

1. (4 points) True or False

For each statement, choose T if the statement is correct, otherwise, choose F.

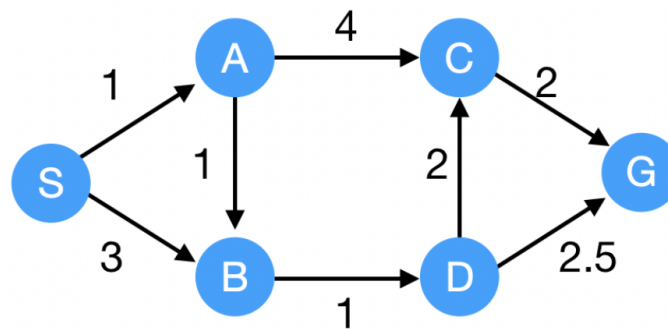
Note: You should write down your answers in the box below.

(a)	(b)	(c)	(d)
F	F	T	T

- (a) (1') The Floyd-Warshall algorithm can return the shortest path between all pairs of nodes in a connected graph with n nodes in $O(n^3)$, while this algorithm only needs $O(n^2)$ to find the shortest path between a given pair of nodes in the graph (in the worst case).
- (b) (1') In any connected graph without negative cycle, A* graph-search algorithm with admissible Heuristics can always find the shortest path between two nodes.
- (c) (1') Floyd-Warshall algorithm can work with positive or negative weights with no negative cycle.
- (d) (1') We can modify Floyd-Warshall algorithm to detect whether there exists a negative cycle or not in a directed graph.

2. (8 points) A* algorithm

Given the heuristic for each node. Consider A* graph search from S to G on the graph below. Arcs are labeled with weights. Assume that ties are broken alphabetically when there is a tie (so a partial plan $S \rightarrow X \rightarrow A$ would be expanded before $S \rightarrow X \rightarrow B$ and $S \rightarrow A \rightarrow Z$ would be expanded before $S \rightarrow B \rightarrow A$).



	S	A	B	C	D	G
heuristic	5.5	4.5	6	2	2.5	0

- (a) (2') The given heuristic values are
(A) Admissible as well as consistent

- (B) Admissible but not consistent
 - (C) Consistent but not admissible
 - (D) Neither admissible nor consistent
- (b) (4') If the heuristic meet admissibility and consistency, write down the path returned by A* graph search. If not, change the heuristic of only one node to make the heuristic meet admissibility and consistency. You should write down the node you choose and the new heuristic value for the choosen node.

Solution:

The node chosen is B with new value 3.5

Considering admissibility, $h(B) \leq 3.5$; Considering consistency, $h(B) \geq 3.5$ (A-B-G), so set $h(B) = 3.5$

- (c) (2') To solve the N Puzzle problems with the A* search, which of following heuristics is/are admissible?
- (A) $h =$ if the state is the goal state, it is 1; otherwise, it is 0
 - (B) $h =$ number of misplaced tiles
 - (C) $h =$ the sum of the minimum number of moves required to put each tile in its correct location
 - (D) $h =$ actual minimum number of moves required to put all tile in its correct location

3. (3 points) Floyd-Warshall's algorithm

- (a) (3') We run Floyd-Warshall algorithm on a graph with n vertices $v_1, v_2, \dots, v_{n/2}, \dots, v_n$ (n is even). Suppose all three loops (k, i, j) are iterated from 1 to n . After running at least(C) iterations of the out-most loop k , it is ensured to find the shortest path between $v_{n/2}$ and v_n .
- (A) $\frac{n}{2} - 1$ (B) $\frac{n}{2}$ (C) $n-1$ (D) n

Algorithm 1 Floyd-Warshall's algorithm

```

1: procedure FLOYD-WARSHALL( $V, E$ )
2:   let dist be a  $|V| \times |V|$  array of minimum distances initialized to  $\infty$  (infinity)
3:   for each edge( $u, v$ )  $\in E$  do
4:     dist[ $u$ ][ $v$ ]  $\leftarrow w(u, v)$  // The weight of the edge ( $u, v$ )
5:   end for
6:   for each vertex  $v \in V$  do
7:     dist[ $v$ ][ $v$ ]  $\leftarrow 0$ 
8:   end for
9:   for  $k$  from 1 to  $|V|$  do
10:    for  $i$  from 1 to  $|V|$  do
11:      for  $j$  from 1 to  $|V|$  do
12:        if dist[ $i$ ][ $j$ ]  $>$  dist[ $i$ ][ $k$ ] + dist[ $k$ ][ $j$ ] then
13:          dist[ $i$ ][ $j$ ]  $\leftarrow$  dist[ $i$ ][ $k$ ] + dist[ $k$ ][ $j$ ]
14:        end if
15:      end for
16:    end for
17:  end for
18:  return dist
19: end procedure

```
