8.3

1. (1) $\frac{x^2}{4}+\frac{y^2}{9}=1$ 绕z / $\frac{x^2}{4}+\frac{z^2}{9}=1$ 绕x

(2) $x^2+y^2=1$ 绕 x/y

或 $x^2+z^2=1$ 绕 x/z.

(4) $x^2-\frac{y^2}{4}=1$ / $z^2-\frac{y^2}{4}=1$ 绕y

(6) $x^2-y^2=1$ / $x^2-z^2=1$ 绕x

(7) $x^2=4z$ / $y^2=4z$ 绕z.

2. (4) 双曲线 双曲柱面 (5) 抛物线 抛物柱面 (6) 点，直线

3. (1) $x^2+y^2-\frac{z^2}{4}=1$ 单叶双曲面

(2) $y^2+z^2=\sin^2 x$ $(0\le x\le\pi)$.

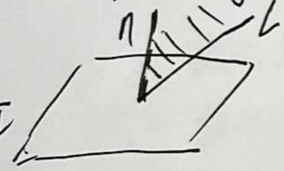
(3) $4x^2+9y^2+4z^2=36$. 椭球面.

4. 设 $(x,y,z)$ 为曲面上一点 P.

P到轴 $\begin{cases} y=z \\ x=0 \end{cases}$ 距离为 $\frac{\sqrt{(z-y)^2+2x^2}}{\sqrt{2}}$ ⇒ 母线长.

由圆锥体几何关系: $\frac{\text{P到顶点}(0,1,1)\text{距离}}{\sqrt{10}}=\frac{\sqrt{x^2+(y-1)^2+(z-1)^2}}{\sqrt{10}}=d$

$\Rightarrow 9x^2+4y^2+4z^2+2y+2z-10yz-2=0$

5. 

$S=(1,1,-1)$, $n=(1,-1,2)$. ~~交点$P_0$~~ L过$P_0(1,0,1)$

$S\times n=(1,-3,-2)$

L与n构成平面: $x-3y-2z+1=0$

⇒ 投影直线 $L_0$: $\begin{cases} x-3y-2z+1=0 \\ x-y+2z-1=0 \end{cases} \Leftrightarrow \frac{x-2}{4}=\frac{y-1}{2}=-z$

曲面为: $4x^2-17y^2+4z^2+2y-1=0$

~~$4x^2+4z^2=17y^2-2y+1$~~

6. 略.

8. $x^2+y^2+z^2=(z-4)^2 \Leftrightarrow x^2+y^2+8z-16=0$. 旋转抛物面.

9. 交线: $\begin{cases} x^2+y^2+4z^2=1 \\ x^2=y^2+z^2 \end{cases} \xrightarrow{\text{消}z} 5x^2-3y^2=1$.

故柱面: $\begin{cases} 5x^2-3y^2=1 \\ \cancel{z=0} \end{cases}$

10. 设球面：$x^2+y^2+(z-z_0)^2=R^2$

代入 $z=0 \Rightarrow x^2+y^2=R^2-z_0^2=16$

代入 $(0,-3,1) \Rightarrow 9+(1-z_0)^2=R^2$

$\Rightarrow \begin{cases} z_0=-3 \\ R=5 \end{cases}$

$\Rightarrow x^2+y^2+(z+3)^2=25$.

11. $\begin{cases} \frac{x^2}{16}+\frac{y^2}{4}-\frac{z^2}{5}=1 \\ x-2z+3=0 \end{cases} \xrightarrow{\text{消}z} x^2-24x+20y^2-116=0$

~~故~~ 故在 $xy$ 平面上~~曲~~投影为：$\begin{cases} x^2-24x+20y^2-116=0 \\ z=0 \end{cases}$

12. (1). $xy=h$. ① $h\neq 0$，双曲线.

② $h=0$，~~x轴或y轴或原点~~ x轴.y轴.

~~(2) $xy=h$ $xy=h$~~

(2) $xy=z$：双曲~~抛物~~抛物面.

第八章综合习题：

2: $\begin{pmatrix} 1 & -2 & 2 \\ -1 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ 线性相关

3: 设 $\forall \vec{a},\vec{b},\vec{c},\vec{d}\in\mathbb{R}^3$.

①若 $\vec{a},\vec{b},\vec{c}$ 线性无关. 则 $\exists\, k_1,k_2,k_3$ 使得 $\vec{d}=k_1\vec{a}+k_2\vec{b}+k_3\vec{c}$

即 $\vec{d}$ 一定与 $\vec{a},\vec{b},\vec{c}$ 线性相关.

② 若 $\vec{a},\vec{b},\vec{c}$ 线性相关. ~~则~~ 显然满足题意

证毕.

4: (1) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ 线性无关.为一组基.

