CS 181 Artificial Intelligence (Fall 2018), Midterm Exam

Name (in Chinese): ______________________

ID#: ______________________

Instructions

- Time: 1–2:40pm (100 minutes)
- This exam is closed-book, but you may bring an A4-size cheat sheet. Put all the study materials and electronic devices into your bag and put your bag in the front, back, or sides of the classroom.
- You can write your answers in either English or Chinese. Try writing all your answers on the front of the paper. If you use the back of the paper, please indicate that on the front.
- Two blank pieces of paper are attached, which you can use as scratch paper. Raise your hand if you need more paper.

1 Multiple choice (10 pt)

1.1 Each question has only one correct answer

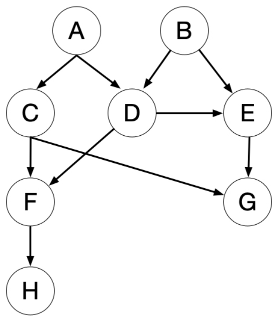
1	2	3	4	5

1. In the context of adversary search, which of the following is correct?
   A. The time complexity and space complexity of minimax search are both $O(b^m)$ when the branching factor is b and the depth is m.
   B. The performance of α-β pruning does not depend on the expanding order of the successor game states.
   C. α-β pruning always returns the same value as minimax for all nodes on the leftmost edge of the tree, assuming successor game states are expanded from left to right.
   D. α-β pruning may prune the direct children of the root node.
   E. None of the above.

2. A and B are propositions. Which of the following statements is correct?
   A. $A \wedge \neg B \equiv \neg(B \Rightarrow A)$
   B. $A \wedge \neg B \equiv \neg(A \Rightarrow \neg B)$
   C. $A \wedge \neg B \equiv \neg(\neg A \vee B)$
   D. $\neg(A \vee \neg B) \equiv \neg B \Rightarrow \neg A$
   E. None of the above statements is correct.

3. Which of the following statements is correct?
   A. The **MGU** (Most general unification) of **Knows(John,x)** and **Knows(John, Jane)** is **{ Jane/ x }** (substituting Jane with x)
   B. The **MGU** (Most general unification) of **Knows(John,x)** and **Knows(y, z)** is **{ y/ John, x/z }**
   C. The statement $(\neg A \vee D) \wedge (A \vee E) \equiv D \vee E$ is true and represents the process of **resolution**
   D. "**IsHuman(x)**" (where x is a variable) is syntactically correct as an expression of first order logic.
   E. None of the above statements is correct.

4. Which of the following statements is true?



   A. $C \perp\!\!\!\perp D | A$
   B. $A \perp\!\!\!\perp E$
   C. $A \perp\!\!\!\perp E | D$
   D. $A \perp\!\!\!\perp G | DE$

E. $C \perp\!\!\!\perp D | AH$

5. Which of the following statements is correct about Markov network (MN)?

   A. Every Bayesian network can be converted into an MN without any loss of conditional independence information.

   B. An MN encodes a joint distribution with a directed graph.

   C. There is no cycle in an MN.

   D. Constraint graphs can be seen as a special case of Markov networks.

   E. None of the above.

Solution:

1. C
2. C
3. B
4. A
5. D

1.2 Each question has one or more correct answers

1	2	3	4	5

1. Which of the following statements of the tree search algorithm is/are correct? (Assuming the best solution has a finite cost and the minimum arc cost is positive)

   A. DFS is complete.

   B. Iterative deepening is complete.

   C. UCS is complete and optimal.

   D. A* is complete and optimal with consistent heuristics.

   E. None of the above.

2. Which of the following statements is/are correct?

   A. An inference algorithm is sound if everything that can be proved is in fact entailed

   B. If a sentence S is unsatisfiable, then $\neg S$ is satisfiable

   C. Forward chaining and backward chaining are sound and complete for propositional logic

   D. $\neg A \vee \neg B \vee C$ is a Horn clause

   E. None of the above statements is correct.

3. Consider the following samples, which of the following is/are true?

   - $-a, +b, +c, -d, +e$ $(w = 0.12)$
   - $+a, +b, -c, +d, +e$ $(w = 0.2)$
   - $+a, +b, -c, +d, +e$ $(w = 0.2)$
   - $-a, +b, +c, -d, +e$ $(w = 0.12)$
   - $+a, +b, +c, +d, +e$ $(w = 0.1)$

