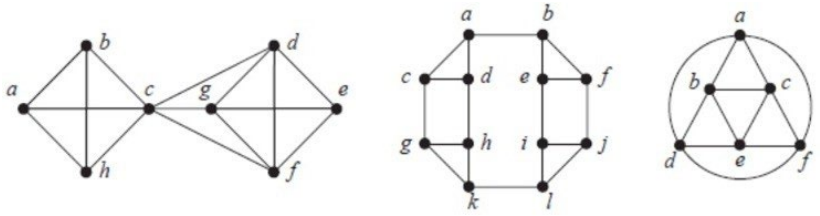
Discrete Mathematics: Homework 12

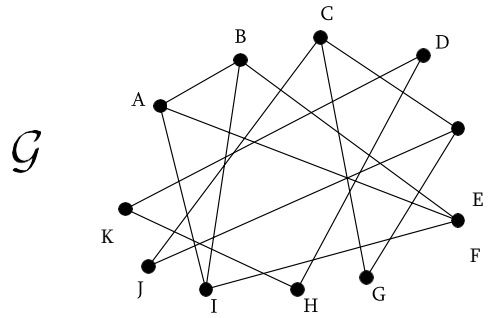
(Deadline: 2022/6/6)

1. (10 points) Determine $\kappa(G_i)$, $\lambda(G_i)$ and $\delta(G_i) = \min_{v \in V} \deg(v)$ for each of the following graphs and verify that $\kappa(G_i) \leq \lambda(G_i) \leq \delta(G_i)$, where $i = 1, 2, 3$.



G_1 G_2 G_3

2. (10 points) Is the graph G below connected? Give the connected components of G.



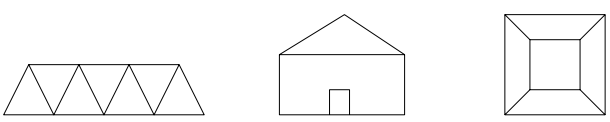
3. (20 points) Let G be connected graph. If e is an edge, such that removing E gives a non-connected graph, then e is called a bridge.

   - Prove that if G contains no vertices of odd degree then G is bridgeless.

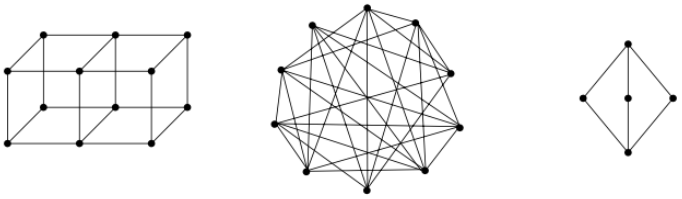
4. (10 points) Determine for which values the complete bipartite graph $K_{m,n}$ has a.) an Euler circuit. b.) an Euler path.

5. (15 points)

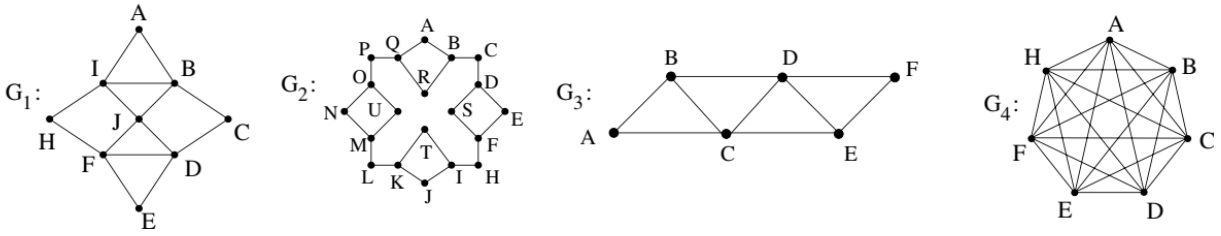
   (a) Can you draw the following pictures without lifting the pen? Explain.



   (b) Do the following graphs admit any Hamilton circuit? If yes, draw one, otherwise, explain why there is no Hamilton circuit.



(c) Do the graphs G_1, G_2, G_3 and G_4 below admit any Euler path or Euler circuit? If yes, draw one, otherwise, explain why there is no Euler path nor Euler circuit.



6. (10+10 points) A simple graph with $2p$ vertices is such that each of its vertices is of degree at least p. Show that this graph is connected. Do we have the same result for a simple graph with n vertices such that each vertex v has $deg(v) \geq (n-1)/2$?

7. (10 points) Draw a simple connected graph with 7 vertices without any Euler path (nor Euler circuit!).

8. (5 points) Let G be a directed graph with n vertices, and let M be its adjacency matrix. Assume M^n is not the zero matrix (i.e. at least one coefficient is not zero). Show that G contains at least one circuit.