EE 150L
Signals and Systems Lab

Lab6 Laplace Transform

Date Performed:

Class Id: 1A-105

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1. Please briefly describe the difference and relationship between Laplace transform and Fourier transform.

The Fourier transform is a special case of the Laplace transform. The Fourier transform is the most basic transformation, derived from the Fourier series. The Fourier series only applies to periodic signals, and the aperiodic signals are treated as periodic signals with period T going to infinity, and then the Fourier transform is derived, which is a good way to deal with the spectrum of aperiodic signals. However, the weakness of the Fourier transform is that the original signal must be absolutely integrable, so it is not widely applicable.

The Laplace transform is a generalization of the Fourier transform, which does not apply to functions that grow exponentially, whereas the Laplace transform is equivalent to the Fourier transform with an exponential convergence factor, which extends the frequency domain to the complex frequency domain, which can analyze a wider range of signals.

2. $y''(t)+3y'(t)+2y(t)=2f'(t)+6f(t), f(t)=u(t), y(0_-)=2, y'(0_-)=1$

a) Find out the transfer function H(s).

b) What is the relationship between H(s) and h(t).

c) Find out the zero state response with H(s).

提示:

➢ 系统的传递函数 H(s) 是指在零初始条件下系统响应（即输出）与激励（即输入）之比。
即当$y(0_-)=0, y'(0_-)=0$时:

$$H(s)=\frac{Y(s)}{F(s)}$$

➢ 要从微分方程获得系统传递函数，需对微分方程两边进行拉普拉斯变换，同时利用拉普拉斯变换的时域微分性质。

a)

$$[s^2Y(s)-sy(0_-)-y'(0_-)]+3\cdot[sY(s)-y(0_-)]+2Y(s)=2[sF(s)-f(0)]+6F(s)$$

$$(s^2+3s+2)Y(s)-2s-1-6=(2s+6)F(s)-2$$

$$Y(s)=\frac{2s+5}{s^2+3s+2}+\frac{2s+6}{s^2+3s+2}F(s)$$

$$F(s)=\frac{1}{s}$$

$$H(s)=\frac{Y(s)}{F(s)}=\frac{2s+6}{s^2+3s+2}$$

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b) $H(s)=\int_0^\infty h(t)e^{-st}dt \quad (s=\beta+j\omega)$

c). $H(s) = \frac{4}{s+1} - \frac{2}{s+2}$

$$h(t) = [4e^{-t} - 2e^{-2t}]u(t)$$

$$y(t) = u(t) * [4e^{-t} - 2e^{-2t}]u(t)$$

$$= \int_{-\infty}^{t} (4e^{-\tau} - 2e^{-2\tau})u(\tau)\,d\tau$$

$$= \int_{0}^{t} (4e^{-\tau} - 2e^{-2\tau})\,d\tau$$

$$= [-4e^{-t} + e^{-2t} + 3]u(t)$$