Discrete Mathematics: Homework 1

(Deadline: 8:00am, Feb 25, 2022)

1. (15 points) Show that $\log_5 7$ is an irrational number.
2. (20 points) Let p be a prime and let k be an integer such that $0 < k < p$. We know that the binomial coefficient
$$\binom{p}{k} = \frac{p!}{k!(p-k)!}$$
is an integer. Show that $\binom{p}{k}$ is a multiple of p.
3. (20 points) Let $a, b > 1$ be relatively prime integers. Show that if $a|n$ and $b|n$, then $ab|n$.
4. (25 points) Let $a, b, c \in \mathbb{Z}^+$. Show that $\gcd(a, bc) = 1$ if and only if $\gcd(a, b) = \gcd(a, c) = 1$.
5. (20 points) Let $\mathbb{R}$ be the set of real numbers. Let $S = (\mathbb{R} \times \mathbb{R}) \setminus \{(0, 0)\}$. Let
$$R = \Big\{((a, b), (c, d)) : (a, b), (c, d) \in S \text{ and } \exists \lambda \in \mathbb{R} \setminus \{0\} \text{ such that } (a, b) = (\lambda c, \lambda d)\Big\}$$
Show that R is an equivalence relation.