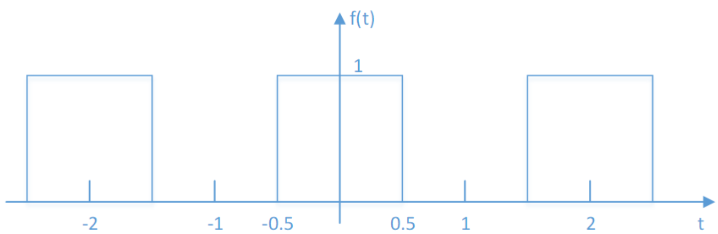
Time Domain and Frequency Domain

```
clear;clf;
dt = 0.01;
t = 0:dt:10;
amp =8;
freq =3;
pha =70;

ft = amp*cos(2*pi*freq*t+pha*pi/180);
T = 1/freq;
Lab3_freq_analyze_matrix(T, ft, dt);
```

Fourier Series of Periodic Signal



Trigonometric format

$$f(t) = a_0 + \sum_{n=1}^{N} (a_n \mathrm{cosn}(n\omega_1 t) + b_n \sin(n\omega_1 t))$$

$$a_0 = \frac{1}{T_1} \int_{t_0}^{t_0+T_1} f(t)\mathrm{dt}$$

$$a_n = \frac{2}{T_1} \int_{t_0}^{t_0+T_1} f(t)\cos(n\omega_1 t)\mathrm{dt}$$

$$b_n = \frac{2}{T_1} \int_{t_0}^{t_0+T_1} f(t)\sin(n\omega_1 t)\mathrm{dt}$$

Numeric method

```
% numeric method: loop function, trapz
clear;clf;
T = 2; f = 1/T; w1 = 2*pi*f;
dt = 0.01;
t = -3:dt:3;         % cover at least 3 cycles
tao = -1:dt:1;       % take one cycle of the periodic signal
ft = 0.5+0.5*square(pi*(tao+0.5),50);   % signal expression corresponding to tao
a0 = trapz(tao,ft)/T;                   % DC component
f = a0;
N = input('N=');     % superposition from DC component to Nth harmonic
an = zeros(1,N);
```

```
bn = zeros(1,N);
for n = 1:N
    fcos = ft.*cos(n*w1*tao); an(n)=trapz(tao,fcos)*2/T;
    fsin = ft.*sin(n*w1*tao); bn(n)=trapz(tao,fsin)*2/T;
    f = f+ an(n)*cos(n*w1*t)+bn(n)*sin(n*w1*t);
end
plot(t,f);xlabel('t(s)');ylabel('ft'); grid on;
title(['Numeric Loop with N=' num2str(N)]);
```

```
% Before performing calculations using matrix operations
% let's review the dot product of vectors and matrices
matrix = ones(3,3)
array = [1 2 3]
array.*matrix       % the row vector multiplies with each row in the matrix in turn
```

```
% numeric method: matrix calculation
clear;clf;
T = 2; f = 1/T; w1 = 2*pi*f;
dt = 0.01;
t = -3:dt:3;
tao = -1:dt:1;
ft = 0.5+0.5*square(pi*(tao+0.5),50);
a0 = trapz(tao,ft)/T;
N = input('N=');
n = 1:N;
```

$$f(t) = a_0 + \sum_{n=1}^{N} (\boldsymbol{a_n} \cos n(n\omega_1 t) + b_n \sin(n\omega_1 t))$$

$$a_n = \frac{2}{T_1} \int_{t_0}^{t_0+T1} f(t) \cos n\omega_1 t dt,$$

$$\omega_n = n\omega_1, \quad \tau \in [\mathrm{t0}, \mathrm{t0} + T], \ \tau_m = (m-1) * d\tau, \ d\tau = \mathrm{dt}$$

$$
\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \frac{2}{\mathrm{T}_1}\int_{t_0}^{t_0+T_1} \begin{bmatrix} f(\tau)\bullet\cos w_1\tau \\ f(\tau)\bullet\cos w_2\tau \\ \vdots \\ f(\tau)\bullet\cos w_n\tau \end{bmatrix} d\tau
$$

