Discrete Mathematics: Homework 8

(Deadline: 11:59pm, May 3, 2022)

1. (**10 points**) Let p, q, r and s be propositional variables. Construct the truth table for the formula $p \to \neg q \vee r \to \neg(\neg r \to s \wedge p)$.

2. (**15 points**) Let p, q, r and s be propositional variables. Determine the types of the following formulas (tautology, contradiction or contingency). Explain your answers.

   (1) $(\neg p \vee q) \wedge (q \to \neg r \wedge \neg p) \wedge (p \vee r)$

   (2) $(p \to q) \wedge (q \to r) \to (p \to r)$

   (3) $(p \to r) \wedge (q \to s) \wedge (p \vee q) \to (r \vee s)$

3. (**20 points**) Let a, b, c and d be propositions as below.

   - a: "Alice attends the meeting."
   - b: "Bob attends the meeting."
   - c: "Charlie attends the meeting."
   - d: "David attends the meeting."

   Translate the following statements into propositional formulas in a, b, c and d.

   (1) "David attends the meeting if and only if Charlie attends and Alice doesn't attend."

   (2) "Charlie attends the meeting provided that David doesn't attend, but, if David attends, then Bob doesn't attend."

   (3) "A necessary condition for Alice attending the meeting, is that, if Bob and Charlie aren't attending, David attends."

   (4) "Alice, Bob and Charlie attend the meeting if and only if David doesn't attend, but, if neither Alice nor Bob attend, then David attends only if Charlie attends."

4. (**20 points**) Let l, q, n and b be the following propositions:

   - l: "The file system is locked."
   - q: "New messages will be queued."
   - n: "The system is functioning normally."
   - b: "New messages will be sent to the message buffer."

   Decide if a system satisfying the following specifications exists using l, q, n and b:

   (1) "If the file system is not locked, then new messages will be queued."

   (2) "If the file system is not locked, then the system is functioning normally, and conversely."

   (3) "If new messages are not queued, then they will be sent to the message buffer."

   (4) "If the file system is not locked, then new messages will be sent to the message buffer."

   (5) "New messages will not be sent to the message buffer."

5. (**15 points**) Let $A_1, A_2, \ldots, A_8$ and A be formulas defined by the following truth table.

p	q	r	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A
T	T	T	T	F	F	F	F	F	F	F	F
T	T	F	F	T	F	F	F	F	F	F	T
T	F	T	F	F	T	F	F	F	F	F	F
T	F	F	F	F	F	T	F	F	F	F	T
F	T	T	F	F	F	F	T	F	F	F	F
F	T	F	F	F	F	F	F	T	F	F	T
F	F	T	F	F	F	F	F	F	T	F	T
F	F	F	F	F	F	F	F	F	F	T	T

Determine the formulas $A_1, A_2, \ldots, A_8$ and A.

6. (**10 points**) Let P, Q, R and S be propositional formulas. Show that

$$(P \wedge Q \wedge S) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge Q \wedge \neg S) \vee \neg(P \wedge R \rightarrow Q) \equiv P$$

using the rule of replacement. (You can use any laws in lec17-18.)

7. (**10 points**) Let Δ be the unary logical connective defined by the follow truth table

p	Δp
T	F
F	F

Represent the following formulas

(a) $\neg p$

(b) $p \wedge q$

(c) $p \vee q$

as formulas that only use the connectives Δ and $\rightarrow$.