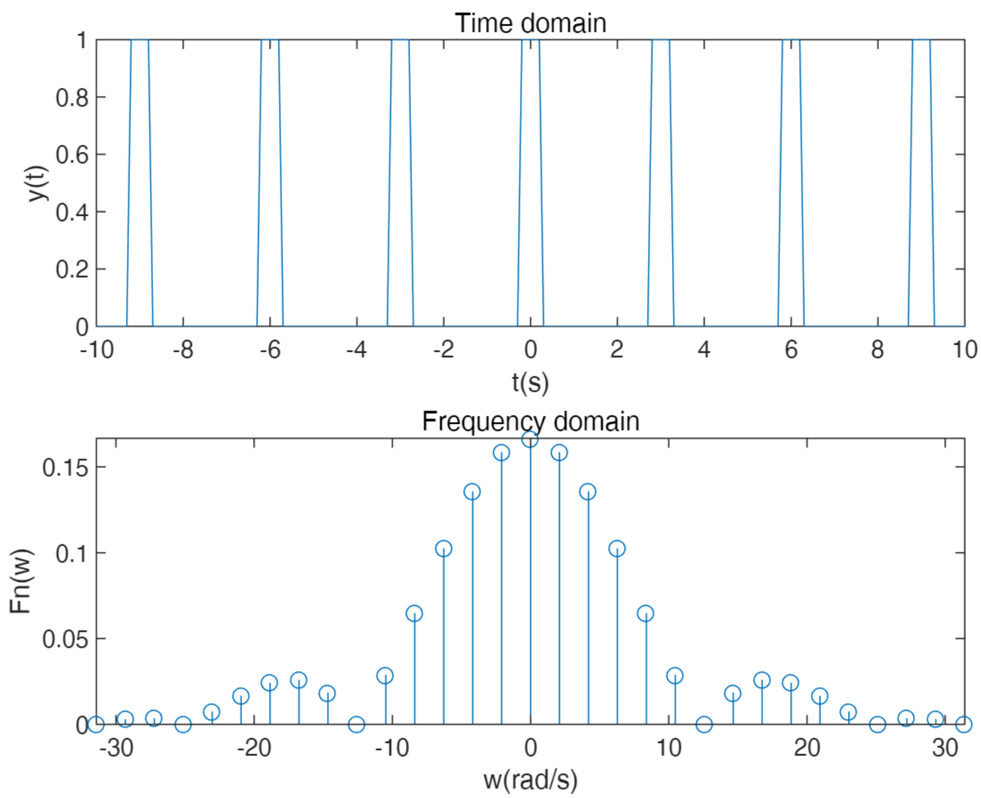
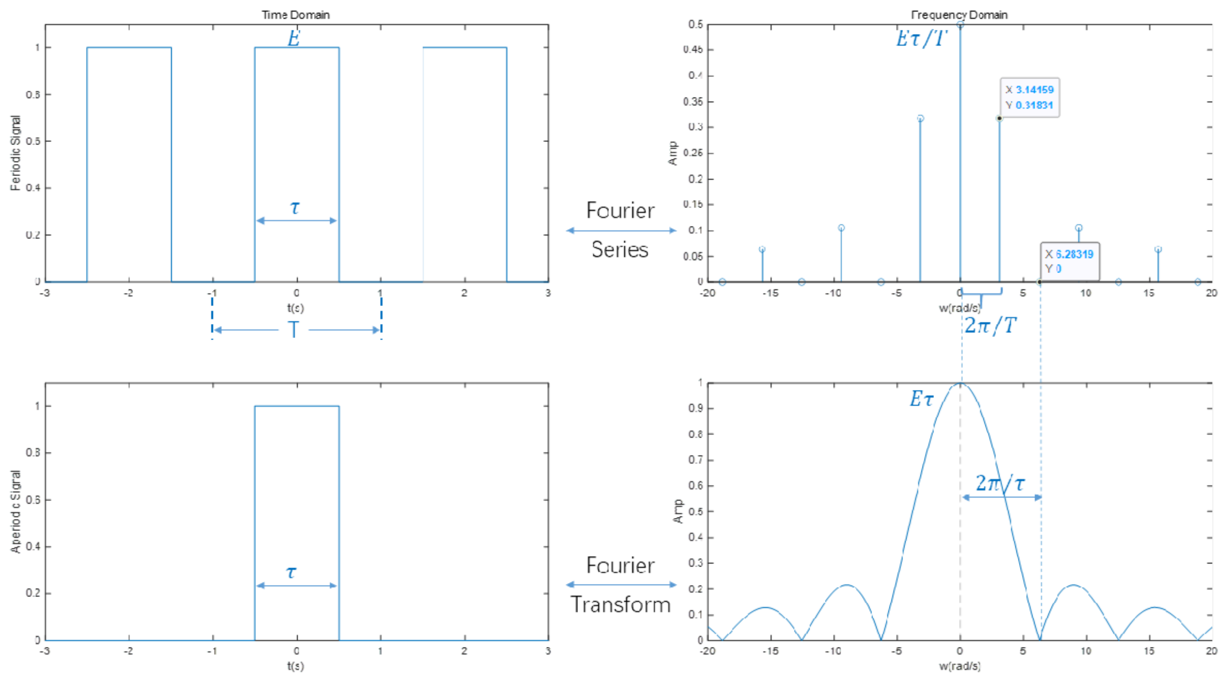
From Periodic to Aperiodic signal

```
clear; clf;
dt = 0.1;
t = -10:dt:10;
T = 3;
tao = 0.5;
f = 0.5*(1+square(2*pi*(t+tao/2)/T,tao/T*100));
subplot(2,1,1); plot(t,f); axis([-10 10 -inf inf]);
title('Time domain');xlabel('t(s)');ylabel('y(t)')

N = 50; n = -N:N;
w1 = 2*pi/T; Wn = n*w1;
t_tao = -tao/2:dt:tao/2;
ft = 1;
F = ft.*exp(-j*n'*w1*t_tao); Fn = trapz(t_tao,F,2)/T;
subplot(2,1,2);stem(Wn,abs(Fn)); axis([-10*pi 10*pi -inf inf]);
title('Frequency domain');xlabel('w(rad/s)');ylabel('Fn(w)')
```



