1. (20 points) Show that $(P \vee Q \rightarrow R \wedge S) \wedge (S \vee W \rightarrow U) \wedge P \Rightarrow U \vee V$ using the tautological implications (and the resulting valid argument forms) on page 6 of lec4.pptx. (**Hint**: see page 7, 8 for an example)

(1) $R \wedge S \equiv S \wedge R$

(2) S using Simplification on (1)

(3) $P \vee Q \rightarrow R \equiv \neg(P \vee Q) \vee R \equiv (\neg P) \wedge (\neg Q) \vee S$

$\equiv ((\neg P) \wedge (\neg Q)) \vee ((\neg P) \wedge S)$

(4) $(\neg P) \vee S$ using Simplification twice on (3)

(5) $(S \vee W \rightarrow U) \Rightarrow (\neg S) \vee U$ in the same way as (3) (4)

(6) $(\neg P) \vee S \wedge (\neg S) \vee U \wedge P$

$\equiv F \vee F \vee U$

$\equiv U \Rightarrow U \vee V$ according to Addition.

So $(P \vee Q \rightarrow R \wedge S) \wedge (S \vee W \rightarrow U) \wedge P \Rightarrow U \vee V$

2. (20 points) Use the tautological implications (and the resulting valid argument forms) on page 6 of lec4.pptx to show that the premises "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained."

let P: rainy Q: foggy R: the sailing race is held

S: the lifesaving demonstration will go on

W: the trophy is awarded

we need to prove $((\neg P \vee \neg Q) \to R \wedge S) \wedge (R \to W) \wedge \neg W \Rightarrow P$

(1) $R \to W$ premise

(2) $\neg W$ premise

(3) $\neg R$ Modus ponens using (1) and (2)

(4) $\neg P \vee \neg Q$ premise

(5) $\neg(P \wedge Q)$ logical equivalence applied to (4)

(6) $R \wedge S$ premise

(7) $\neg(P \wedge Q) \to (R \wedge S) \equiv (P \wedge Q) \vee (R \wedge S) \wedge \neg R$

(8) $(R \wedge S) \wedge \neg R \equiv (\neg R \wedge R) \wedge S \equiv F \wedge S \equiv F$

(9) $(P \wedge Q) \vee F \equiv P \wedge Q$

(10) P Simplification using (9)

So the premises imply the conclusion.

3. (20 points) Show that the argument form with premises $p_1, p_2, \ldots, p_n$ and conclusion $q \to r$ is valid if the argument form with premises $p_1, p_2, \ldots, p_n, q$ and conclusion r is valid.

let $p_1 \wedge p_2 \wedge \cdots \wedge p_n = P$

then since the argument form with premises $p_1, p_2, \ldots, p_n, q$ and conclusion r is valid

we can know that $P \wedge q \Rightarrow r$

let $A = P \wedge q$, $B = r$

$A \to B \equiv T$ $\quad A \wedge \neg B \equiv F$

let $C = P$. $D = q \to r$.

$$C \to D \equiv \neg P \vee (q \to r)$$
$$\equiv P \to (q \to r)$$
$$\equiv (P \wedge q) \to r$$
$$\equiv A \to B$$
$$\equiv T$$

$C \to D$ is a tautology so $C \Rightarrow D$

So the argument form with premises $p_1, p_2, \cdots, p_n$ and conclusion $q \to r$ is valid.

4. (40 points) Suppose that the following two premises are true:

P $\quad$ Q

- "Math is hard or Leibniz doesn't like Math";
- "If SI120 is easy, then Math is not hard".

R $\quad$ $\neg P$

Which of the following conclusions are true under the above premises?

(a) "If Leibniz likes Math, then SI120 is not easy."

(b) "If SI120 is not easy, then Leibniz doesn't like Math."

(c) "Math is not hard or SI120 is not easy."

(d) "If Leibniz doesn't like Math, then either SI120 is not easy or Math is not hard."

Justify your answers.

let $(P \vee Q) \wedge (R \to \neg P) = A$

(a) let $B = (\neg Q \to \neg R)$

(1) $R \to \neg P$ premise

(2) $\neg R \vee \neg P$ logical equivalence using (1)

(3) $P \vee Q$ premise

(4) $Q \vee \neg R$ Resolution using (2) and (3)

(5) $\neg Q \to \neg R$ logical equivalence using (4).

So (a) is true.

(b) let $B = (\neg R \to Q)$

from (a) (5) we can get $\neg Q \to \neg R$

still using logical equivalence on it

we can get $R \to Q$

So (b) is not true

(c) let $B = (\neg P \vee \neg R)$

from (a) (2) we have know that

$\neg R \vee \neg P \equiv \neg P \vee \neg R$

so (c) is true

(d) let $B = Q \to (\neg R \vee \neg P) \equiv Q \to \neg (R \wedge P)$

$\equiv (R \wedge P) \to \neg Q$

(1) $R \to P$ premise

(2) R Simplification using (1)

(3) $R \to \neg Q$ logical equivalence using (2)

(4) $\neg R \vee \neg Q$ logical equivalence using (3)

(d) (4) is different from (a) (4)

so (d) is not true

Conclusion: (a) and (c) are true.