CS 101   Fall 2022 - Quiz 5-A

24/10/2022 - 20 Minutes   Name: ID number:

1. **(8 points) True or False**

   For each statement, choose T if the statement is correct, otherwise, choose F.

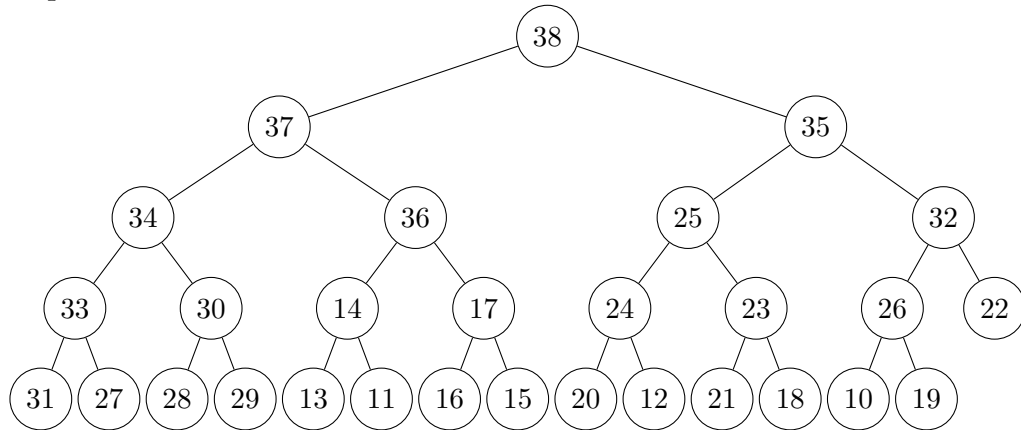   *Note: You should write down your answers in the box below.*

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
T	F	T	F	T	T	T	F

   (a) (1') A binary tree of height $h = 0$ is perfect.

   (b) (1') Every binary tree has at least one node.

   (c) (1') If the pre-order traversal and in-order traversal of two binary trees are equal respectively, then the two binary trees are exactly the same.

   (d) (1') BFS and DFS on a binary tree always give different traversal sequences.

   (e) (1') In a binary max-heap containing n numbers, the smallest element can be found in time $O(n)$.

   (f) (1') If the post-order traversal of a complete binary tree is descending, then the binary tree is a min-heap.

   (g) (1') Every complete binary tree has a perfect binary sub-tree.

   (h) (1') Every full binary tree is also a complete binary tree.

2. **(11 points) Fill-in-the-blanks**

   (a) (2') A full binary tree with n non-leaf nodes contains $\underline{2n + 1}$ total nodes.

   (b) (2') There are $\underline{42}$ distinct shapes of binary trees with 5 nodes.

   (c) (4') In a binary min-heap with n elements and duplicated elements are not allowed, the 7th smallest element can be found in time $O(\underline{\log n})$ if we can only access the top of the heap.$(n \gg 7)$ And the 7th smallest element can be found in time $O(\underline{1})$ if we can access the array storing the heap.

   (d) (3') Suppose we have an initial heap stored in an array as $(120, 140, 40, 50, 80, 70, 60, 90, 20, 100)$, we will construct a min-heap from the initial heap using Floyd's method. After the construction is completed, we delete the root from the heap. Then the post-order traversal of the heap will be $\underline{(120, 140, 90, 80, 50, 70, 100, 60, 40)}$.

3. (6 points) Heap

38
37   35
34   36   25   32
33 30 14 17 24 23 26 22
31 27 28 29 13 11 16 15 20 12 21 18 10 19

(a) (1') Is this heap a max-heap or a min-heap?

max-heap

(b) (3') Suppose that you pop the key from the heap above. Write down all the elements that are involved in one (or more) compares.

14,17,19,34,35,36,37. The compares are 35-37, 19-37, 34-36, 19-36, 14-17, 19-17

(c) (2') Suppose that inserting the key x was the last operation performed in the binary heap in the figure. That is, after inserting x, the heap is shown as the figure above. Write down all possible value of x.

19,26,32,35. To insert a node in a binary heap, we place it in the next available lead node and swim it up. Thus, 19,26,32,35, and 38 are the only keys that we might move. But, the last inserted key could not have been 38, because, then, 35 would have been the old root (which would violate heap order because the left child of the root is 37).