From Periodic to Aperiodic



Symbolic Method

Fourier Transform (function: fourier)

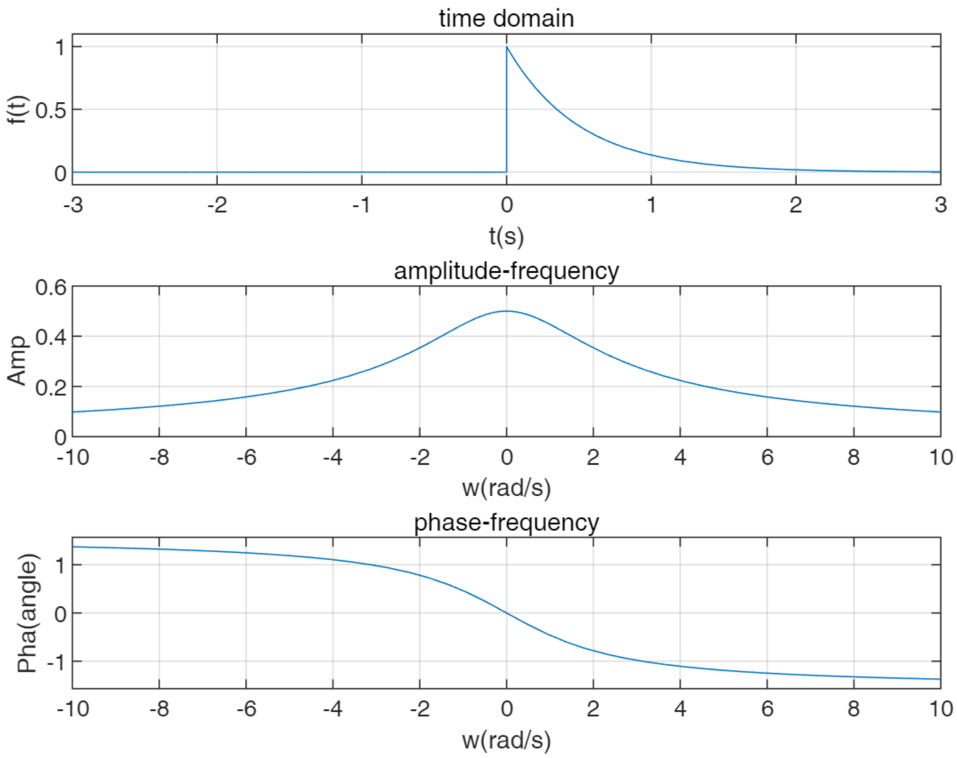
Find out Fourier transform for $f(t) = \begin{cases} e^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}$

```
clear; clf;
syms t
a = 2;
f = exp(-(a*t))*heaviside(t);
subplot(3,1,1);fplot(f);axis([-3,3,-0.1,1.1]);grid on;
title('time domain'); xlabel('t(s)');ylabel('f(t)');
F = fourier(f)
```

F =

$$\frac{1}{2 + w\,\mathrm{i}}$$

```
subplot(3,1,2);fplot(abs(F));% fplot 绘制图形横轴默认为角频率
axis([-10,10,0,.6]);grid on;
title('amplitude-frequency'); xlabel('w(rad/s)');ylabel('Amp');
subplot(3,1,3);fplot(angle(F));
axis([-10,10,-pi/2,pi/2]);grid on;
title('phase-frequency'); xlabel('w(rad/s)');ylabel('Pha(angle)');
```



Inverse Fourier Transform (function: fourier)

