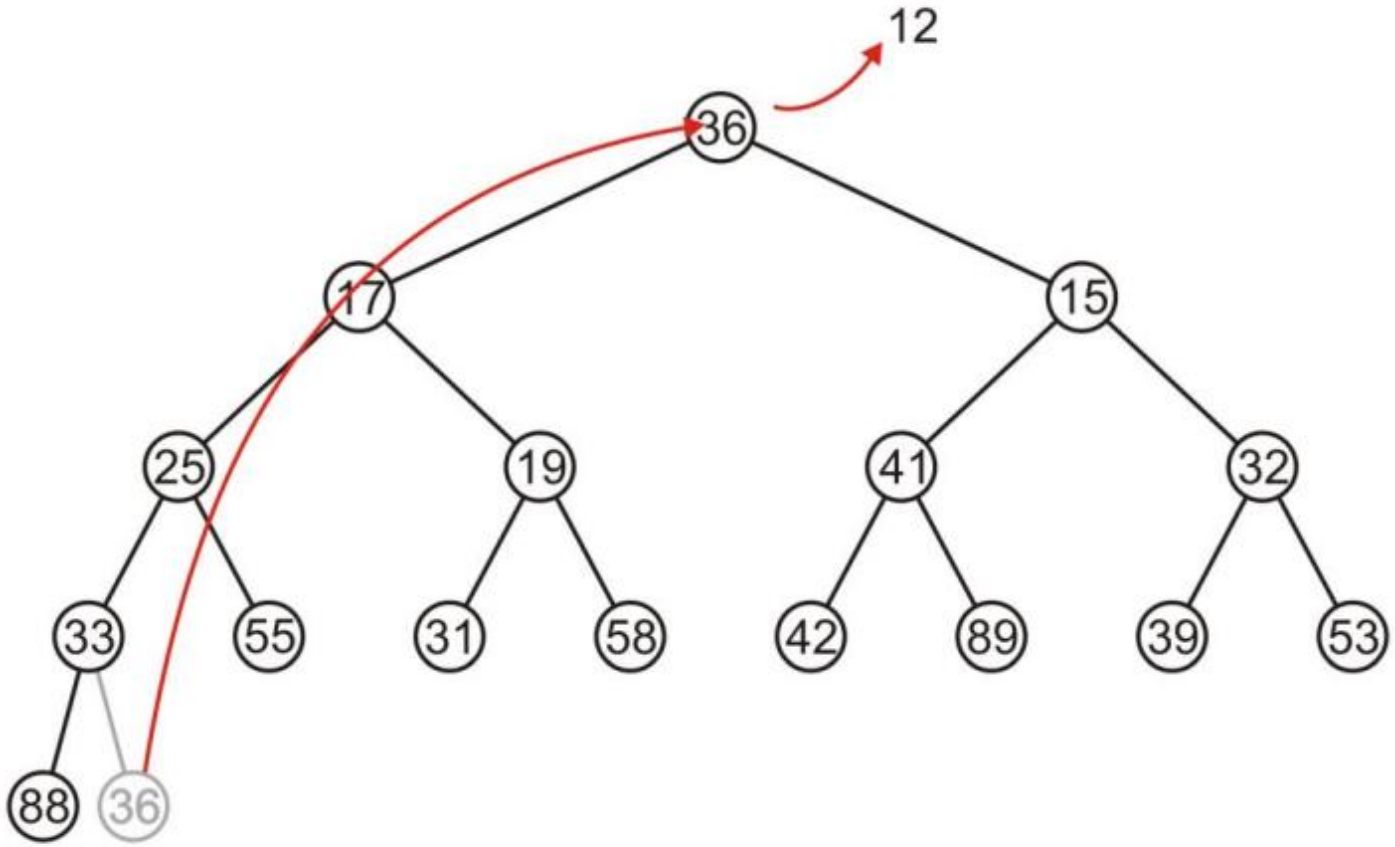
Push



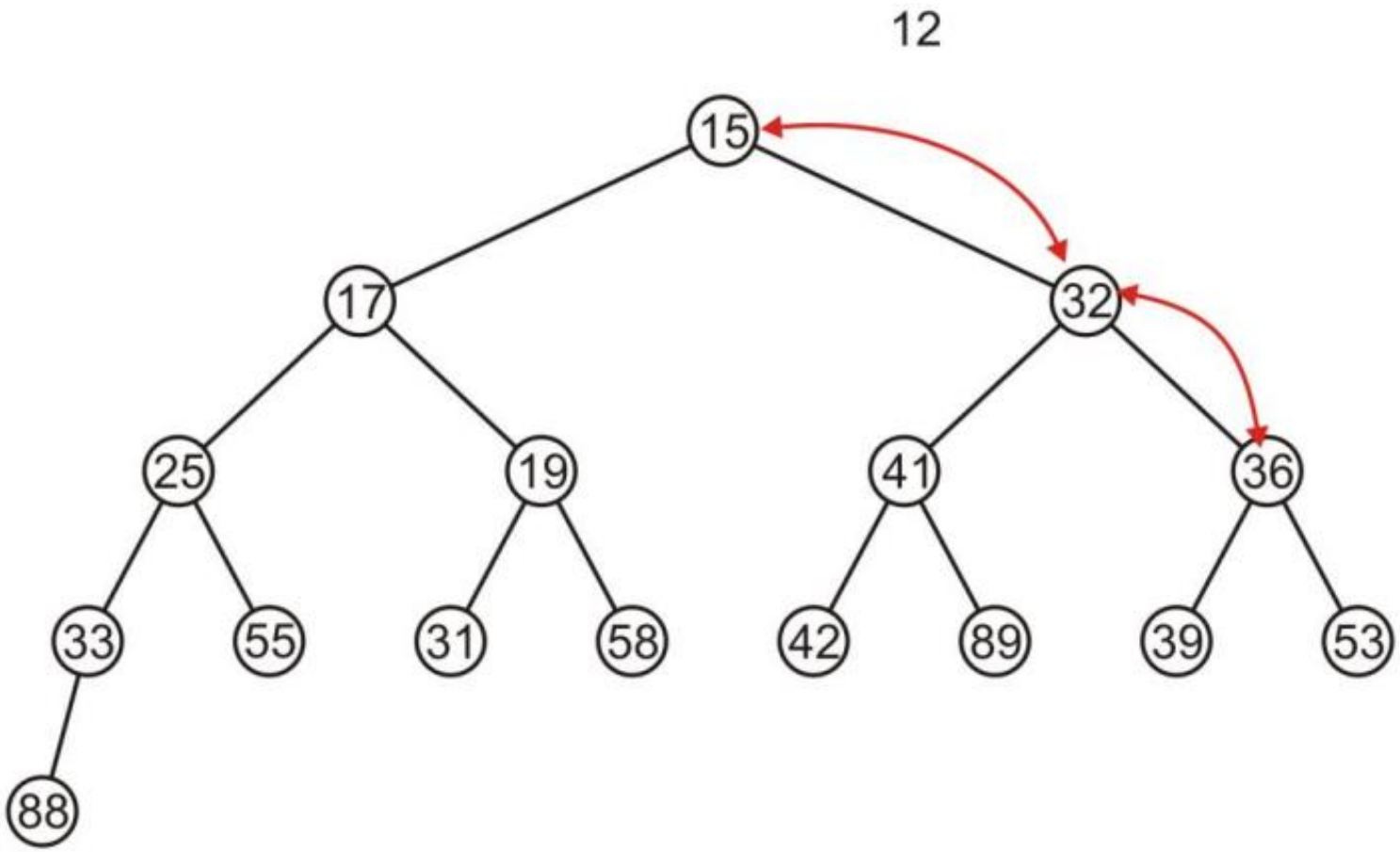
Pop



Pop

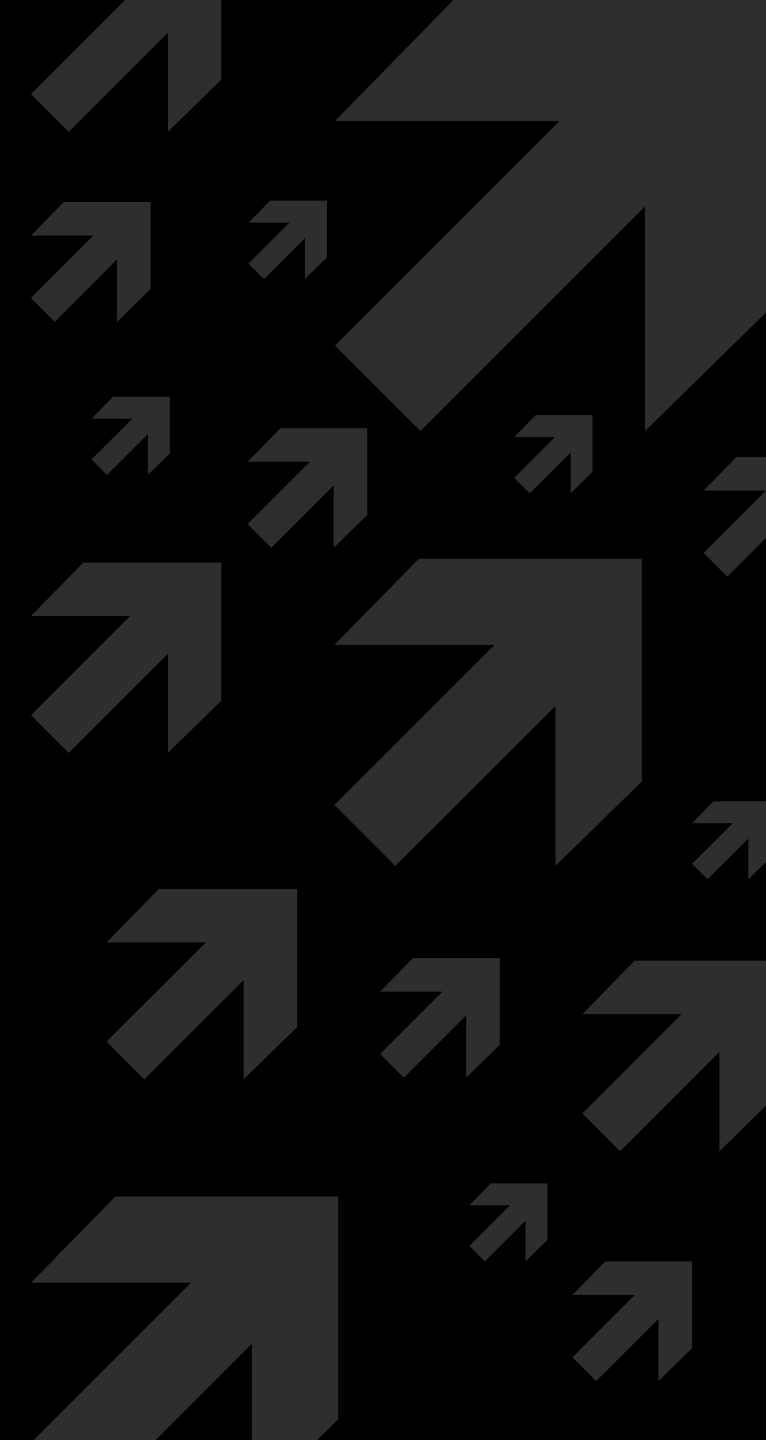


Pop



2.
BST

BST concepts/properties



BST Concepts

- Father > left child; Father < right child
- Subtrees are BSTs
- **Notice**! height: O(n). In best case height: logn. That is why we sometimes write O(h) for search elements.
- in-order gives a sequence of sorted elements

(c) (2') Which of the following statements about the binary heap is/are true? Note that binary heaps mentioned in this problem are implemented by complete binary trees.

