

王柯皓 2021533025

1. (a) F

(f) F

(b) T

(g) F

(c) T

(h) T

(d) T

(i) F

(e) F

(j) T



王 柯 皓 2021533025

2. (a) B

(d) B

(b) B

(e) D

(c) C



王柯皓 2021533025

3. (a) ACD (d) BCD
(b) AC (e) C
(c) D



4. (a) i. $S-A-B-D-T$

ii. $S-A-B-C-D-T$

(b) i. $\alpha) S-A-B-C-D-T$

$\beta) S \rightarrow A \rightarrow C \rightarrow T \quad 4$

ii $\alpha) S-A-C-D-T$

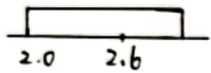
$\beta) S \rightarrow A \rightarrow C \rightarrow T \quad 4$

$\gamma) \text{ both admissible and consistent}$

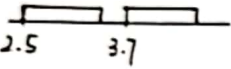


(a) let the unit-interval start at the given points

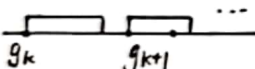
① if the next given point has been included in the former unit-interval then there is no need to cover a new unit-interval

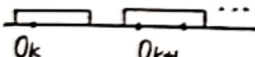
i.e.  2 points are 2.0 and 2.6
the only unit-interval is $[2, 3]$

② if the next given point is not in the former unit-interval then we need 2 new unit-interval separately.

i.e.  2 points are 2.5 and 3.7
we need two unit-interval $[2.5, 3.5]$ $[3.7, 4.7]$

(b) suppose g_i, o_i are the i -th unit-interval in greedy algorithm and optimal algorithm and $g_1 = o_1, g_2 = o_2, \dots, g_k = o_k$ for $i \in (1, k)$. the i -th point is x_i

Greedy: 

Optimal: 

$$\text{for } i \in (k+1, +\infty) \quad \underbrace{x_k - d_k \leq o_{k+1}}_{\text{optimal}} \leq \underbrace{x_k + d_k = g_{k+1}}_{\text{greedy}}$$

it shows that the greedy algorithm may not be so feasible as the optimal algorithm but it still works optimally to find the minimum number of unit-intervals to cover all the given points.



王柯皓 2021533025

6. For k -color problem, it can always be reduced to the sum of 3-color problems and 4-color problems

since k can be represented as $k = n_1 \times 3 + n_2 \times 4$

where n_1, n_2 are all intergers

Since 3-color problem and 4-color problem are both in NP-Complete

k -color problem is also in NP-Complete

王柯皓 2021533025



7. (a) if slots $j=1, \dots, i-1$ are all incompatible with slot i

$$p(i) = 0$$

else

if slot $j=i-1$ is compatible with slot i

$$p(i) = p(i-1) + 1$$

else

$$p(i) = p(i-1)$$

(b) $OPT(i)$ = the maximum total revenue of placing i billboards which are all compatible to each other.

$$(c) \quad OPT(i) = \begin{cases} r_1 & (i=1) \\ OPT(i-1) + r_{p(i)} & (i > 1) \end{cases}$$

(d) $\Theta(n^2)$

