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§ 8.1 (Page 791)

27. Describe the kernel and range of

(a) the orthogonal projection on the xz-plane.

(b) the orthogonal projection on the yz-plane.

(c) the orthogonal projection on the plane defined by the equation $y = x$.

(a) y-axis xz-plane

(b) x-axis yz-plane

(c) the line through the origin which is orthogonal to plane y=x plane y=x

28. Let V be any vector space, and let $T: V \to V$ be defined by $T(\mathbf{v}) = 3\mathbf{v}$.

(a) What is the kernel of T?

(b) What is the range of T?

(a) $\vec{0}$

(b) V

30. Let A be a 7×6 matrix such that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, and let $T: R^6 \to R^7$ be multiplication by A. Find the rank and nullity of T.

Since A is a 7x6 matrix such that $Ax=0$ has only the trivial solution

$\operatorname{rank}(A) = 0$ so $\operatorname{rank}(T) = 0$

the nullity $(T) = 6$

38. For a positive integer $n > 1$, let $T: M_{nn} \to R$ be the linear transformation defined by $T(A) = \operatorname{tr}(A)$, where A is an $n \times n$ matrix with real entries. Determine the dimension of $\ker(t)$.

$\ker(T) = \{A \in M_{nn} ; tr(A) = 0\}$

$\dim \ker(T) = n^2 - 1$

40. (***Calculus required***) Let $V = C[a, b]$ be the vector space of functions continuous on $[a, b]$, and let $T: V \to V$ be the transformation defined by

$$T(\mathbf{f}) = 5f(x) + 3\int_a^x f(t)dt$$

Is T a linear operator?

$T(kf) = 5kf(x) + 3k\int_0^x f(t)\,dt = kT(f)$

$\therefore T(\alpha f + \beta g) = 5\alpha f(x) + 5\beta g(x) + 3\int_a^x (\alpha f(t) + \beta g(t))dt$

$= 5\alpha f(x) + \alpha 3\int_a^x f(t)dt + 5\beta g(x) + 3\beta\int_a^x g(t)dt$

$= \alpha T(f) + \beta T(g)$ So T is a linear operator.

41. (***Calculus required***) Let $D: P_3 \to P_2$ be the differentiation transformation $D(\mathbf{p}) = p'(x)$. What is the kernel of D?

ker(D) is a set of all constants.

§ 8.2 (Page 806)

5. As indicated in the accompanying figure, let $T: R^2 \to R^2$ be the orthogonal projection on the line $y = x$.

(a) Find the kernel of T.

(b) Is T one-to-one? Justify your conclusion.

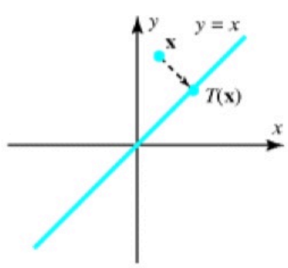

Figure Ex-5

(a) $\ker(T) = (k(-1,1)) = \{k(1,-1)\}$

(b) since $\ker(T) \neq \{\vec{0}\}$

then T is not one-to-one

13. (***Calculus required***) Let V be the vector space $C^1[0, 1]$ and let $T: V \to R$ be defined by

$$T(\mathbf{f}) = f(0) + 2f'(0) + 3f'(1)$$

Verify that T is a linear transformation. Determine whether T is one-to-one, and justify your conclusion.

Since $T(f)$ consists of all constants

then $T(\alpha f + \beta g) = \alpha T(f) + \beta T(g)$ holds

so T is a linear transformation

T is not one to one

eg. $f(x) = x^2(x-1)^2$

§ 8.4 (Page 832)

12. Let $T_1: P_1 \to P_2$ be the linear transformation defined by

$$T_1(p(x)) = xp(x)$$

and let $T_2: P_2 \to P_2$ be the linear operator defined by

$$T_2(p(x)) = p(2x+1)$$

Let $B = \{1, x\}$ and $B' = \{1, x, x^2\}$ be the standard bases for P_1 and P_2.

(a) Find $[T_2 \circ T_1]_{B',B}$, $[T_2]_{B'}$, and $[T_1]_{B',B}$.

(b) State a formula relating the matrices in part (a).

(c) Verify that the matrices in part (a) satisfy the formula you stated in part(b).

(a) $(T_2 \circ T_1)(P(x)) = (2x+1)P(2x+1)$

$(T_2 \circ T_1)(1) = 2x+1$

$(T_2 \circ T_1)(x) = (2x+1)(2x+1) = 1 + 4x + 4x^2$

$[T_2 \circ T_1]_{B',B} = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 0 & 4 \end{bmatrix}$

$T_1(1) = x$
$T_1(x) = x^2$

$[T_1]_{B',B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

$T_2(1) = 1$
$T_2(x) = 2x+1$

$T_2(x^2) = 1 + 4x + 4x^2$

$[T_2]_{B'B} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{bmatrix}$

(b) $[T_2 \circ T_1]_{B'B} = [T_2]_{B',B}[T_1]_{B',B}$

(c) $[T_2]_{B',B}[T_1]_{B',B}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 0 & 4 \end{bmatrix} = [T_2 \circ T_1]_{B',B}$

13. Let $T_1 : P_1 \to P_2$ be the linear transformation defined by

$$T_1(c_0 + c_1 x) = 2c_0 - 3c_1 x$$

and let $T_2 : P_2 \to P_3$ be the linear transformation defined by

$$T_2\left(c_0 + c_1 x + c_2 x^2\right) = 3c_0 x + 3c_1 x^2 + 3c_2 x^3$$

Let $B = \{1, x\}$, $B'' = \left\{1, x, x^2\right\}$, and $B' = \left\{1, x, x^2, x^3\right\}$.

(a) Find $[T_2 \circ T_1]_{B',B}$, $[T_2]_{B',B''}$, and $[T_1]_{B'',B}$.

(b) State a formula relating the matrices in part (a).

(c) Verify that the matrices in part (a) satisfy the formula you stated in part(b).

(a) $T_1(1) = 2$

$T_1(x) = -3x$

$$[T_1]_{B'',B} = \begin{bmatrix} 2 & 0 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$

$T_2(1) = 3x$

$T_2(x) = 3x^2$

$T_2(x^2) = 3x^3$

$$[T_2]_{B',B''} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$[T_2 \circ T_1](1) = 6x$

$[T_2 \circ T_1](x) = -9x^2$

$$[T_2 \circ T_1]_{B',B} = \begin{bmatrix} 0 & 0 \\ 6 & 0 \\ 0 & -9 \\ 0 & 0 \end{bmatrix}$$

(b). $[T_2 \circ T_1]_{B'B} = [T_2]_{B',B''}[T_1]_{B'',B}$

(c). $$[T_2]_{B',B''}[T_1]_{B'',B} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -3 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 6 & 0 \\ 0 & -9 \\ 0 & 0 \end{bmatrix}$$