Find out inverse Fourier Transform of $F(j\omega) = \dfrac{1}{2 + i\omega}$

```
syms t w
ifourier(1/(2+1i*w),t)
```

ans =

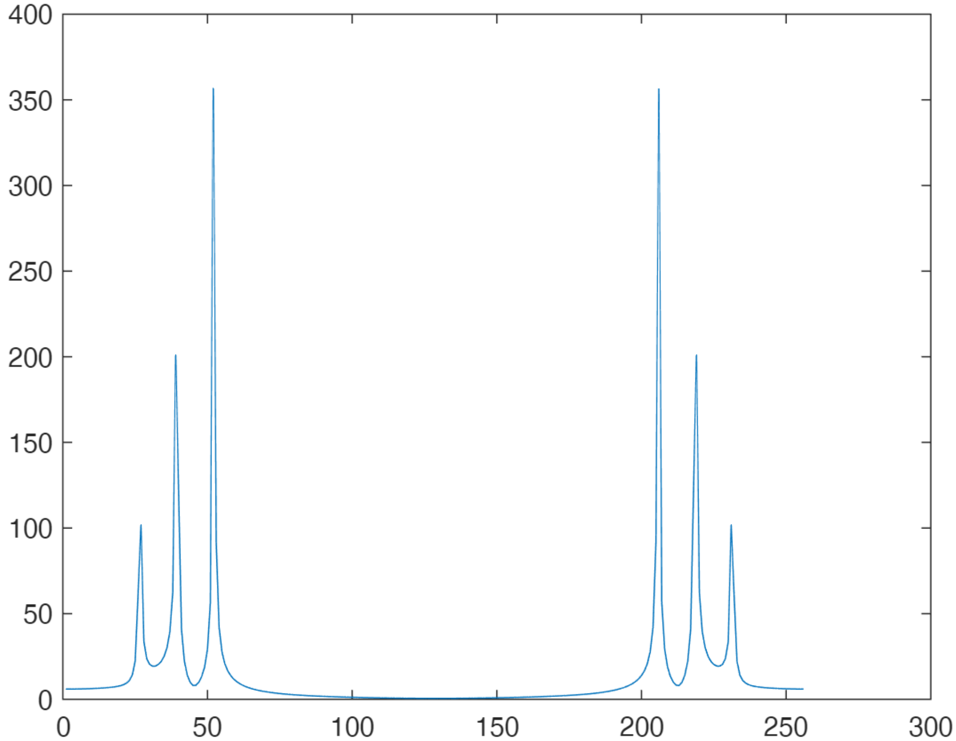
$$\frac{e^{-2t}\,(\mathrm{sign}(t) + 1)}{2}$$

Numeric Method

fft for periodic signal

$x = \cos(2\pi \cdot 10 \cdot t) + 2\sin(2\pi \cdot 15 \cdot t) + 3\cos(2\pi \cdot 20 \cdot t)$

```
clear; clf;
N = 256;
Fs = 100; dt = 1/Fs;        % ts=1/Fs;   t = 0:ts:5;
t = (0:N-1)*dt;    % t=(0:N-1)*ts
x = 1*cos(2*pi*10.*t)+2*sin(2*pi*15.*t)+3*cos(2*pi*20.*t);
X = fft(x);
plot(abs(X));
```