   A. Ignore the weights, $P(+a) = 0.5$.

B. Ignore the weights, $P(+b|+c) = 1$.

C. With likelihood weighting, $P(-c|+d, +e) = 0.8$.

D. With likelihood weighting, $P(-d|+e) = 0.4$.

E. None of the above.

4. Which of the following statements is/are correct about VE (Variable Elimination)?

   A. The size of the largest factor only determines the time complexity and does not determine the space complexity.

   B. The elimination ordering can affect the size of the largest factor.

   C. There always exists an elimination ordering that results in polynomial-size factors.

   D. Inference by enumeration is always faster than VE.

   E. The computational complexity of VE can be linear in the BN size for poly-tree BNs.

5. Which of the following statements is/are correct about BN (Bayesian Networks)?

   A. A BN expresses a set of conditional independence relationships.

   B. The size of a BN is exponential in the number of variables.

   C. There is no cycle in a BN.

   D. The size of a BN is $O(kd^n)$ where k is the maximum domain size, d is the maximum number of parents, and n is the number of variables.

   E. A node in a BN can only have one parent in its Markov blanket.

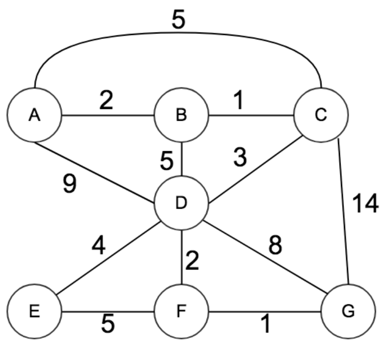
Solution:

1. BCD
2. ABD
3. BC
4. BE
5. AC

2 Search (10 pt)

2.1

Consider the state space graph shown below. A is the start state and G is the goal state. The cost for each edge is shown on the graph. Each edge can be traversed in both directions.



2.1.1 (4 pt)

In each cell of the table below, write "Yes" if the corresponding *graph search* algorithm can possibly return the corresponding path; leave the cell blank otherwise.

	A-D-G	**A-B-D-G**	**A-B-C-D-F-G**
Depth first search			
Breadth first search			
Uniform cost search			
A* search with consistent heuristic			

2.1.2 (3 pt)

Suppose you have a heuristic function h shown below. All the values are given except $h(B)$.

Node	A	B	C	D	E	F	G
h	8	?	5	2	4	0.5	0

If A* with heuristic h returns **A-C-D-F-G**, what is the range of $h(B)$? ________

If heuristic h is consistent, what is the range of $h(B)$? ________

2.2 (3 pt)

Let H_1 be a consistent heuristic and H_2 be an inconsistent one.

(i) (true or false) $(H_1 + H_2)/2$ is necessarily consistent.

(ii) (true or false) $\max(H_1, H_2)$ is necessarily consistent.

(iii) (true or false) $\min(H_1, H_2)$ is necessarily consistent.

Solution:

(1) (1,2,3), (1), (3), (3)

(2) $h(B) \geq 8, h(B) = 6$.

(3) false, false, false

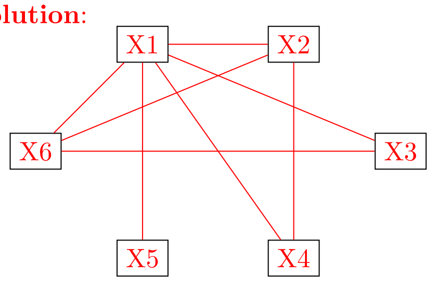
3 CSP (10 pt)

Consider a CSP with variables $X_1, X_2, \cdots, X_6$, domain $\{c_1, c_2, c_3\}$ and two constraints: (1) $X_i \neq X_j$ if i divides j or j divides i, and (2) $X_i \in \{c_j \mid i * j + 1 \text{ is prime}\}$.

3.1 (2 pt)

Draw the constraint graph that shows constraint (1).

Solution:



3.2 (3 pt)

What is the domain of each variable after enforcing constraint (2)?

Solution:

$X_1 : c_1, c_2$

$X_2 : c_1, c_2, c_3$

$X_3 : c_2$

$X_4 : c_1, c_3$

$X_5 : c_2$

$X_6 : c_1, c_2, c_3$

3.3 (2 pt)

Which arcs in the constraint graph are not consistent after enforcing constraint (2)?

Solution:

$X_1 \to X_3$, $X_1 \to X_5$, $X_6 \to X_3$

3.4 (3 pt)

What is the solution? If there are multiple solutions, you can write any one. If there is no solution, write "No solution". (Hint: try enforcing arc consistency by "deleting from the tail".)