(2) $\det\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & x \\ -1 & 1 & 2 \end{pmatrix}=3-3x=0 \Rightarrow x=1$

5. $|a\times b| = |a||b|\sin\langle \vec{a},\vec{b}\rangle = \sqrt{3}$

$|(a+b)\times(a-b)| = |\vec{a}\times\vec{a} - \vec{a}\times\vec{b} + \vec{b}\times\vec{a} - \vec{b}\times\vec{b}| = 2|\vec{a}\times\vec{b}| = 2\sqrt{3}$

6. $a\times(b\times c) = b(a\cdot c) - c(a\cdot b)$

$\Rightarrow$ ~~$a\times(b\times c)+$~~

$(a\times b)\times c + (b\times c)\times a + (c\times a)\times b$

$= b(a\cdot c) - a(b\cdot c) + c(a\cdot b) - b(a\cdot c) + a(c\cdot b) - c(a\cdot b)$

$= 0$

7. 设准线上任一点 $M_0(x_0, y_0, z_0)$. 作方向为 $(2,1,1)$ 的直线:

$$\begin{cases} x = x_0 + 2t \\ y = y_0 + t \\ z = z_0 + t \end{cases},\ t\in\mathbb{R}.$$

又 $M_0$ 在准线上 $\Rightarrow \begin{cases} (y-t)^2 + (z-t)^2 = 1 \\ x - 2t = 1 \end{cases}$

$\Rightarrow x^2 + 2y^2 + 2z^2 - 2xy - 2xz - 2x + 2y + 2z - 1 = 0$

8. 设 $M_0(x_0, y_0, z_0)$ 为准线上任一点 $M_0$.

过 $M_0$ 与 $(2,1,1)$ 的直线为:

$$\frac{x-2}{x_0-2} = \frac{y-1}{y_0-1} = \frac{z-1}{z_0-1} \iff \begin{cases} x = (x_0-2)t + 2 \\ y = (y_0-1)t + 1 \\ z = (z_0-1)t + 1 \end{cases}\ t\in\mathbb{R}.$$

故 $\begin{cases} x_0 = \frac{x-1}{t} + 2 \\ y_0 = \frac{y-1}{t} + 1 \\ z_0 = \frac{z-1}{t} + 1 \end{cases}$ 代入准线方程, 消 $t$

$\Rightarrow x^2 + y^2 + z^2 - 2xy - 2zx + 2y + 2z - 2 = 0$

10: 设 $\forall M_0 \in$ 圆. $\begin{cases} (x_0-2)^2 + y_0^2 = 1 \\ z_0 = 0 \end{cases}$

旋转后, $M_0$ 形成一个圆. 此圆的方程为: $\begin{cases} x^2 + z^2 = x_0^2 \\ y = y_0 \end{cases}$

消 $(x_0, y_0, z_0)$

$\Rightarrow$ 一般式: ~~$\sqrt{x^2+z^2} = \sqrt{x_0^2+y_0^2+z_0^2}$~~

$(\pm\sqrt{x^2+z^2} - 2)^2 + y^2 = 1$

参数式: 令 $x^2 + z^2 = (2+\cos\theta)^2$, $y^2 = \sin^2\theta$

$$\Rightarrow \begin{cases} x = (2+\cos\theta)\cos\varphi \\ y = \sin\theta \\ z = (2+\cos\theta)\sin\varphi \end{cases}$$

12: 三条轴公共点 $O'(2,2,2)$.

方向分别为 $n_1=(-1,2,2)$, $n_2=(2,-1,2)$, $n_3=(2,2,-1)$

取一组新的基: $i'=(-\frac{1}{3},\frac{2}{3},\frac{2}{3})$, $j'=(\frac{2}{3},-\frac{1}{3},\frac{2}{3})$, $k'=(\frac{2}{3},\frac{2}{3},-\frac{1}{3})$

进行坐标变换: (注: $(x',y',z')$ 为 $[o';i',j',k']$ 下, $(x'',y'',z'')$ 为 $[o';i,j,k]$ 下.

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix}$$

再平移:

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} x-2 \\ y-2 \\ z-2 \end{pmatrix}.$$

代入 $[o';i',j',k']$ 下椭球面: $\frac{x'^2}{a^2}+\frac{y'^2}{b^2}+\frac{z'^2}{c^2}=1$

$$\Rightarrow \frac{(-x+2y+2z-6)^2}{9a^2}+\frac{(2x-y+2z-6)^2}{9b^2}+\frac{(2x+2y-z-6)^2}{9c^2}=1$$

(另解: 标准方程: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ 中, 分别将 $x,y,z$ 替换为

椭球上一点 $(x,y,z)$ 到 $boc$, $aoc$, $aob$ 三个轴所成平面的距离.)

