EE 150L
Signals and Systems Lab

Lab5 Sampling and Reconstruction

Date Performed: 11•17

Class Id: 1A-105

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1. (a) If $\mathcal{F}[f(t)] = F(\omega)$, verify that $\mathcal{F}[f(t)\cos(\omega_0 t)] = \frac{1}{2}[F(\omega+\omega_0) + F(\omega-\omega_0)]$.

(b) Read the **Nyquist Sampling Theorem** section in the PDF material and briefly explain how the conclusions in (a) relates to Nyquist's Sampling Theorem.

(a)

$$F\left[f(t)\cdot\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right] = F\left[\frac{e^{j\omega_0 t}}{2}f(t)\right] + F\left[\frac{e^{-j\omega_0 t}}{2}f(t)\right]$$

$$= \frac{1}{4\pi}F(e^{j\omega_0 t}) * F[f(t)] + \frac{1}{4\pi}F(e^{-j\omega_0 t}) * F[f(t)]$$

$$= \frac{1}{4\pi}\cdot 2\pi\,\delta(\omega-\omega_0)F(\omega) + \frac{1}{4\pi}\cdot 2\pi\,\delta(\omega+\omega_0)F(\omega)$$

$$= \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$$

(b)

Time Domain $x_s(t)$	Frequency Domain $X_s(\omega)$
$2x(t)\cos\omega_s t$	$X(\omega+\omega_s) + X(\omega-\omega_s)$

let $x(t) = f(t)$ $\omega_s = \omega_0$

the conclusions in (a) relates to Nyquist's Sampling Theorem.

2. According to the Nyquist sampling theorem, the sampling frequency must not be less than Nyquist rate in order to accurately represent the signal before sampling. Read the PDF material to understand the Nyquist rate. Find the Nyquist rate of the following signals and give reasons for your judgment.

(a) $x(t) = \cos(2000\pi t) + \sin(5000\pi t)$

(b) $x(t) = \frac{\sin(500\pi t)}{\pi t}$

(a) $X(j\omega) = \pi\delta(\omega + 2000\pi) + \pi\delta(\omega - 2000\pi) + j\pi\delta(\omega + 5000\pi) - j\pi\delta(\omega - 5000\pi)$

$X(j\omega) = 0 \quad |\omega| > 5000\pi$

So Nyquist rate is $\omega = 10000\pi$ rad/s.

(b) $X(j\omega) = u(\omega + 500\pi) - u(\omega - 500\pi)$

$X(j\omega) = 0 \quad |\omega| > 500\pi$

So Nyquist rate is $\omega = 1000\pi$ rad/s.