$$
= \frac{2}{\mathrm{T}_1}\int_{t_0}^{t_0+T_1} \begin{bmatrix} f(\tau_1)\bullet\cos w_1\tau_1 & f(\tau_2)\bullet\cos w_1\tau_2 & \cdots & f(\tau_m)\bullet\cos w_1\tau_m \\ f(\tau_1)\bullet\cos w_2\tau_1 & f(\tau_2)\bullet\cos w_2\tau_2 & \cdots & f(\tau_m)\bullet\cos w_2\tau_m \\ \vdots & \vdots & \ddots & \vdots \\ f(\tau_1)\bullet\cos w_n\tau_1 & f(\tau_2)\bullet\cos w_n\tau_2 & \cdots & f(\tau_m)\bullet\cos w_n\tau_m \end{bmatrix} d\tau
$$

$$
= \frac{2}{\mathrm{T}_1}\int_{t_0}^{t_0+\mathrm{T}_1} \begin{bmatrix} f(\tau_1) & f(\tau_2) & \cdots & f(\tau_m) \end{bmatrix} .* \cos\left(\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} * \begin{bmatrix} \tau_1 & \tau_2 & \cdots & \tau_m \end{bmatrix} \right) d\tau
$$

```
fcos = ft.*cos(n'*w1*tao); an = trapz(tao,fcos,2)*2/T;
fsin = ft.*sin(n'*w1*tao); bn = trapz(tao,fsin,2)*2/T;
```

$$
f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \mathrm{cosn}(n\omega_1 t) + b_n \sin(n\omega_1 t)) = a_0 + \sum_{\boldsymbol{n}=1}^{\infty}(\boldsymbol{a_n}\mathbf{cosn}(\boldsymbol{n}\omega_1\boldsymbol{t})) + \sum_{n=1}^{\infty}(b_n \sin(n\omega_1 t))
$$

$\omega_n = n\omega_1,\ \ t_k = (k-1) * \mathrm{dt}$

$$\sum_{n=1}^{\infty}\left(a_n \cos w_n t\right)$$

$$=\left[a_1 \cos w_1 t + a_2 \cos w_2 t + \cdots a_n \cos w_n t\right]$$

$$=\begin{bmatrix} a_1 \cos w_1 t_1 + a_2 \cos w_2 t_1 + \cdots a_n \cos w_n t_1 \\ a_1 \cos w_1 t_2 + a_2 \cos w_2 t_2 + \cdots a_n \cos w_n t_2 \\ \vdots \\ a_1 \cos w_1 t_k + a_2 \cos w_2 t_k + \cdots a_n \cos w_n t_k \end{bmatrix}'$$

$$=\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} * \begin{bmatrix} \cos w_1 t_1 & \cos w_1 t_2 & \cdots & \cos w_1 t_k \\ \cos w_2 t_1 & \cos w_2 t_2 & \cdots & \cos w_2 t_k \\ \vdots & \vdots & \ddots & \vdots \\ \cos w_n t_1 & \cos w_n t_2 & \cdots & \cos w_n t_k \end{bmatrix}$$

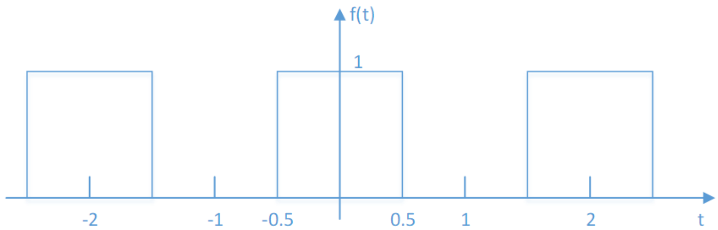
$$=\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} * \cos\left(\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} * \begin{bmatrix} t_1 & t_2 & \cdots & t_k \end{bmatrix}\right)$$

```
f = a0 + an'*cos(n'*w1*t) + bn'*sin(n'*w1*t);
plot(t,f); xlabel('t(s)');ylabel('ft'); grid on;
title(['Numeric Matrix with N=' num2str(N)]);
```

Symbolic method

```
% Symbolic method: loop function, int
clear; clf;
N = input('N=');
syms t a0 an bn n
T1 = 2; freq = 1/T1; w1 = 2*pi*freq;
range = [-1.5,0.5];          % cycle range changes
ft = 0.5+0.5*sign(t+0.5);    % signal expression changes following the cycle range
a0 = 1/T1*int(ft,t,range);
f = a0;
for n=1:N
    an = 2/T1*int(ft*cos(n*w1*t),t,range);
    bn = 2/T1*int(ft*sin(n*w1*t),t,range);
    f = f+an*cos(n*w1*t)+bn*sin(n*w1*t);
end
fplot(f); xlabel('t');ylabel('y(t)');
title(['Symbolic Loop with N=' num2str(N)]);
axis([-3,3,-0.2,1.2]); grid on;
```