13. $M_1:(\rho_1\sin\theta_1\cos\varphi_1, \rho_1\sin\theta_1\sin\varphi_1, \rho_1\cos\theta_1)$

$M_2:(\rho_2\sin\theta_2\cos\varphi_2, \rho_2\sin\theta_2\sin\varphi_2, \rho_2\cos\theta_2)$

$$d=\|M_1-M_2\|=\sqrt{(\rho_1-\rho_2)^2+2\rho_1\rho_2[1-\cos(\theta_1-\theta_2)\sin\varphi_1\sin\varphi_2-\cos\varphi_1\cos\varphi_2}$$

14. $\overrightarrow{M_1}:(a\sin\theta_1\cos\varphi_1, a\sin\theta_1\sin\varphi_1, a\cos\theta_1)$

$\overrightarrow{M_2}:(a\sin\theta_2\cos\varphi_2, a\sin\theta_2\sin\varphi_2, a\cos\theta_2)$

两向量夹角 $\gamma$:

$$\cos\gamma=\frac{\overrightarrow{M_1}\cdot\overrightarrow{M_2}}{|M_1||M_2|}=\frac{a^2\sin\theta_1\sin\theta_2\cos\varphi_1\cos\varphi_2+a^2\sin\theta_1\sin\theta_2\sin\varphi_1\sin\varphi_2+a^2\cos\theta_1\cos\theta_2}{a^2}$$

$$=\sin\theta_1\sin\theta_2(\cos(\varphi_1-\varphi_2))+\cos\theta_1\cos\theta_2$$

弧长 $d=a\gamma$

8.4

4: (1). 柱: $r^2\cos^2\theta-r^2\sin^2\theta=25$. 球: $r^2\sin^2\theta\cos^2\varphi-r^2\sin^2\theta\sin^2\varphi=25$.

(2) 球: ~~$2r^2\sin^2\theta-4r^2\cos^2\theta=0$~~ $2r^2\sin^2\theta-4r^2\cos^2\theta=0$

(4) 球: $r^2\sin^2\theta\cos2\varphi-r^2\cos^2\theta=1$

(5) 球: $r^2+r^2\cos\theta=4$

(8) 球: $r\sin\theta\cos\varphi+r\sin\theta\sin\varphi+r\cos\theta=1$

(10) 直: $x^2-y^2=z$

(11) 直: $r\sin\varphi=1 \Rightarrow r\sin\theta\sin\varphi=\sin\theta \Rightarrow y=\sin\theta \Rightarrow (1-y^2)(x^2+y^2+z^2)=z^2$

$\Rightarrow y^2(x^2+y^2+z^2)=x^2+y^2$.

5. 直角坐标: $z=2(x^2+y^2)$

柱面坐标: $z=2r^2$

6. 直角坐标: $2(x^2+y^2)-z^2=2$

柱〃: $2r^2-z^2=2$

8. $$\begin{cases} x = x_0 + a\sin u\cos v \\ y = y_0 + a\sin u\sin v \\ z = z_0 + a\cos u \end{cases} \quad u\in[0,\pi],\ v\in[0,2\pi)$$

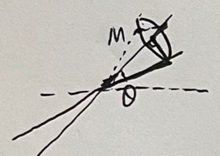
8.3

4: 轴: $\begin{cases} y=z \\ x=0 \end{cases}$. 方向为 $\vec{S}=(0,1,1)$ 过 $O(0,0,0)$

直线 $l$: $\begin{cases} y-2z+1=0 \\ x=0 \end{cases}$, 轴与 $l$ 交点 $M'(0,1,1)$ 为锥面顶点.

设 $(x,y,z)$ 为所成平面上一点 $M$.

点到轴距离 $d=\dfrac{|\vec{S}\times\vec{MO}|}{|\vec{S}|}=\dfrac{\sqrt{(z-y)^2+2x^2}}{\sqrt{2}}$



~~点到~~

$|MM'|=\sqrt{x^2+(y-1)^2+(z-1)^2}$.

由于锥面母线与轴夹角已确定.

$\cancel{\tan\theta=}\ \sin\theta=\dfrac{d}{|MM'|}=\dfrac{\sqrt{10}}{10} \Rightarrow 9x^2+4y^2+4z^2+2y+2z-10yz=0$

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使用习题课上讲的一般方法: $\rightarrow M_1:(x_1,y_1,z_1)$

取 $M_1\in l$, $M_1$ 绕轴所成曲线方程为:

$$\begin{cases} (y-y_1)+(z-z_1)=0 \\ (x-0)^2+(y-0)^2+(z-0)^2=(x_1-0)^2+(y_1-0)^2+(z_1-0)^2 \end{cases}$$

$$\Rightarrow \begin{cases} y+z=y_1+z_1 & ① \\ x^2+y^2+z^2=x_1^2+y_1^2+z_1^2 & ② \end{cases}$$

又 $\because M_1\in l \Rightarrow \begin{cases} y_1-2z_1+1=0 \\ x_1=0 \end{cases}$ 代入①,②

$\Rightarrow 9x^2+4y^2+4z^2+2y+2z-10yz=0$