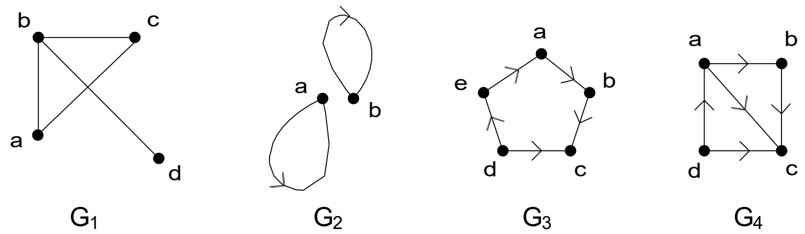
Discrete Mathematics: Homework 11

(Deadline: 2022/5/31)

1. (10 points) Draw the graph such that the vertices are the integers 7, 1, 5, 35, 13, 11, 65 and such that there is a directed edge that starts at n and ends at m if n divides m.

2. (10 points) In a group with 20 children is it possible that 7 of them have exactly 3 friends, 9 of them have exactly 4 friends, and 4 of them have exactly 5 friends? Why?

3. (20 points) Write the adjacency matrices of:
   (a) the graphs G1, G2, G3 and G4 below, with respect to the alphabetical ordering of the vertices.



   (b) the complete graph K_5, the cycle C_6 and the complete bipartite graph $K_{2,3}$.

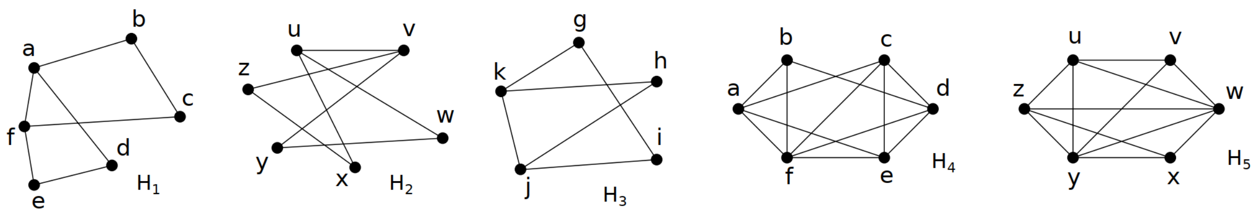
4. (20 points) Consider the following matrices:

$$M_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} ;\ M_2 = \begin{pmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{pmatrix} ;\ M_3 = \begin{pmatrix} 0 & 3 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{pmatrix} ;\ M_4 = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

   (a) When it is possible, draw the undirected graphs whose matrices are the adjacency matrices.

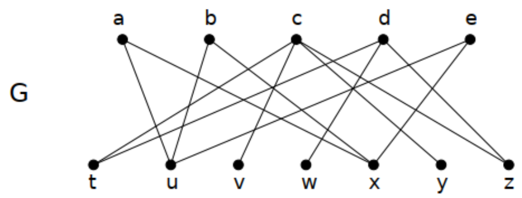
   (b) Consider the remaining matrices (the ones you haven't used above). When it is possible, draw the directed graphs whose matrices are the adjacency matrices.

5. (10 points) Are the graphs H1 and H2 isomorphic? What about H1 and H3? And what about H4 and H5?



6. (10 points) For which values of n the graphs Kn, Cn, Wn, Qn are bipartite?

7. (20 points) Consider the following graph G:



a) Give an example of a matching M for G such that |M | = 3.

b) Is G bipartite? If yes, describe the partition of vertices into two disjoints sets V1 and V2. Does there exist a complete matching from V1 to V2 ? From V2 to V1 ?