

1. (8 points) True or False

For each statement, choose T if the statement is correct, otherwise, choose F.

Note: You should write down your answers in the box below.

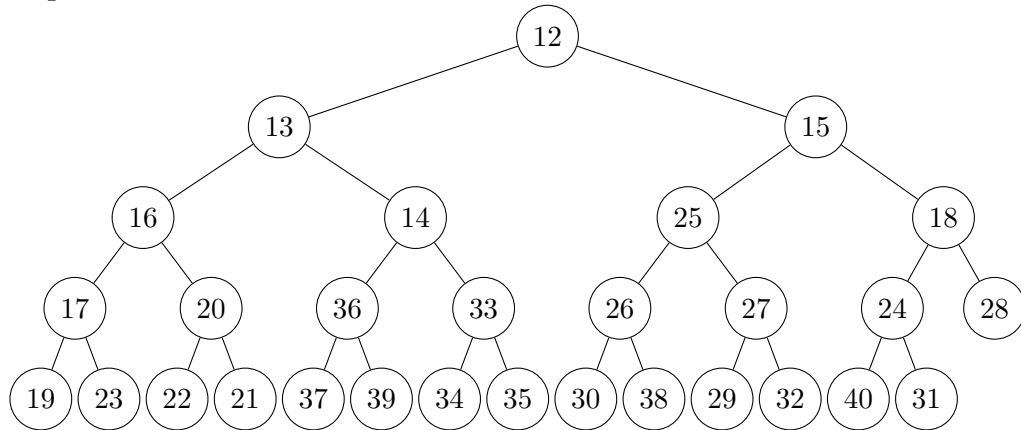
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
F	T	F	T	F	T	T	F

- (a) (1') Every binary tree has at least one node.
- (b) (1') A binary tree of height $h = 0$ is complete.
- (c) (1') BFS and DFS on a binary tree cannot give the same traversal sequence.
- (d) (1') If the post-order traversal of a complete binary tree is ascending, then the binary tree is a max-heap.
- (e) (1') Every complete binary tree is also a full binary tree.
- (f) (1') Every complete binary tree has a perfect binary sub-tree.
- (g) (1') In a binary min-heap containing n numbers, the largest element can be found in time $O(n)$.
- (h) (1') If the pre-order traversal and post-order traversal of two binary trees are equal respectively, then the two binary trees are exactly the same.

2. (11 points) Fill-in-the-blanks

- (a) (2') A full binary tree with n leaf nodes contains $2n - 1$ total nodes.
- (b) (2') There are 42 distinct shapes of binary trees with 5 nodes.
- (c) (4') In a binary max-heap with n elements and duplicated elements are not allowed, the 6^{th} largest element can be found in time $O(\log n)$ if we can only access the top of the heap. ($n \gg 6$)
And the 6^{th} largest element can be found in time $O(\underline{1})$ if we can access the array storing the heap.
- (d) (3') Suppose we have an initial heap stored in an array as (120, 140, 40, 50, 80, 70, 60, 90, 20, 100), we will construct a min-heap from the initial heap using Floyd's method. After the construction is completed, we delete the root from the heap. Then the post-order traversal of the heap will be (120, 140, 90, 80, 50, 70, 100, 60, 40).

3. (6 points) Heap



- (a) (1') Is this heap a max-heap or a min-heap?

min-heap

- (b) (3') Suppose that you pop the key from the heap above. Write down all the elements that are involved in one (or more) compares.

13,14,15,16,31,33,36. The compares are 31-13, 13-15, 31-16, 16-14, 31-36, 31-33

- (c) (2') Suppose that inserting the key x was the last operation performed in the binary heap in the figure. That is, after inserting x , the heap is shown as the figure above. Write down all possible value of x .

31,24,18,15. To insert a node in a binary heap, we place it in the next available leaf node and swim it up. Thus, 31,24,18,15, and 12 are the only keys that we might move. But, the last inserted key could not have been 12, because then 15 would have been the old root (which would violate heap order because the left child of the root is 13).