王柯皓 2021533025

1. (a) for $x_1(t)=1$, $y_1(t)=10$

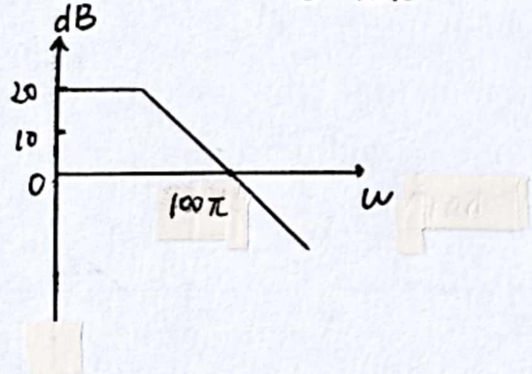
$$a \times 10 = 1000\pi \qquad a = 100\pi$$

So $$\frac{dy(t)}{dt} + 100\pi\, y(t) = 1000\pi\, x(t)$$

$$j\omega Y(j\omega) + 100\pi Y(j\omega) = 1000\pi X(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1000\pi}{100\pi + j\omega}$$

(b) $$H(j\omega) = 10\,\frac{1}{1+j\omega\frac{1}{100\pi}}$$



(c). $$y_2(t) = 50\,e^{j10\pi t} + 10\,e^{j1000\pi t}\,e^{j\frac{\pi}{4}}$$

王柯皓 2021533025

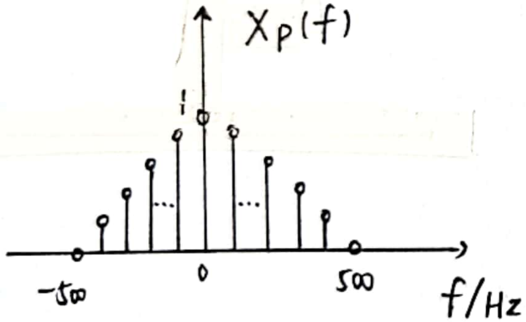
2. (a) $x_p(t) = \sum x_a(nT_1) \dfrac{\sin[\frac{\pi}{T_1}(t-nT_1)]}{\frac{\pi}{T_1}(t-nT_1)}$