Solution:

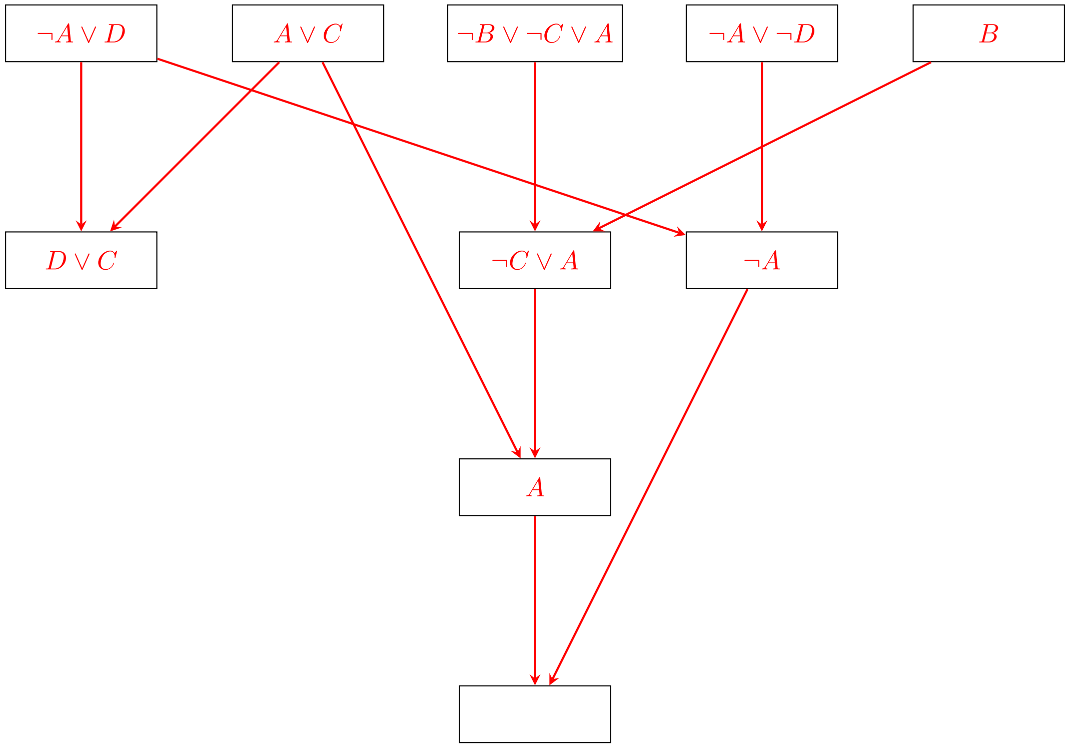
$X_1 = c_1, X_2 = c_2, X_3 = c_2, X_4 = c_3, X_5 = c_2, X_6 = c_3$

4 Logic (10 pt)

4.1 (5 pt)

Use resolution to prove $(A \Rightarrow D) \wedge (A \vee C) \wedge (\neg B \vee \neg C \vee A) \wedge (\neg A \vee \neg D) \models \neg B$.

Solution:



4.2 (5 pt)

Use first-order logic to express the statement: *Anyone who has any cats will not have any mice.* Then convert it to CNF. You can only use the following three predicates: use CAT(x) and MOUSE(x) to indicate x is a cat and a mouse respectively, and use HAVE(x, y) to indicate x has y.

Solution:

- $\forall x[\forall y(HAVE(x,y) \wedge CAT(y) \Rightarrow \neg\exists z, (HAVE(x,z) \wedge MOUSE(z)))]$
- or equivalently, $\forall x[\exists y(HAVE(x,y) \wedge CAT(y)) \Rightarrow \neg\exists z, (HAVE(x,z) \wedge MOUSE(z))]$

Simplification:

- $\forall x[\forall y(HAVE(x,y) \wedge CAT(y) \Rightarrow \forall z, \neg(HAVE(x,z) \wedge MOUSE(z)))]$
- $\forall x[\forall y(\neg(HAVE(x,y) \wedge CAT(y)) \vee (\forall z, \neg(HAVE(x,z) \wedge MOUSE(z))))]$
- $\forall x, \forall y, \forall z, [(\neg(HAVE(x,y) \wedge CAT(y))) \vee (\neg(HAVE(x,z) \wedge MOUSE(z)))]$
- $\forall x, \forall y, \forall z, [(\neg HAVE(x,y) \vee \neg CAT(y)) \vee (\neg HAVE(x,z) \vee \neg MOUSE(z))]$
- Final CNF: $[\neg HAVE(x,y) \vee \neg CAT(y) \vee \neg HAVE(x,z) \vee \neg MOUSE(z)]$

5 Winner, winner, chicken dinner (10 pt)

Armin is playing a game named PUBG where he has to look for airdrops to enrich his equipment. Can you help him?