Exponantial format



$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_1 t} = \sum_{n=-N}^{N} F_n e^{jn\omega_1 t}$$

$$F_n = \frac{1}{T_1}\int_{t_0}^{t_0+T} f(t)e^{-jn\omega_1 t}\mathrm{dt}$$

$$F_0 = \frac{1}{T_1}\int_{t_0}^{t_0+T} f(t)e^{-j0\omega_1 t}\mathrm{dt} = \frac{1}{T_1}\int_{t_0}^{t_0+T} f(t)\mathrm{dt}$$

```
% Symbolic method: loop function, int
clear; clf;
N = input('N=');
syms t1 a0 an bn n
T1 = 2; freq = 1/T1; w1 = 2*pi*freq;
range = [-1.5,0.5];
ft = 0.5+0.5*sign(t1+0.5);
Wn = (-N:N)*w1;
f = 0; k = 1;
for n=-N:N
    Fn(k) = 1/T1*int(ft*exp(-1j*n*w1*t1),t1,range);
    f = f+Fn(k)*exp(1j*n*w1*t1);
    k = k+1;
end
subplot(2,2,[1 3]);fplot(f);title(['N=' num2str(N)]);xlabel('t');ylabel('y(t)');axis([-3,3,-0.
subplot(2,2,2);stem(Wn/(2*pi),abs(Fn)); xlabel('f(Hz)');ylabel('Amplitude')
subplot(2,2,4);stem(Wn/(2*pi),angle(Fn)*180/pi); xlabel('f(Hz)');ylabel('Phase(angle)')
```

```
% numeric method: loop function, trapz
clear;clf;
T = 2; freq = 1/T; w1 = 2*pi*freq;
dt = 0.01;
t = -3:dt:3;
tao = -1:dt:1;
ft = 0.5+0.5*square(pi*(tao+0.5),50);
N = input('N=');
Wn = (-N:N)*w1;
Fn = zeros(1,2*N+1);
i = 1; f=0;
for n = -N:N
```

```
    F =ft.*exp(-1j*n*w1*tao);
    Fn(i) =  trapz(tao,F)/T;
    f = f+Fn(i).*exp(1j*n*w1*t);
    i = i+1;
end
subplot(2,2,[1 3]);plot(t,real(f));
xlabel('t(s)');ylabel('ft'); grid on;
title(['Numeric Matrix with N=' num2str(N)]);
subplot(2,2,2);stem(Wn/(2*pi),abs(Fn));
xlabel('f(Hz)');ylabel('Amplitude'); title('Frequency-Amplitude');
subplot(2,2,4);stem(Wn/(2*pi),angle(Fn)); %.*(abs(Fn)>1e-10)
xlabel('f(Hz)');ylabel('Phase(radian)'); title('Frequency-Phase');
```

```
% numeric method: matrix function, trapz
clear;clf;
T = 2; f = 1/T; w1 = 2*pi*f;
dt = 0.01;
t = -2:dt:2;
tao = -1:dt:1;
ft = 0.5+0.5*square(pi*(tao+0.5),50);
N = input('N=');
n = -N:N;
Wn = n*w1;
```

$F_n = \frac{1}{T_1}\int_{t_0}^{t_0+T} f(t)e^{-\mathrm{j}nw_1 t}\mathrm{dt}, \quad w_n = nw_1, \quad \tau_m = (m-1)*d\tau, \; d\tau = \mathrm{dt}$

$$\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} = \frac{1}{\mathrm{T}_1}\int_{t_0}^{t_0+T_1} \begin{bmatrix} f(\tau)\bullet e^{\;jw_1\tau} \\ f(\tau)\bullet e^{-jw_2\tau} \\ \vdots \\ f(\tau)\bullet e^{-jw_n\tau} \end{bmatrix} d\tau$$

