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§ 4.9 (Page 472):

In Exercises 1–2, find the domain and codomain of the transformation $T_A(\mathbf{x}) = A\mathbf{x}$.

2. (a) A has size 4×5. domain : R^5 codomain : R^4

(b) A has size 5×4. domain: R^4 codomain : R^5

(c) A has size 4×4.

(d) A has size 3×1.

4. If $T(x_1, x_2, x_3) = (x_1 + 2x_2, x_1 - 2x_2)$, then the domain of T is R^3, the codomain of T is R^2, and the image of $\mathbf{x} = (0, -1, 4)$ under T is $(-2, 2)$.

8. Find the standard matrix for the transformation defined by the equations.

(a)
$$\begin{aligned} w_1 &= 2x_1 - 3x_2 + x_4 \\ w_2 &= 3x_1 + 5x_2 - x_4 \end{aligned}$$

$$\begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix}$$

(b)
$$\begin{aligned} w_1 &= 7x_1 + 2x_2 - 8x_3 \\ w_2 &= -x_2 + 5x_3 \\ w_3 &= 4x_1 + 7x_2 - x_3 \end{aligned}$$

(c)
$$\begin{aligned} w_1 &= -x_1 + x_2 \\ w_2 &= 3x_1 - 2x_2 \\ w_3 &= 5x_1 - 7x_2 \end{aligned}$$

$$\begin{bmatrix} -1 & 1 \\ 3 & -2 \\ 5 & -7 \end{bmatrix}$$

10. Find the standard matrix for the operator T defined by the formula.

(a) $T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)$

(b) $T(x_1, x_2) = (x_1, x_2)$

(c) $T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3, x_1 + 5x_2, x_3)$

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

12. In each part, find $T(x)$, and express the answer in matrix form.

(a) $\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$; $\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$

$$T(x) = [T]x = \begin{bmatrix} -1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \end{bmatrix}$$

14. Use matrix multiplication to find the reflection of $(-1, 2)$ about

(a) the x-axis.

(b) the y-axis.

(c) the line $y = x$.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ $(-1, -2)$

(b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $(1, 2)$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $(2, -1)$

16. Use matrix multiplication to find the orthogonal projection of $(2, -5)$ on

(a) the x-axis.

(b) the y-axis.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $(2, 0)$

(b) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -5 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \end{bmatrix}$ $(0, -5)$

(d) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $(4, 3)$

18. Use matrix multiplication to find the image of the vector $(3, -4)$ when it is rotated through an angle of

(a) $\theta = 30^\circ$.

(b) $\theta = -60^\circ$.

(c) $\theta = 45^\circ$.

(d) $\theta = 90^\circ$.

(a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{3\sqrt{3}+4}{2} \\ \frac{3-4\sqrt{3}}{2} \end{bmatrix}$ $\left(\frac{3\sqrt{3}+4}{2}, \frac{3-4\sqrt{3}}{2}\right)$

(b) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{3-4\sqrt{3}}{2} \\ \frac{-3\sqrt{3}-4}{2} \end{bmatrix}$ $\left(\frac{3-4\sqrt{3}}{2}, \frac{-3\sqrt{3}-4}{2}\right)$

(c) $\begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} \frac{7\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$ $\left(\frac{7\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

20. Find the standard matrix for the operator that rotates a vector in R^3 through an angle of -60° about

(a) the x-axis.

(b) the y-axis.

(c) the z-axis.

(a). $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & \cos\theta \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

23. Use Formula 15 to derive the standard matrices for the rotations about the x-axis, y-axis, and z-axis in R^3.

x-axis
$a=1$
$b=c=0$
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

y-axis
$a=c=0$
$b=1$
$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

z-axis
$a=b=0$
$c=1$
$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

24. Use Formula 15 to find the standard matrix for a rotation of $\pi/2$ radians about the axis determined by the vector $\mathbf{v} = (1, 1, 1)$. [Note: Formula 15 requires that the vector defining the axis of rotation have length 1.]

$\vec{u} = (\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$

$\cos\theta = 0 \quad \sin\theta = 1$

$\begin{bmatrix} \frac{1}{3} & \frac{1-\sqrt{3}}{3} & \frac{1+\sqrt{3}}{3} \\ \frac{1+\sqrt{3}}{3} & \frac{1}{3} & \frac{1-\sqrt{3}}{3} \\ \frac{1-\sqrt{3}}{3} & \frac{1+\sqrt{3}}{3} & \frac{1}{3} \end{bmatrix}$

30. In words, describe the geometric effect of multiplying a vector $\mathbf{x}$ by the matrix A.

(a) $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(b) $A = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

(a) expansion of R^2 in the x-direction with factor 2 and y-direction with factor 3

(b) rotation through an angle $\frac{\pi}{6}$