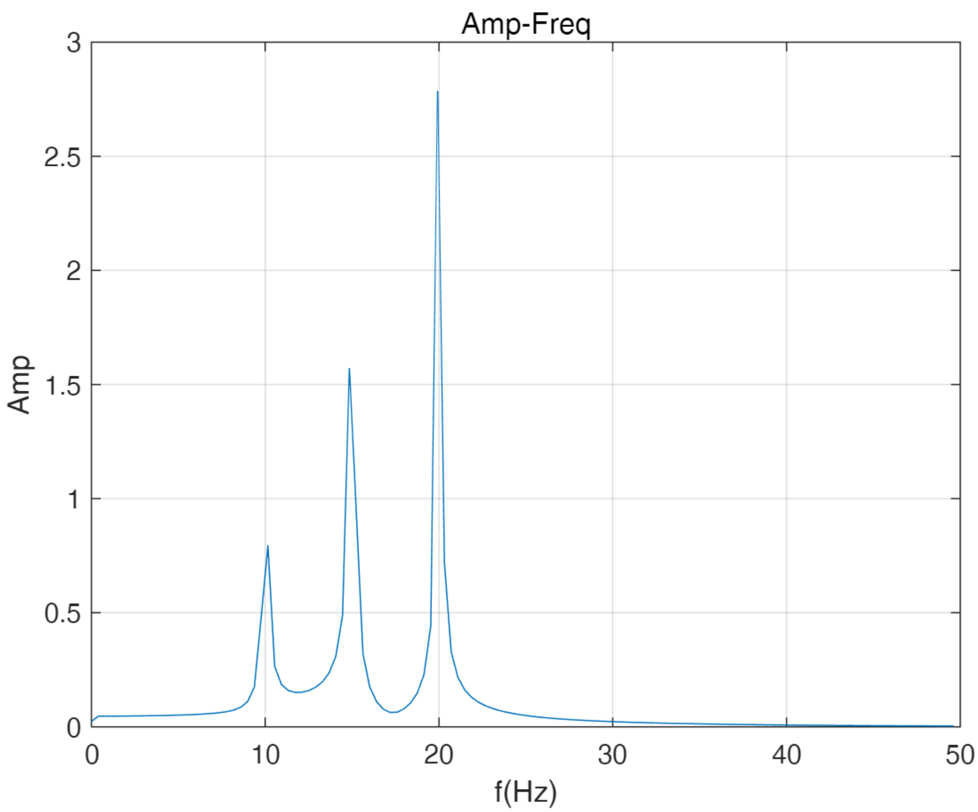


Adjustment

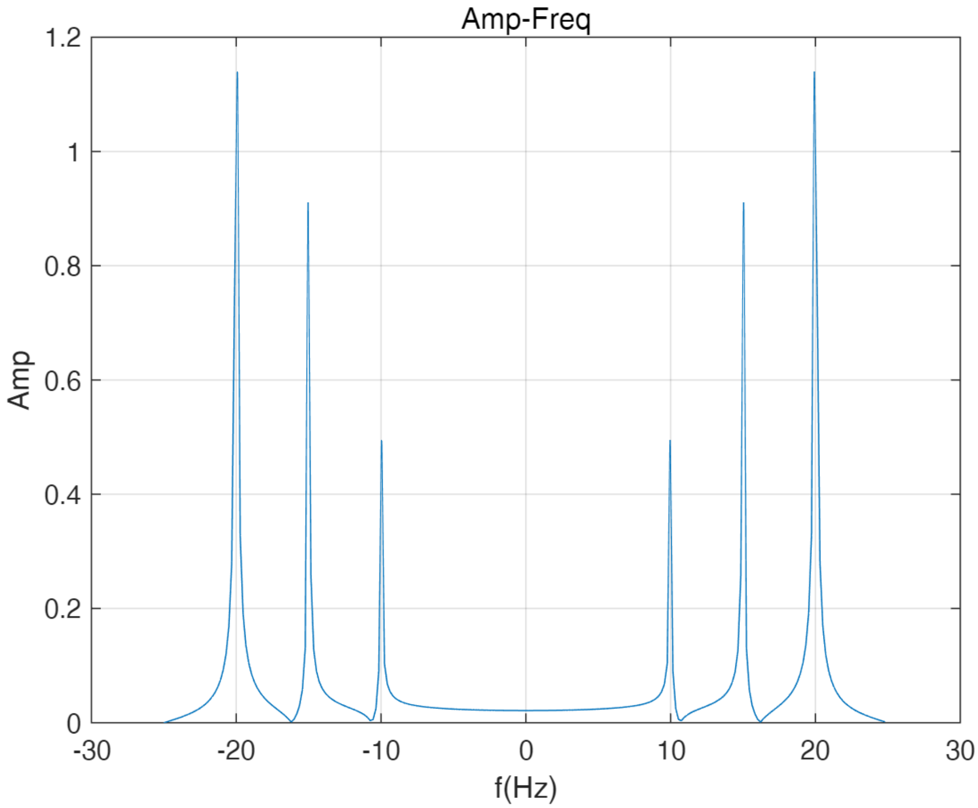
$x = \cos(2\pi \cdot 10 \cdot t) + 2\sin(2\pi \cdot 15 \cdot t) + 3\cos(2\pi \cdot 20 \cdot t)$

1@10Hz, 2@15Hz, 3@20Hz

```
% 单边谱, 动态演示
clear; clf;
N = 256;
Fs = 100;
dt = 1/Fs;
df = Fs/N; %
t = [0:N-1]*dt;
f = [0:N-1]*df; %
x = 1*cos(2*pi*10.*t)+2*sin(2*pi*15.*t)+3*cos(2*pi*20.*t);
X = abs(fft(x))/N; %去一半
X = [X(1),2*X(2:N/2)]; %
f = f(1:N/2);   %
plot(f,X)
grid on;xlabel("f(Hz)");ylabel("Amp");title("Amp-Freq");grid on
```