A. If a binary tree is a min-heap, then the post-order traversal of this tree is a descending sequence.

B. We have a binary heap of n elements and wish to add n more elements into it while maintaining the heap property. It can be done in $O(n)$.

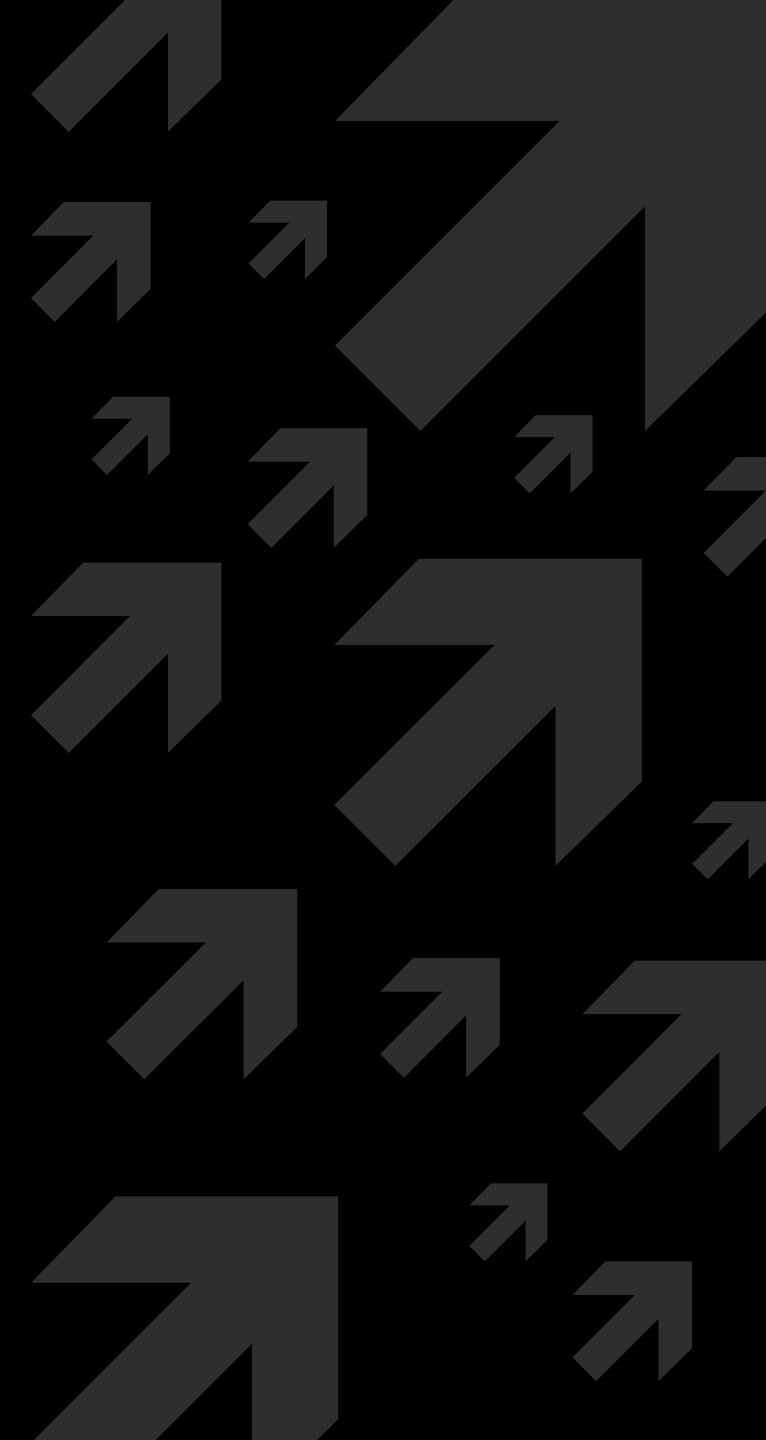
C. There exists a heap with seven distinct elements so that the in-order traversal gives the element in sorted order.

D. If item A is an ancestor of item B in a heap, then it must be the case that the Push operation for item A occurred before the Push operation for item B.

