1、a (1). $(B \vee A) \to C$

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(2). $(A \to C) \wedge (\neg A \to (\neg B \vee \neg C))$

b. $\exists \varepsilon > 0 \ \forall \delta > 0 \ \exists x \ (|x - x_0| < \delta \to |f(x) - A| < \varepsilon)$

2. (1) tautology. Its truth table is always T.

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(2) contingency. It has both T and F in truth table.

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3. a). $A \equiv (p \wedge q) \vee (\neg p \wedge q) \wedge r$

$\equiv q \wedge (p \vee \neg p) \wedge r$

$\equiv q \wedge T \wedge r$

$\equiv q \wedge r$

$B \equiv (p \vee q) \wedge (p \vee r)$

(b) ① $\neg \exists x \forall y (F(x) \wedge (G(y) \to H(x,y))) \equiv \forall x \exists y (F(x) \wedge (G(y) \to H(x,y)))$

② $F(x) \wedge G(y) \to H(x,y)$ doesn't equal to $(F(x) \wedge G(y)) \to H(x,y)$

4. $(\neg p \vee r) \wedge (\neg q \vee s) \wedge (p \wedge q) \equiv (\neg p \vee r) \wedge (p \wedge q) \wedge (\neg q \vee s)$

using Resolution on first two premises.

$(\neg p \vee r) \wedge (p \wedge q) \Rightarrow r \vee q$.

then $(q \vee r) \wedge (\neg q \vee s)$

using Resolution again

we can get $r \vee s$

$t \to (r \wedge s) \equiv \neg (r \wedge s) \to \neg t \equiv (r \wedge s) \vee \neg t$

let $r \vee s = A$ $\quad (r \wedge s) \vee \neg t = B$

then $A \wedge \neg B \equiv (r \vee s) \wedge \neg (r \wedge s) \wedge t$

using Simplification $A \wedge \neg B \equiv (r \vee s) \wedge \neg (r \wedge s)$

$\equiv F$

so $A \wedge \neg B$ is a contradiction

so $A \Rightarrow B$

thus. $(\neg p \vee r) \wedge (\neg q \vee s) \wedge (p \wedge q) \Rightarrow (t \to (r \wedge s))$

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5. a) 原式 $\equiv (\neg \forall x F(x) \vee \exists y G(y) \wedge \forall x F(x)) \rightarrow \exists y G(y)$

$\equiv \exists y G(y) \rightarrow \exists y G(y)$

$\equiv T$

so $((\forall x F(x) \rightarrow \exists y G(y) \wedge \forall x F(x)) \rightarrow \exists y G(y)$ is logically valid.

b). 原式 $\equiv \neg(\neg \forall x F(x) \vee \exists y G(y)) \wedge \exists y G(y)$

$\equiv \forall x F(x) \wedge \neg \exists y G(y) \wedge \exists y G(y)$

$\equiv \forall x F(x) \wedge F$

$\equiv F$

so $\neg(\forall x F(x) \rightarrow \exists y G(y)) \wedge \exists y G(y)$ is unsatisfiable.

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6. a) $\begin{cases} 2|E| = \sum_{v\in V} \deg(v) = 2\times 2 + 2\times 3 + (|V|-4)\times 1 \\ |V| = |E| \end{cases}$

解得 $|V| = 6$

b). They are isomorphic because they have the same graph invariants (the number of vertices, the number of edges, the number of vertices of each degree.)

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7. a). G_1 Yes. $g \to d \to e \to a \to f \to b \to g \to c \to d \to a \to b \to c$

G_2. Yes. $d \to e \to b \to g \to f \to e \to a \to d \to c \to a \to b \to c \to g$.

b) G_3. cut vertices: e, f, h, a bridge: ed.

$\kappa(G_3) = 1$ $\lambda(G_3) = 1$

G_4. cut vertices. f, a, b, c, d, g, h, m, j bridge. cf, df, gf, hf

$\kappa(G_4) = 2$ $\lambda(G_4) = 3$.