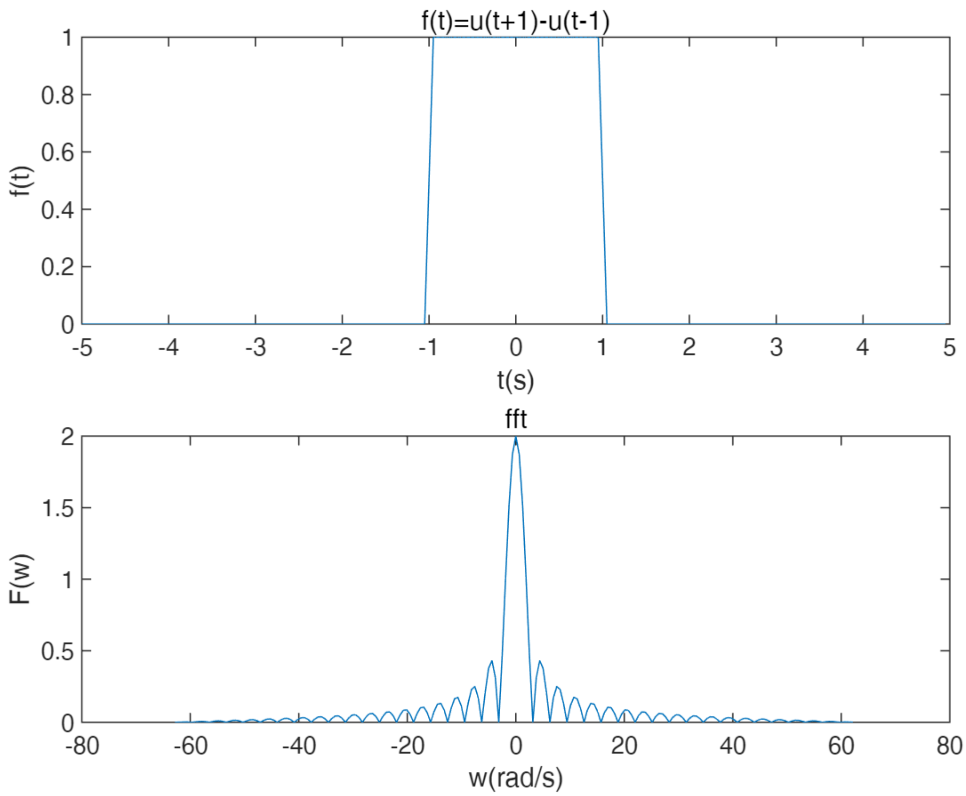

```
% 双边谱
clear; clf;
N = 256;
Fs = 50;              % 采样频率至少为信号最高频率的两倍
df = Fs/N;
t = (0:N-1)/Fs;
x = 1*cos(2*pi*10.*t)+2*sin(2*pi*15.*t)+3*cos(2*pi*20.*t);
X = fft(x);
Y = abs(fft(x))/N;
Yshift = fftshift(Y);    % 将直流分量作为中心点，两侧对称
f = (-N/2:N/2-1)*df;
plot(f,Yshift);xlabel("f(Hz)");ylabel("Amp");title("Amp-Freq");grid on
```



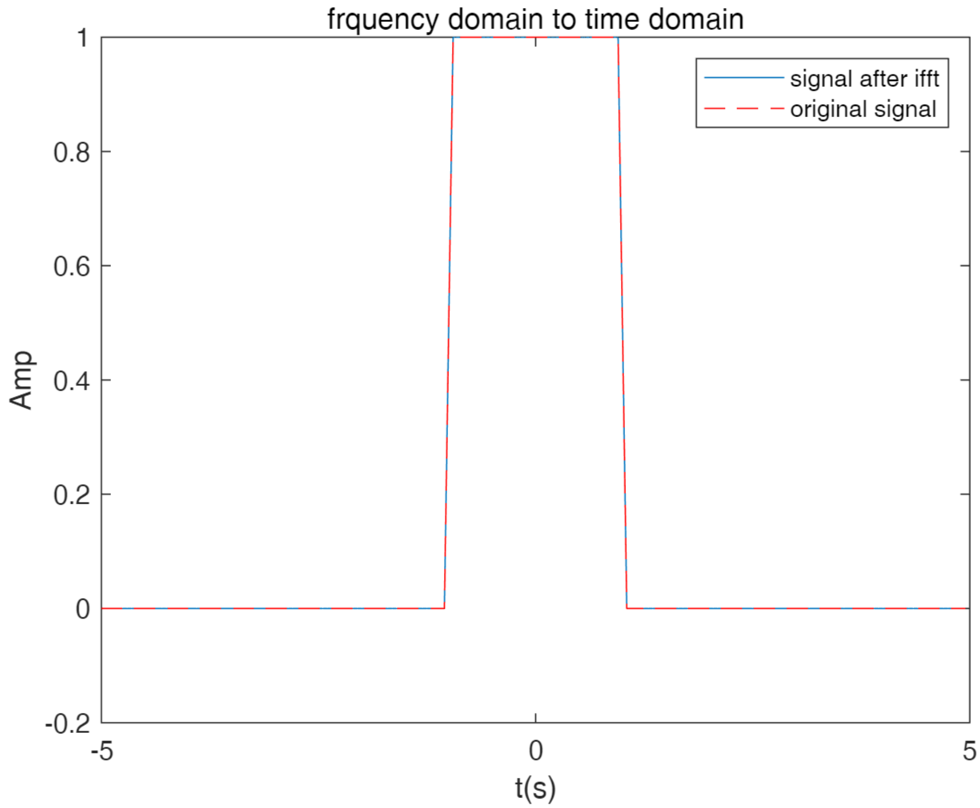
fft for aperiodic signal

$$f(t) = G(t) = \begin{cases} 1 & |t| \le 1 \\ 0 & |t| > 1 \end{cases}$$

```
clf; clear;
% 使用 fft 计算
dt = 0.05;
t = -5:dt:5-dt;
f = heaviside(t+1)-heaviside(t-1);
Fs = 1/dt;
N = length(t);
df = Fs/N;
F1 = fft(f,N);
F = abs(F1)*dt;      % 非周期信号幅度调整*dt
F = fftshift(F);
W = (-N/2:N/2-1)*df*2*pi;
subplot(2,1,1);plot(t,f);xlabel('t(s)');ylabel('f(t)');title('f(t)=u(t+1)-u(t-1)');
subplot(2,1,2);plot(W,F);xlabel('w(rad/s)');ylabel('F(w)');title('fft');
```




```
% ifft
figure
f1 = ifft(F1);
plot(t,f1,t,f,'r--');
xlabel("t(s)");ylabel("Amp");title("frquency domain to time domain");
legend('signal after ifft','original signal')
```