E. None of the above.

3.
Huffman Coding

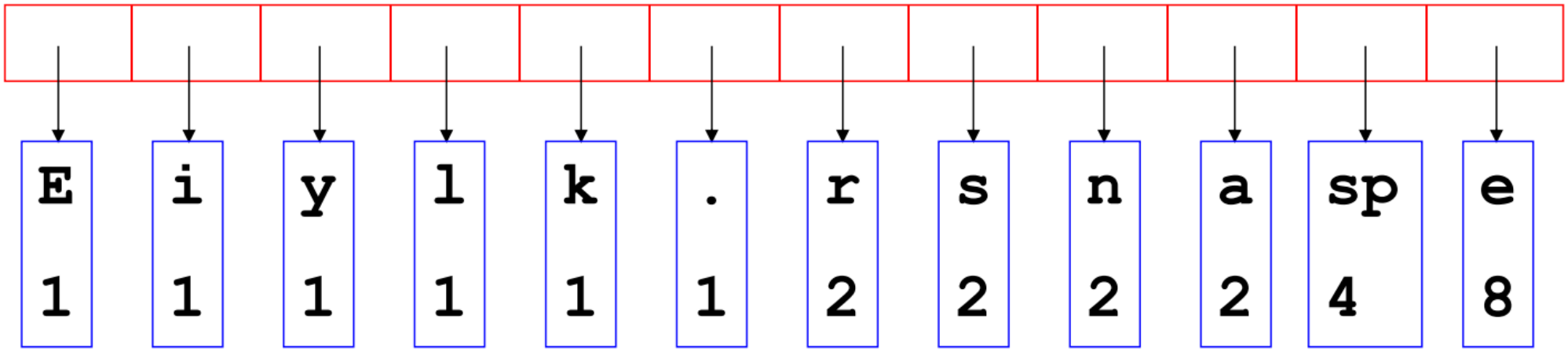
concepts



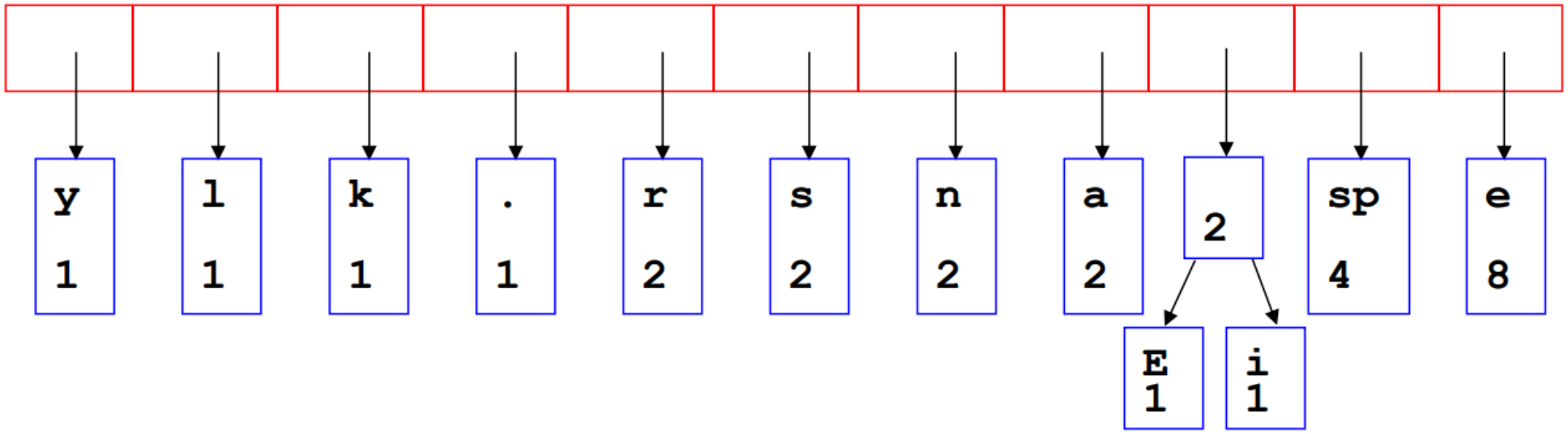
Concepts

- It is usually created through a heap-implemented priority queue in O(nlogn) time.
- More frequent, higher in the tree. That is to say, shorter codes