$$= \frac{1}{\mathrm{T}_1}\int_{t_0}^{t_0+T_1} \begin{bmatrix} f(\tau_1)\bullet e^{-jw_1\tau_1} & f(\tau_2)\bullet e^{-jw_1\tau_2} & \cdots & f(\tau_m)\bullet e^{-jw_1\tau_m} \\ f(\tau_1)\bullet e^{-jw_2\tau_1} & f(\tau_2)\bullet e^{-jw_2\tau_2} & \cdots & f(\tau_m)\bullet e^{-jw_2\tau_m} \\ \vdots & \vdots & \ddots & \vdots \\ f(\tau_1)\bullet e^{-jw_n\tau_1} & f(\tau_2)\bullet e^{-jw_n\tau_2} & \cdots & f(\tau_m)\bullet e^{-jw_n\tau_m} \end{bmatrix} d\tau$$

$$= \frac{1}{\mathrm{T}_1}\int_{t_0}^{t_0+\mathrm{T}_1} [f(\tau_1) \quad f(\tau_2) \quad \cdots \quad f(\tau_m)] \cdot * e^{-j\left(\begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} * [\tau_1 \quad \tau_2 \quad \cdots \quad \tau_m]\right)} d\tau$$

```
F = ft.*exp(-1j*n'*w1*tao); Fn = trapz(tao,F,2)/T;
```

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{\mathrm{j}nw_1 t}=\sum_{n=-N}^{N}F_n e^{\mathrm{j}nw_1 t},\ \ w_n=nw_1\ ,\ \ \tau_k=(k-1)*\mathrm{dt},$$

$$\sum_{n=-\infty}^{\infty}F_n e^{jnw_1 t}$$

$$=\left[F_1e^{jw_1t}+F_2e^{jw_2t}+\cdots F_ne^{jw_nt}\right]$$

$$=\begin{bmatrix}F_1e^{jw_1t_1}+F_2e^{jw_2t_1}+\cdots F_ne^{jw_nt_1}\\ F_1e^{jw_1t_2}+F_2e^{jw_2t_2}+\cdots F_ne^{jw_nt_2}\\ \vdots\\ F_1e^{jw_1t_k}+F_2e^{jw_2t_k}+\cdots F_ne^{jw_nt_k}\end{bmatrix}'$$

$$=\begin{bmatrix}F_1 & F_2 & \cdots & F_n\end{bmatrix}*\begin{bmatrix}e^{jw_1t_1} & e^{jw_1t_2} & \cdots & e^{jw_1t_k}\\ e^{jw_2t_1} & e^{jw_2t_2} & \cdots & e^{jw_2t_k}\\ \vdots & \vdots & \ddots & \vdots\\ e^{jw_nt_1} & e^{jw_nt_2} & \cdots & e^{jw_nt_k}\end{bmatrix}$$

$$=\begin{bmatrix}F_1 & F_2 & \cdots & F_n\end{bmatrix}*e^{j\left(\begin{bmatrix}w_1\\ w_2\\ \vdots\\ w_n\end{bmatrix}*\begin{bmatrix}t_1 & t_2 & \cdots & t_k\end{bmatrix}\right)}$$

```
f = Fn.'*exp(1j*n'*w1*t);   % 注意：这里 Fn 是复数, 在对 Fn 进行转置时会同时变成共轭复数, 因此需要用.'
subplot(2,2,[1 3]);plot(t,real(f));
xlabel('t(s)');ylabel('ft'); grid on;
title(['Numeric Matrix with N=' num2str(N)]);
subplot(2,2,2);stem(Wn/(2*pi),abs(Fn));
xlabel('f(Hz)');ylabel('Amplitude'); title('Frequency-Amplitude');
subplot(2,2,4);stem(Wn,angle(Fn)); % .*(abs(Fn)>1e-10)
xlabel('w(rad)');ylabel('Phase(radian)'); title('Frequency-Phase');
```

Gibbs Phenomenon

```
clear;clf;
t = -2:0.001:2;
N = [4,16,32];
for n = 1:length(N)
    a0 = 0.5;
    f = a0*ones(1,length(t));
    for i=1:2:N(n)
        f = f+cos(i*pi*t)*sinc(i/2);
    end
    plot(t,f); hold on;
```

```
end
grid minor; hold off; legend({'N=4','N=8','N=16'},'Location','northwest');
axis([-1,-0.1,-0.2,1.2]);
```