Spectrum Leak

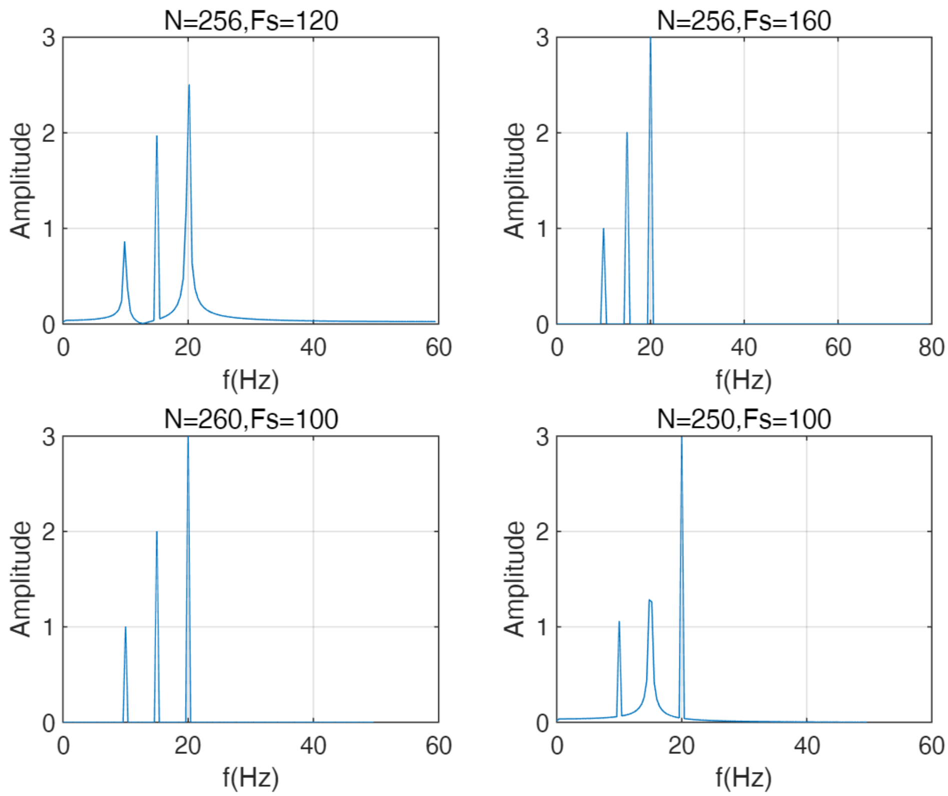
$$f_{\text{sig}} = \frac{\text{Fs}}{N} \cdot k = d_f \cdot k$$

```
clf;clear;
N = 256; Fs = 120;  % df=Fs/N=0.46875   10/0.46875=21.33  15/0.46875=32   20/0.46875=42.67
[f, Yabs] = Lab4_frequency_leak(N,Fs);
subplot(2,2,1); plot(f,Yabs);
title('N=256,Fs=120'); xlabel('f(Hz)'); ylabel('Amplitude'); grid on;

N = 256; Fs = 160;  % df=Fs/N=0.625   10/0.625=16  15/0.625=24   20/0.625=32
[f, Yabs] = Lab4_frequency_leak(N,Fs);
subplot(2,2,2); plot(f,Yabs);
title('N=256,Fs=160'); xlabel('f(Hz)'); ylabel('Amplitude'); grid on;

N = 260; Fs = 100;  % df=Fs/N=100/260   10*260/100=26  15*260/100=39   20*260/100=52
[f, Yabs] = Lab4_frequency_leak(N,Fs);
subplot(2,2,3); plot(f,Yabs);
title('N=260,Fs=100'); xlabel('f(Hz)'); ylabel('Amplitude'); grid on;

N = 250; Fs = 100;  % df=Fs/N=0.4   10/0.4=25  15/0.4=37.5   20/0.4=50
[f, Yabs] = Lab4_frequency_leak(N,Fs);
subplot(2,2,4); plot(f,Yabs);
title('N=250,Fs=100'); xlabel('f(Hz)'); ylabel('Amplitude'); grid on;
```



Matrix for aperiodic sinal

$$\mathrm{F}\,(j\omega)=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt=\lim_{\tau\to 0}\sum_{n=-\infty}^{\infty} f(n\tau)e^{-jwn\tau}\bullet\tau$$

