Discrete Mathematics: Homework 3

(Deadline: 2022/5/22)

1. (10 points) Let $P(x)$ = "x is a person", $L(x, y)$ = "x likes y" and $E(x, y)$ ="$x = y$". Translate the following statements into formulas:

   (a) "Every person likes some other person."

   (b) "There is a person who is liked by every other person."

2. (10 points) Let A be the formula $\forall x \big(\forall y \big((x \neq y) \rightarrow \forall z \big((z = x) \vee (z = y)\big) \big) \big)$.

   (a) Find a domain $D_1 = \emptyset$ such that A is true when x, y, z are taken over D_1.

   (b) Find a domain D_2 such that A is false when x, y, z are taken over D_2.

3. (10 points) Determine if the following formulas are logically valid, satisfiable or unsatisfiable.

   (a) $(\exists x P(x) \leftrightarrow \exists x Q(x)) \rightarrow \exists x (P(x) \leftrightarrow Q(x))$

   (b) $\exists x (\mathbf{T} \vee P(x) \rightarrow \mathbf{F})$

   (c) $\forall x (P(x) \vee \neg \exists y (Q(y) \wedge \neg Q(y)))$

4. (20 points) Show the following statements with interpretations of the formulas.

   (a) $\forall x (P(x) \vee Q(x))$ and $\forall x P(x) \vee \forall x Q(x)$ are not logically equivalent.

   (b) $\exists x (P(x) \wedge Q(x))$ and $\exists x P(x) \wedge \exists x Q(x)$ are not logically equivalent.

5. (10 points) Show that $\exists x (P(x) \vee Q(x)) \equiv \exists x P(x) \vee \exists x Q(x)$.

6. (20 points) Show that $\forall x \big(P(x) \rightarrow Q(x)\big) \Rightarrow \forall x P(x) \rightarrow \forall x\, Q(x)$.

7. (20 points) Show that $\exists x P(x) \wedge \forall x Q(x) \Rightarrow \exists x (P(x) \wedge Q(x))$.