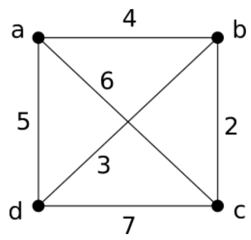
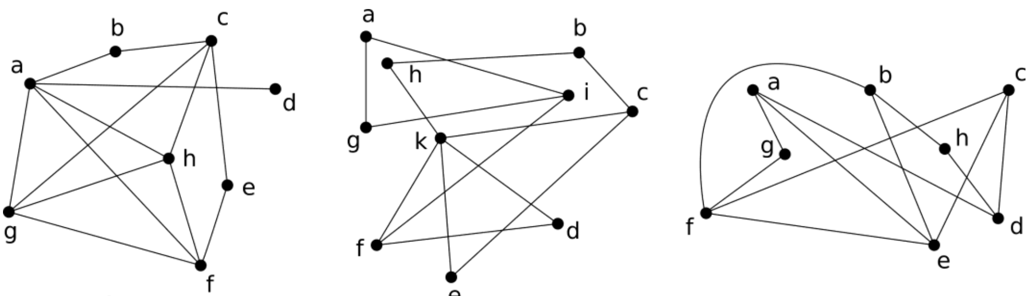
Discrete Mathematics: Homework 13

(Deadline: 2022/6/12)

1. (10 points) Find a planar graph G with $\chi(G) = 4$.
2. (20 points) Let G be a planar graph and $d(v) = 3$ for any vertex v. Show there is a face with at most 5 edges.
3. (10 points) Solve the traveling salesperson problem for this graph.



4. (10 points) Are the graphs G, H, K below planar?



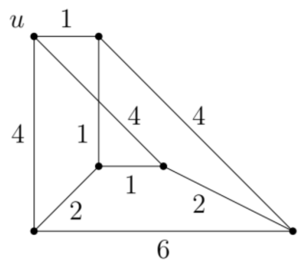
G H K

5. (10 points) The letters A, B, C, D, E, F, G and H denote 8 fishes. In the table below, a circle means that the fishes can cohabit in the same aquarium, a cross means that they cannot.

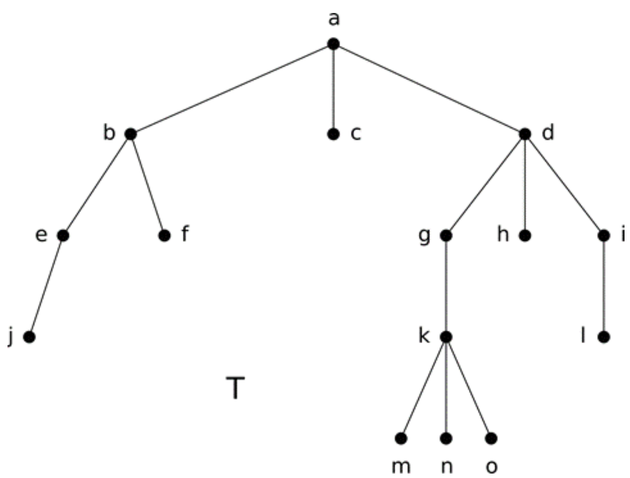
	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*
A	○	×	×	×	○	○	×	×
B	×	○	○	○	×	×	×	○
C	×	○	○	×	○	×	×	×
D	×	○	×	○	×	○	○	×
E	○	×	○	×	○	×	×	○
F	○	×	×	○	×	○	○	○
G	×	×	×	○	×	○	○	○
H	×	○	×	×	○	○	○	○

(a) Model this problem by a graph.
(b) Find the chromatic number of the graph.
(c) Deduce the minimal number of aquarium needed for the fishes.

6. (30 points) For the weighted graph shown in the figure use Dijkstra's algorithm to compute the distance $d(u, v)$ for every $v \in V$. For each step k of the algorithm write down explicitly the set S_k and the labels $L_k(v)$ for every $v \in V$.



7. (10 points)



(a) Given the rooted tree T, answer the following questions:

i. Is T a m-ary tree for some positive integer m? If not, what is the minimal number of edges to add to T to make it a m-ary tree?

ii. Is T a full m-ary tree for some positive integer m? If not, what is the minimal number of edges to add to T to make it a full m-ary tree? Draw the corresponding m-ary tree.

iii. Is T balanced? If not, what is the minimal number of edges to add to T to make it balanced? Draw the corresponding balanced tree.

(b) Let n be a power of 2. How many steps are needed to add n numbers using a tree-connected network of $n - 1$ processors? Explain your answer.

8. (20 points) Show that a connected simple graph G is a tree $\iff$ every edge e of G is a bridge (i.e. $G \setminus e$ is not connected).