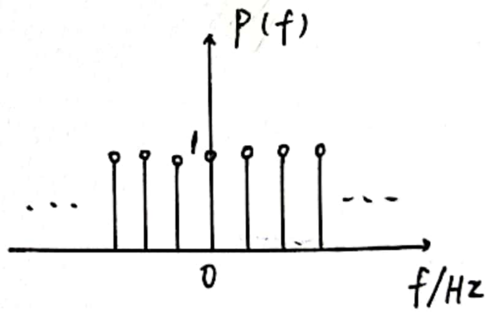
$x_d[n] = \sum x_a(nT_1)$

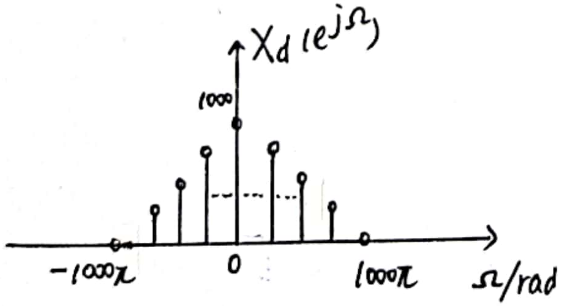
$y[n] = \sum x_a(\frac{nT_1}{2})$

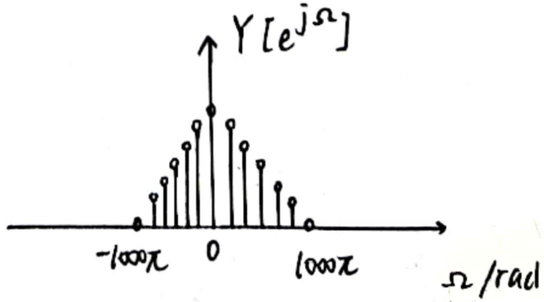
(b) $0 < T_1 \le \frac{1}{1000}$

(c) $T_1 = \frac{1}{1000}$ s









王柯皓 2021533025

3. (a) from $x(t)=15$, $y(t)=-29$

$-15\times 29 = A\times 15$

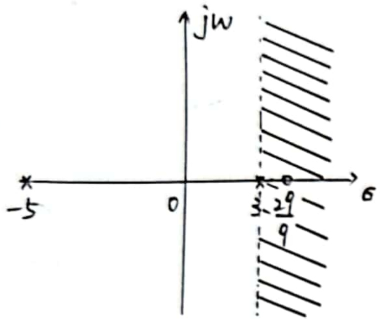
so $A=-29$

$$\frac{d^2y(t)}{dt^2}+2\frac{dy(t)}{dt}-15y(t)=9\frac{dx(t)}{dt}-29x(t)$$

$$s^2Y(s)+2sY(s)-15Y(s)=9sX(s)-29X(s)$$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{9s-29}{s^2+2s-15}=\frac{9s-29}{(s+5)(s-3)}=\frac{\frac{37}{4}}{s+5}-\frac{\frac{1}{4}}{s-3}$$

(b)

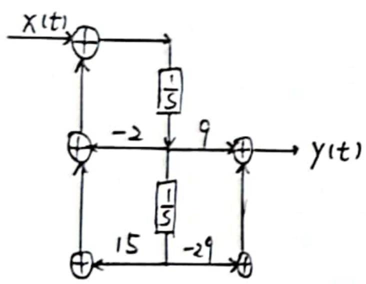


ROC: $Re\{s\}>3$ and $Re\{s\}\neq\frac{29}{9}$

(c) $H(s)$ is not stable (（b) doesn't include $j\omega$ axis)

$H(s)$ is causal

(d)



(e) $H(s)=\frac{\frac{37}{4}}{s+5}-\frac{\frac{1}{4}}{s-3}$

so $h(t)=\frac{37}{4}e^{-5t}u(t)-\frac{1}{4}e^{3t}u(t)$

王柯皓 2021533025

4. (a) for $y[n]=[-\frac{7}{4}(-7)^n-\frac{1}{4}]u[n]$ $\quad Y(z)=-\frac{7}{4}\cdot\frac{1}{1+7z^{-1}}-\frac{1}{4}\cdot\frac{1}{1-z^{-1}}$

for $x[n]=\alpha u[n]$ $\quad X(z)=\frac{\alpha}{1-z^{-1}}$

for $y[n]+7y[n-1]=x[n]-2x[n-1]$

$$Y(z)+7z^{-1}Y(z)+7y[-1]=X(z)-2z^{-1}X(z)-2x[-1]$$

$$(1+7z^{-1})Y(z)+7y[-1]=(1-2z^{-1})X(z)$$

$$-\frac{7}{4}-\frac{1}{4}\cdot\frac{1+7z^{-1}}{1-z^{-1}}+7\beta=\frac{\alpha(1-2z^{-1})}{1-z^{-1}}$$

we can get $\beta=\frac{4}{7}$ $\quad \alpha=2$

(b). for zero-state. $y[-1]=0$

then $(1+7z^{-1})Y(z)=(1-2z^{-1})X(z)$

since $X(z)=\frac{\alpha}{1-z^{-1}}=\frac{2}{1-z^{-1}}$

$$Y(z)=\frac{1-2z^{-1}}{1+7z^{-1}}\cdot\frac{2}{1-z^{-1}}=\frac{\frac{9}{4}}{1+7z^{-1}}-\frac{\frac{1}{4}}{1-z^{-1}}$$

so $y_{zs}[n]=\frac{9}{4}\cdot(-7)^n u[n]-\frac{1}{4}u[n]$

for zero-input $x[n]=0$

so $(1+7z^{-1})Y(z)+7y[-1]=0$

$$Y(z)=\frac{-7y[-1]}{1+7z^{-1}}=\frac{-4}{1+7z^{-1}}$$

so $y_{zi}[n]=-4\cdot(-7)^n u[n]$

(it's true that $y_{zs}[n]+y_{zi}[n]=y[n]$).