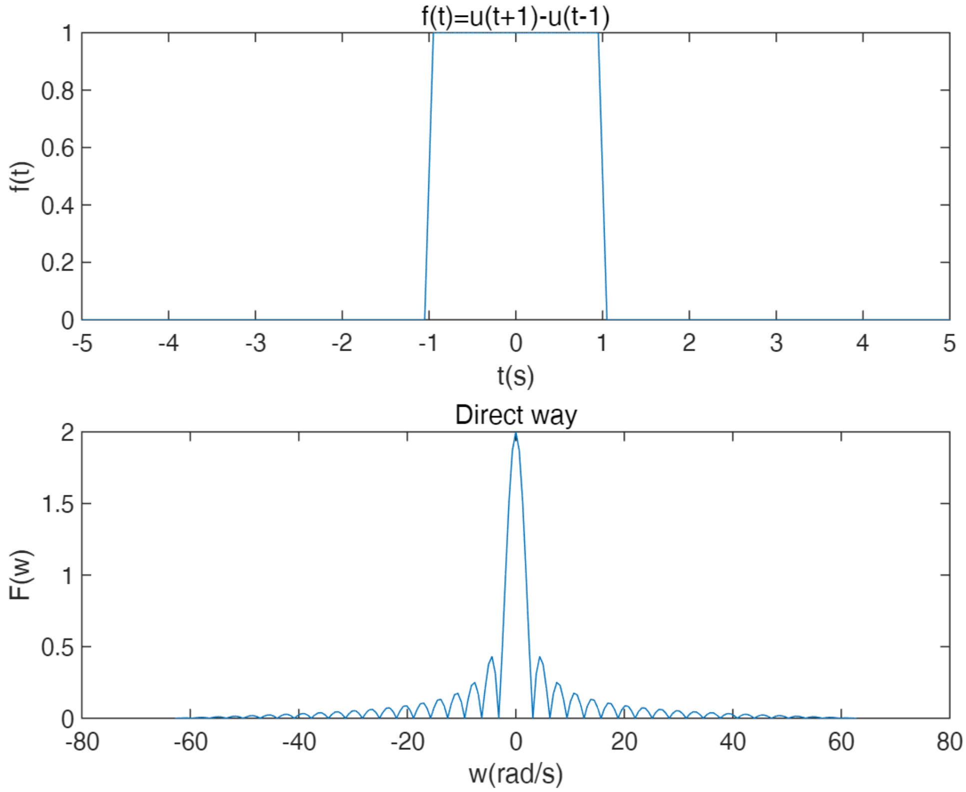
$$F\,(\omega_k)=\tau\sum_{n=-N}^{n=N} f\left(n\tau\right)e^{-j\omega_k n\tau},\ \ -\mathrm{N}\le \mathrm{n}\le \mathrm{N},\ \ -\mathrm{M}\le \mathrm{k}\le \mathrm{M}$$

$$
\mathrm{F}(\mathrm{w}_k) = \begin{bmatrix} F(w_{-M}) \\ \vdots \\ F(w_0) \\ \vdots \\ F(w_M) \end{bmatrix} = \tau * \begin{bmatrix} f(-N\tau)\cdot e^{-jw_{-M}(-N\tau)} + \cdots + f(0\cdot\tau)\cdot e^{jw_{-M}(0\cdot\tau)} + \cdots + f(N\tau)\cdot e^{-jw_{-M}(N\tau)} \\ \vdots \\ f(-N\tau)\cdot e^{-jw_0(-N\tau)} + \cdots + f(0\cdot\tau)\cdot e^{jw_0(0\cdot\tau)} + \cdots + f(N\tau)\cdot e^{-jw_0(N\tau)} \\ \vdots \\ f(-N\tau)\cdot e^{-jw_M(-N\tau)} + \cdots + f(0\cdot\tau)\cdot e^{jw_M(0\cdot\tau)} + \cdots + f(N\tau)\cdot e^{-jw_M(N\tau)} \end{bmatrix}
$$

$$
= \tau * \begin{bmatrix} f(-N\tau) & \cdots & f(0\tau) & \cdots & f(N\tau) \end{bmatrix} * \begin{bmatrix} e^{-jw_{-M}(-N\tau)} & \cdots & e^{-jw_0(-N\tau)} & \cdots & e^{-jw_M(-N\tau)} \\ \vdots & & \vdots & & \vdots \\ e^{-jw_{-M}(0\cdot\tau)} & \cdots & e^{-jw_0(0\cdot\tau)} & \cdots & e^{-jw_M(0\cdot\tau)} \\ \vdots & & \vdots & & \vdots \\ e^{-jw_{-M}(N\tau)} & \cdots & e^{-jw_0(N\tau)} & \cdots & e^{-jw_M(N\tau)} \end{bmatrix}
$$