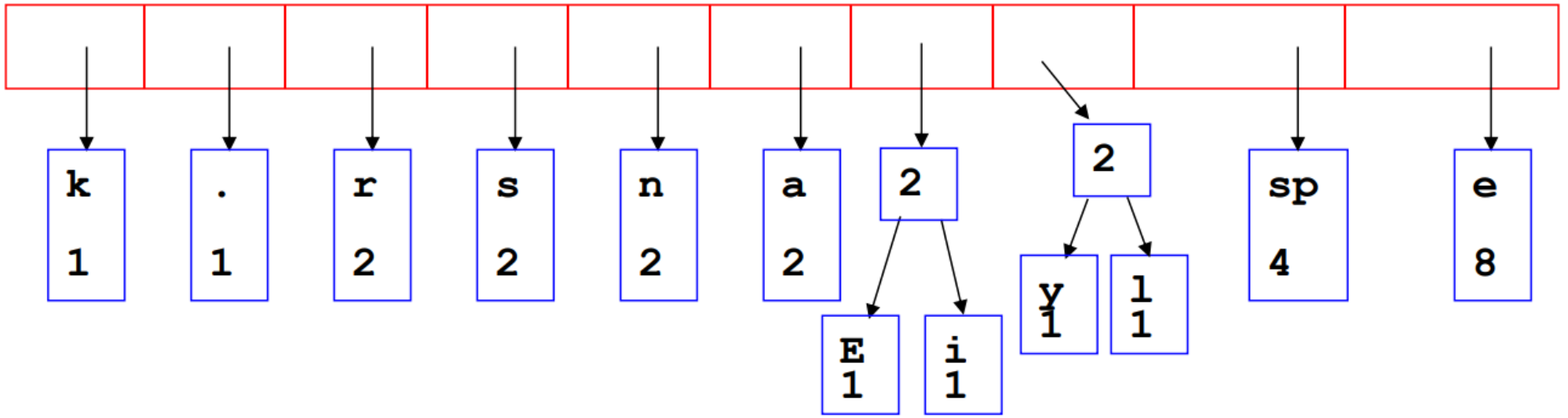
Build a tree



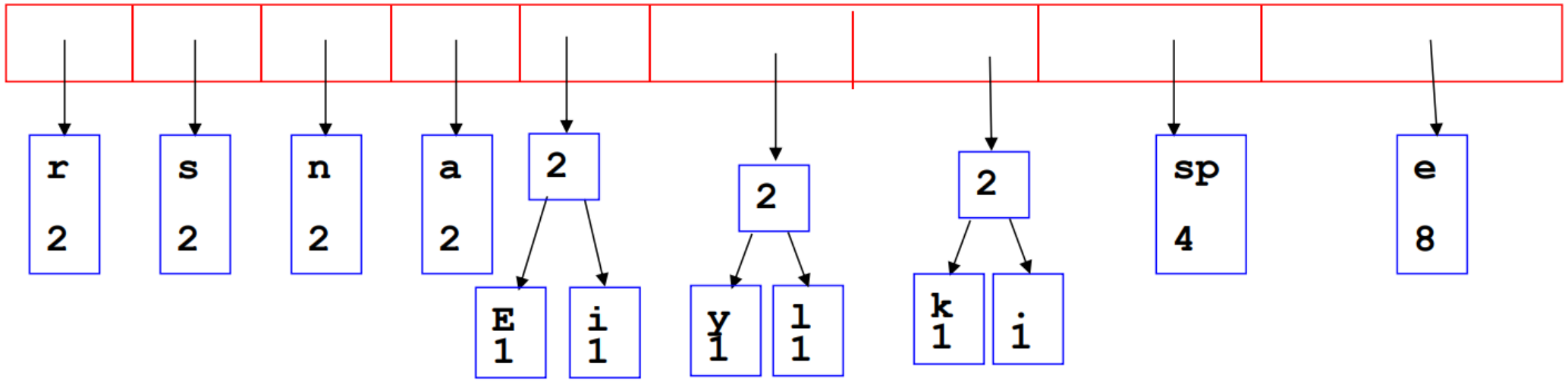
Build a tree



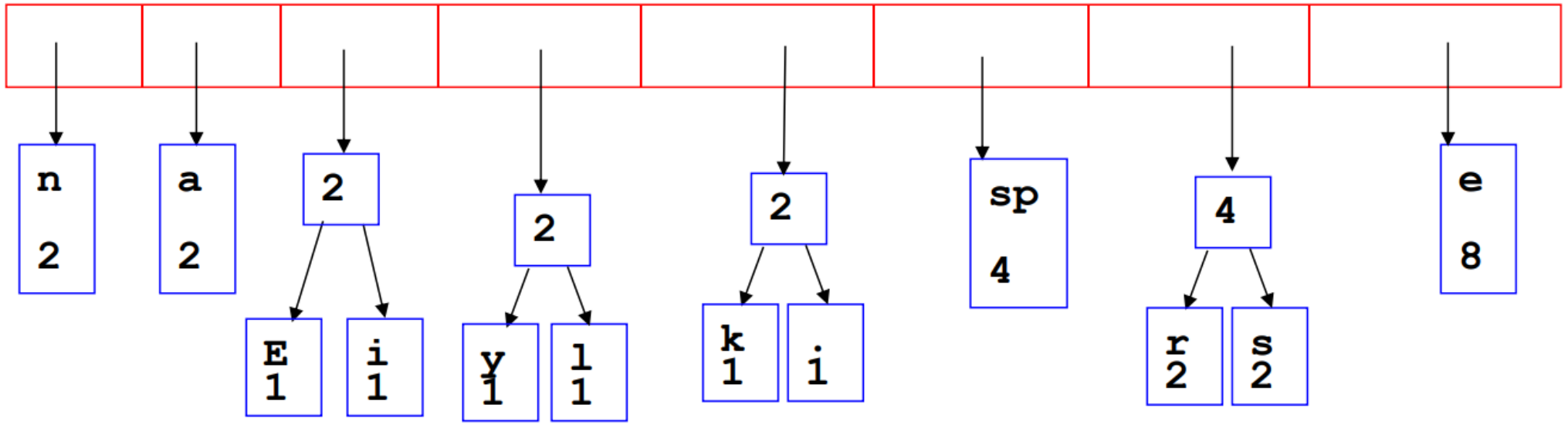
Build a tree



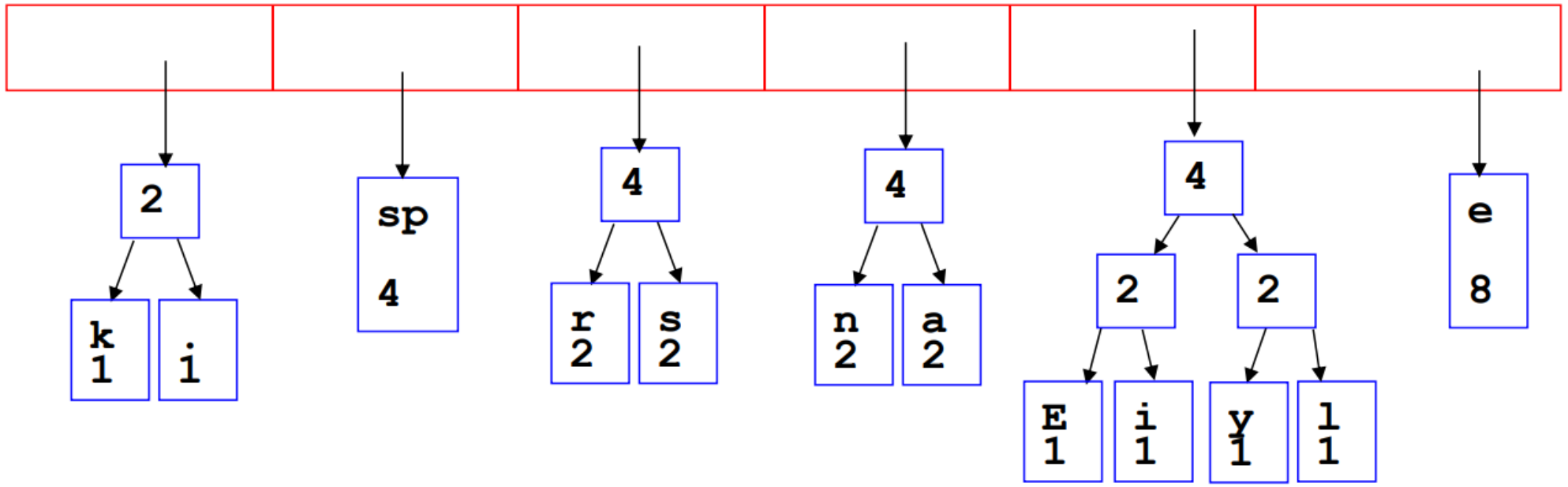
Build a tree



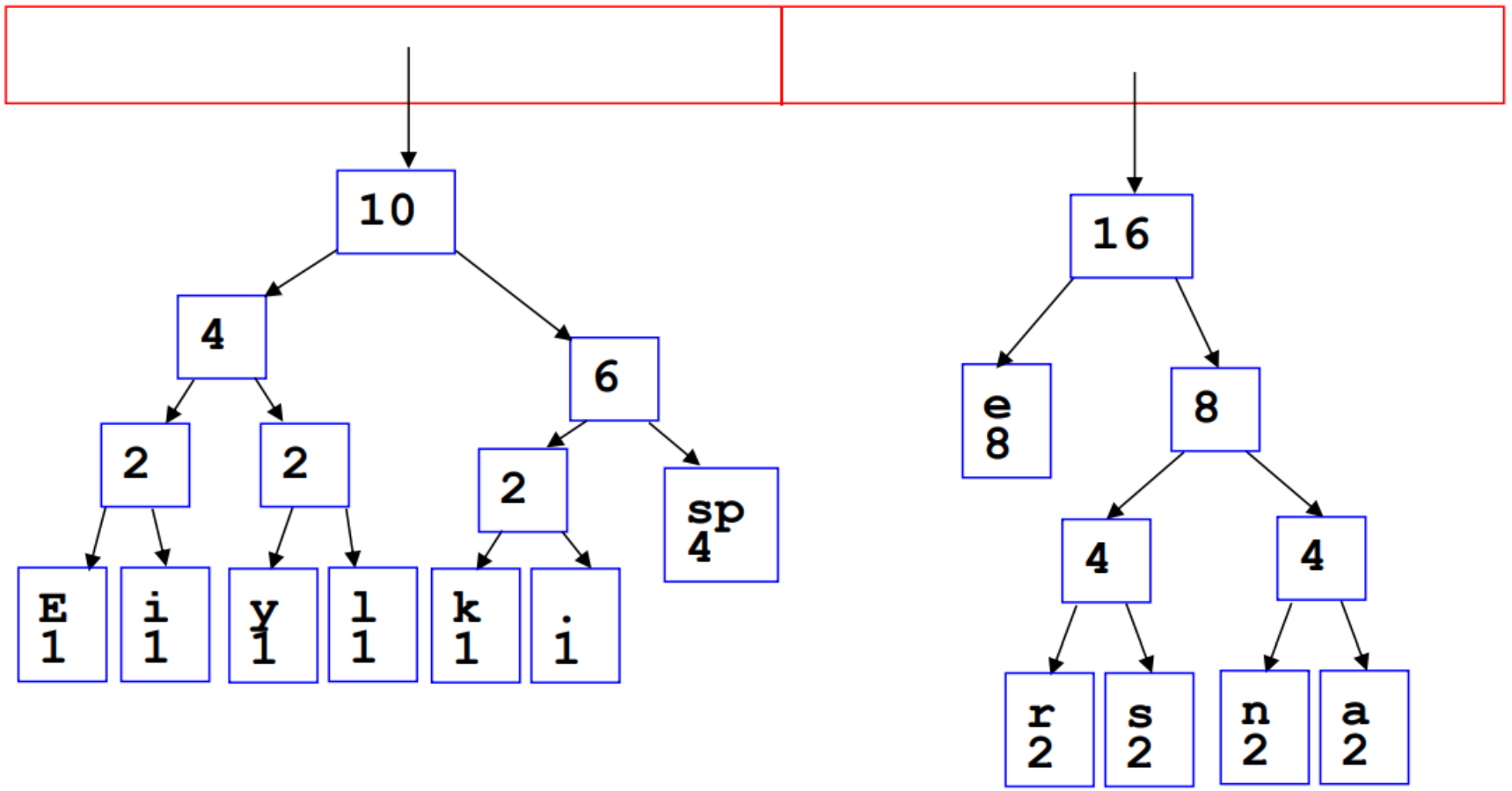
Build a tree



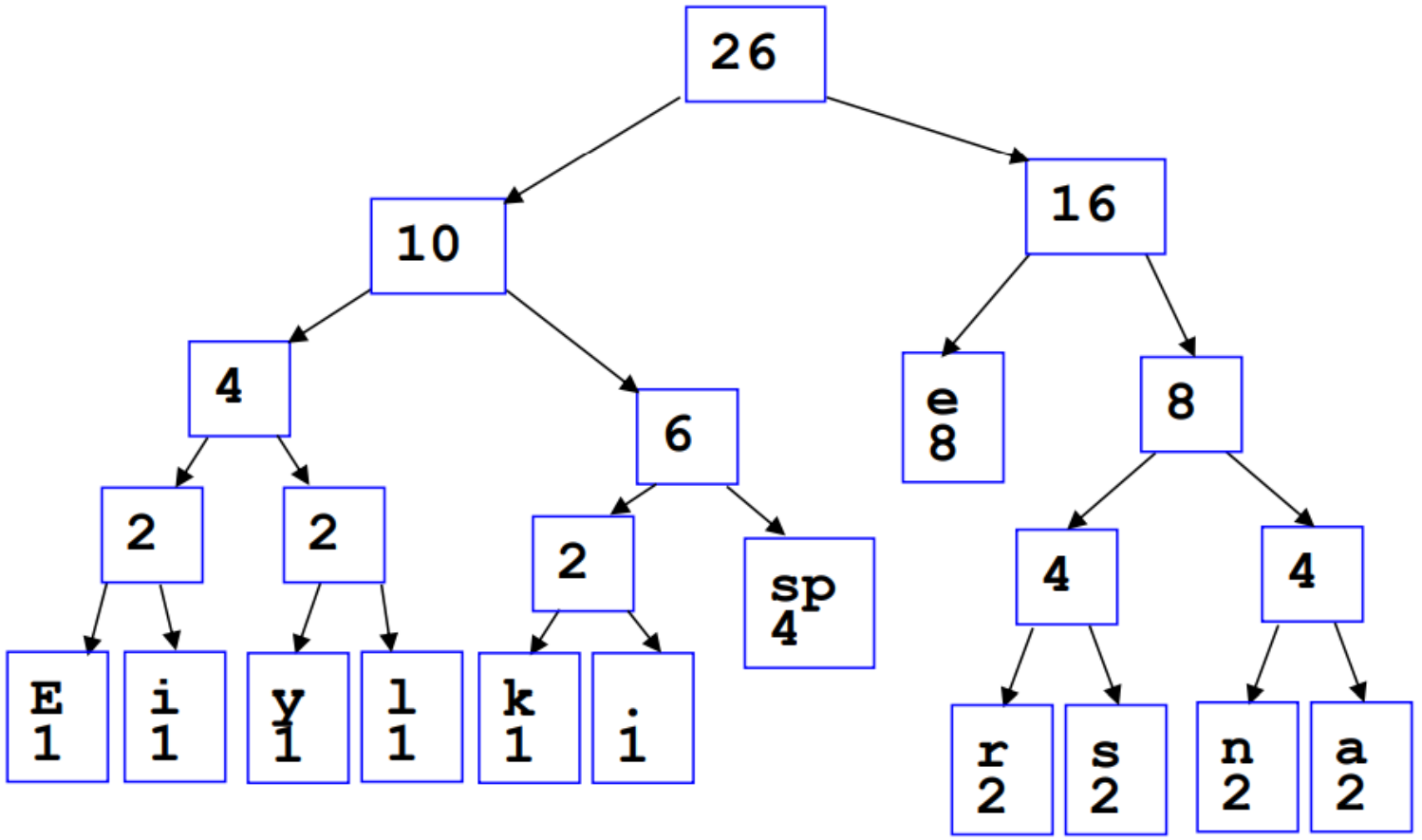