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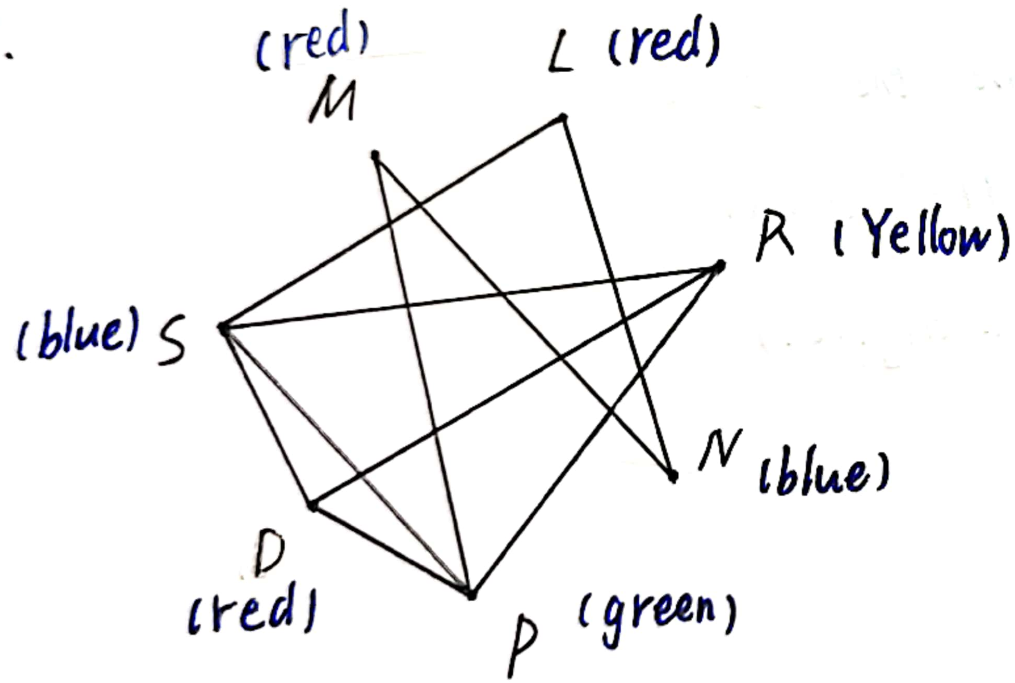
20215330 25

8. a) $P = |V(G)| - |E(G)| + |R(G)| - 1$

$= 10 - 8 + 3 - 1$

$= 4$

b).



So the minimal time is 4 hours.

9. $k=0$. $S_0=\phi$

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$L_0(v_1)=0$, $L_0(v_2)=L_0(v_3)=\cdots=L_0(v_8)=+\infty$.

$k=1$: $u:=v_1 \rightsquigarrow S_1=\{v_1\}$

$L_0(v_1)+d(v_1,v_2)=6<L_0(v_2) \rightsquigarrow L_1(v_2)=6$

$L_0(v_1)+d(v_1,v_3)=3<L_0(v_3) \rightsquigarrow L_1(v_3)=3$

$k=2$: $u:=v_3 \sim S_1=\{v_1,v_3\}$.

$L_1(v_3)+d(v_3,v_4)=5<L_1(v_4) \rightsquigarrow L_2(v_4)=5$

$L_1(v_3)+d(v_3,v_5)=8<L_1(v_5) \rightsquigarrow L_2(v_5)=8$

$L_1(v_3)+d(v_3,v_7)=11<L_1(v_7) \rightsquigarrow L_2(v_7)=11$.

$k=3$. $u:=v_2 \sim S_1=\{v_1,v_2,v_3\}$

$L_2(v_2)+d(v_2,v_6)=12<L_2(v_6) \rightsquigarrow L_3(v_6)=12$

$L_2(v_2)+d(v_2,v_4)=8>L_2(v_4$.

$k=4$. $u:=v_6 \sim S_1=\{v_1,v_2,v_3,v_6\}$

$L_3(v_6)+d(v_6,v_8)=15<L_3(v_8) \rightsquigarrow L_4(v_8)=15$

$k=5$: $u:=v_4 \sim S_1=\{v_1\ v_2,v_3,v_4,v_6\}$

$L_4(v_4)+d(v_4,v_5)=6<L_5(v_5) \rightarrow L_5(v_5)=6$

$L_4(v_4)+d(v_6,v_8)=11<L_4(v_8) \rightarrow L_5(v_8)=11$

$k=6$: $u:=v_5 \sim S_1=\{v_1,v_2,v_3,v_4,v_5,v_6\}$

$L_5(v_5)+d(v_5,v_7)=7<L_5(v_7) \rightarrow L_6(v_7)=7$

$k=7$. $u:=v_7 \sim S_1=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$

$L_6(v_7)+d(v_7,v_8)=10<L_6(v_8) \rightsquigarrow L_7(v_8)=10$

$k=8$ $u:=v_8 \sim S_1=\{v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8\}$.

return: $L(v_8)=10$.

10. a) suppose the number of vertices of degree 4 is n

$$(n+1+6+7)-1 = 1\times3+6\times2+7\times1+4n$$

$$n=$$

(b) $(a*b-c)\div(d+e*f)*g+(h*j)\div(j*(k-l))$

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