There are three types of airdrop supply – red, blue, and green. The airdrop time is picked uniformly at random throughout the 24 hours of a day. The probabilities of getting a blue, green, and red airdrop are 50%, 30% and 20% respectively. A red airdrop is light in 9 out of 10 cases during the daytime and is always light at night. A blue airdrop is light in 1 out of 10 cases at night and is always heavy in the daytime. A green airdrop is heavy in 8 out of 10 cases regardless of the time. Armin wants an AWM gun which can only be obtained by airdrop. The probability of getting AWM in a red, blue, and green airdrop is 0.1, 0.1, and 0.9 respectively. Night is from 9pm to 5am and the rest of a day is daytime.

5.1 (2 pt)

Draw a **correct and causal** Bayesian network structure with the least number of edges. You should use the variables below.

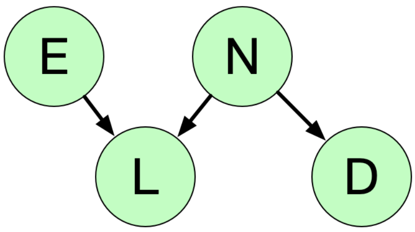
E: Day or night

L: The airdrop is light or heavy

N: The airdrop color

D: An AWM gun is in the airdrop or not

Solution:



5.2 (2 pt)

Write down the conditional probability table for each variable.

Solution:

Pr(E=‘night’) = 1/3
Pr(E=‘day’) = 2/3

Pr(N=‘red’) = 1/5
Pr(N=‘blue’) = 1/2
Pr(N=‘green’) = 3/10

Pr(L=‘light’| E=‘night’, N=‘red’) = 1
Pr(L=‘light’|E=‘day’, N=‘red’) = 9/10
Pr(L=‘light’|E=‘night’, N=‘blue’) = 1/10
Pr(L=‘light’|E=‘day’, N=‘blue’) = 0
Pr(L=‘light’|E=‘night’, N=‘green’) = 1/5
Pr(L=‘light’|E=‘day’, N=‘green’) = 1/5

Pr(D=‘AWM’|N=‘red’) = 1/10
Pr(D=‘AWM’|N=‘blue’) = 1/10
Pr(D=‘AWM’|N=‘green’) = 9/10

5.3 (3 pt)

Armin gets a light airdrop at night. What is the probability of having an AWM gun inside? (You can write your answer in either decimal or fraction form.)

Solution:

$$\begin{aligned}
Pr(D = AWM|E = night, L = light) &= \frac{Pr(D = AWM, E = night, L = light)}{Pr(E = night, L = light)} \\
&= \frac{\sum_N Pr(E = night, N, L = light, D = AWM)}{\sum_{N,D} Pr(E = night, N, L = light, D)} \\
&= \frac{Pr(E = night) \sum_N Pr(N) Pr(L = light|E = night, N) Pr(D = AWM|N)}{Pr(E = night) \sum_N Pr(N) Pr(L = light|E = night, N) \sum_D Pr(D|N))} \\
&= \frac{\sum_N Pr(N) Pr(L = light|E = night, N) Pr(D = AWM|N)}{\sum_N Pr(N) Pr(L = light|E = night, N)} \\
&= \frac{\frac{1}{5} \cdot 1 \cdot \frac{1}{10} + \frac{1}{2} \cdot \frac{1}{10} \cdot \frac{1}{10} + \frac{3}{10} \cdot \frac{1}{5} \cdot \frac{9}{10}}{\frac{1}{5} \cdot 1 + \frac{1}{2} \cdot \frac{1}{10} + \frac{3}{10} \cdot \frac{1}{5}} \\
&= 0.2548
\end{aligned}$$

5.4 (3 pt)

Armin sees a blue airdrop. What is the probability that it is heavy? (You can write your answer in either decimal or fraction form.)

Solution:

$$\begin{aligned}
Pr(L = heavy|N = blue) &= \sum_E Pr(L = heavy, E|N = blue) \\
&= \sum_E Pr(E|N = blue) \cdot Pr(L = heavy|E, N = blue) \\
&= \sum_E Pr(E) \cdot Pr(L = heavy|E, N = blue) \\
&= \frac{1}{3} \cdot \frac{9}{10} + \frac{2}{3} \cdot 1 \\
&= 0.9667
\end{aligned}$$