Build a tree



Build a tree

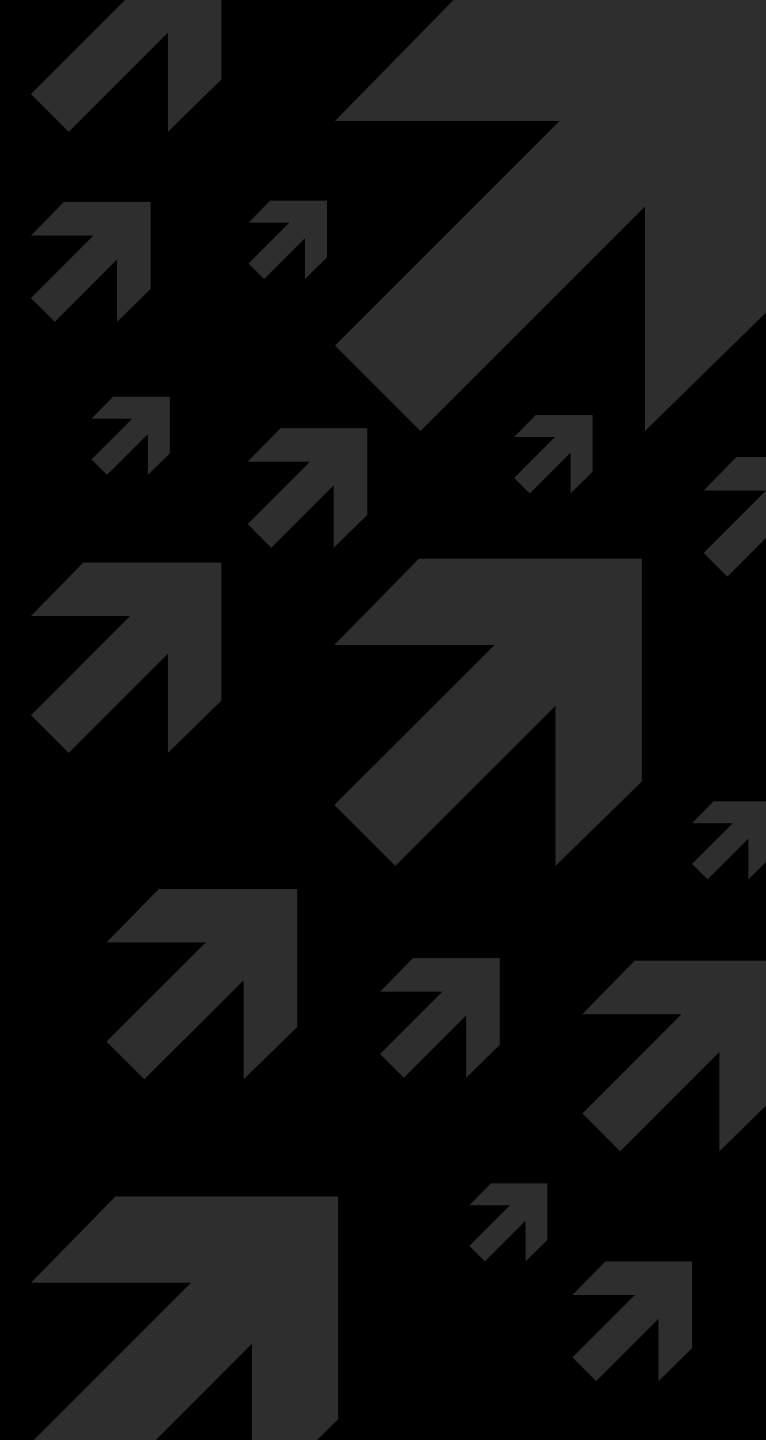


Build a tree



3.
AVL trees

concepts

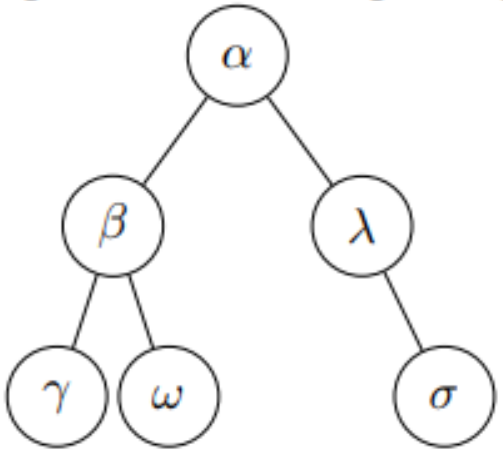


AVL tree Concepts

- balanced BST(with height diff 0 or 1)
- height h with at least $(\frac{1+\sqrt{5}}{2})^h$ nodes, at most $2^{h+1} - 1$
- operations(insert, erase)