$$
= \tau * \begin{bmatrix} f(-N\tau) & \cdots & f(0\tau) & \cdots & f(N\tau) \end{bmatrix} * e^{-j\cdot\begin{bmatrix} -N\tau \\ \vdots \\ 0\cdot\tau \\ \vdots \\ N\tau \end{bmatrix} * \begin{bmatrix} w_{-M} & \cdots & w_0 & \cdots & w_M \end{bmatrix}}
$$

```
clear; clf;
% 矩阵计算
dt = 0.05;
t = -5:dt:5;
f = heaviside(t+1)-heaviside(t-1);
M = 100; k = -M:M;
W = 2*pi*10;  % the range of the frequency domain to observe
dw = W/M;     % w1/M determines the resolution of the frequency
Wk = k*dw;
F = f*exp(-1i*t'*Wk)*dt;
F = abs(F);
subplot(2,1,1);plot(t,f);xlabel('t(s)');ylabel('f(t)');title('f(t)=u(t+1)-u(t-1)');
subplot(2,1,2);plot(Wk,F);xlabel('w(rad/s)');ylabel('F(w)');title('Direct way');
```

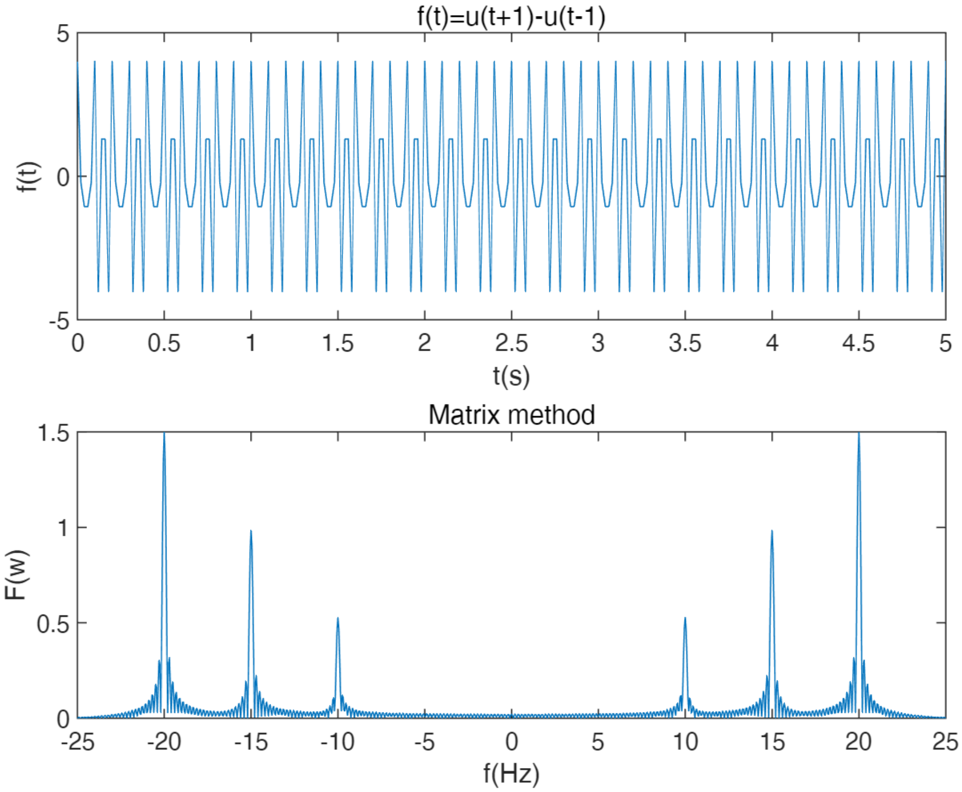


Matrix for periodic signal

$x = \cos(2\pi \cdot 10 \cdot t) + 2\sin(2\pi \cdot 15 \cdot t) + 3\cos(2\pi \cdot 20 \cdot t)$

```
% 矩阵法计算周期信号傅里叶变换
clear;clf;
N = 256;
Fs = 50;  % at least twice the bandwidth or maximum component frequency of the signal
dt = 1/Fs;
t = (0:N-1)/Fs;
f = 1*cos(2*pi*10.*t)+2*sin(2*pi*15.*t)+3*cos(2*pi*20.*t);

M = 500; k = -M:M;
f_range = Fs/2; % indicates the range of the frequency domain to observe
W = 2*pi*f_range;
dw = W/M;       % w/M determines the resolution of the frequency
Wk = k*dw;
F = f*exp(-1i*t'*Wk)*dt;
F = abs(F)/(t(end)-t(1));  % divided by the signal duration
subplot(2,1,1);plot(t,f);
xlabel('t(s)');ylabel('f(t)');title('f(t)=u(t+1)-u(t-1)');xlim([0 5])
subplot(2,1,2);plot(Wk/(2*pi),F);
xlabel('f(Hz)');ylabel('F(w)');title('Matrix method');
```


f(t)=u(t+1)-u(t-1)
f(t)
t(s)
Matrix method
F(w)
f(Hz)

Fourier(F)		
	fft	matrix
aperiodic	abs(F)*dt	abs(F)
periodic	abs(F)/N	abs(F)/t