Discrete Mathematics: Homework 5

(Deadline: March 25, 2022)

1. (20 points) Let A and B be any sets. Show that if $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.

   (**Remark:** $\mathcal{P}(A)$ is the power set of A, i.e., the set of all subsets of A)

2. (20 points) Construct a bijection from $A = (0, 1) \cup [2, 3) \cup (4, 5]$ to $B = (6, 7) \cup [8, +\infty)$.

3. (20 points) Prove or disprove $|\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}| = |\mathbb{R}|$.

4. (20 points) Prove or disprove $|\{(a_1, a_2, a_3, \ldots) : a_i \in \{1, 2, 3\} \text{ for all } i = 1, 2, 3, \ldots\}| = |\mathbb{Z}^+|$.

5. (20 points) Find a countably infinite number of subsets of $\mathbb{Z}^+$, say $A_1, A_2, \ldots \subseteq \mathbb{Z}^+$ such that the following requirements are simultaneously satisfied:

   - $|A_i| = |\mathbb{Z}^+|$ for all $i = 1, 2, \ldots$;
   - $A_i \cap A_j = \emptyset$ for all $i \neq j$;
   - $\cup_{i=1}^{\infty} A_i = \mathbb{Z}^+$.