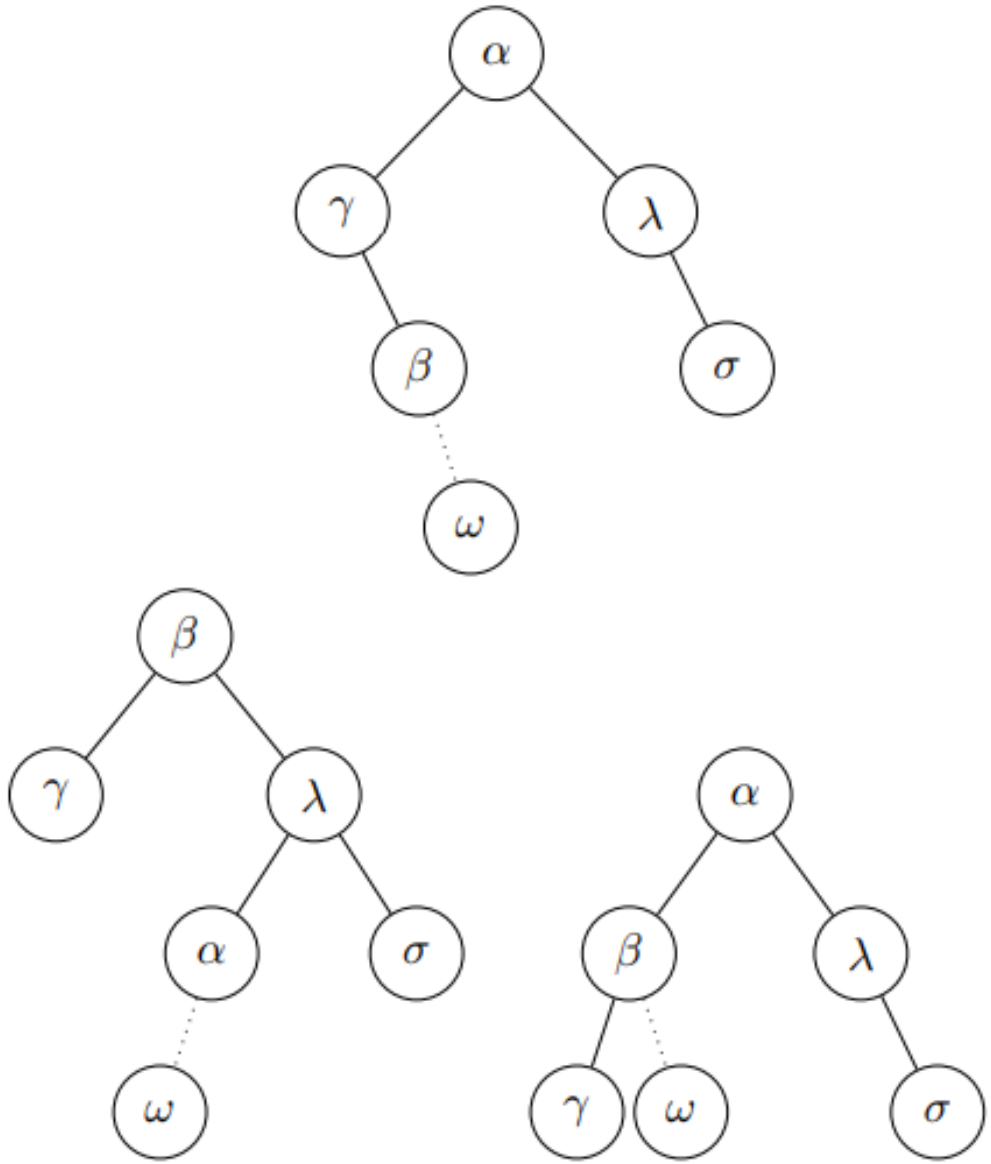
4. (10 points) Magic AVL

Consider the AVL tree below. Each symbol represents an unique object stored in the AVLtree.



The following 4 questions are **independent** of each other, i.e. for each question, your answer should be build on the original AVL Tree above, instead of the AVL Tree from the answer of the last question.

(c) (2 pt) If we know the last object we insert is ω. What is the tree before inserting it? Symbols must be unique. You may only use the 6 printed symbols. If there are more than 3 correct answers, give only 3 correct answers would lead to a full credit.


α
γ
λ
β
σ
ω
β
γ
λ
α
σ
ω
α
β
λ
γ
ω
σ